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A PROMPT SEQUENTIAL METHOD FOR SUBSURFACE FLOW MODELING USING THE MODIFIED MULTI-SCALE FINITE VOLUME AND STREAMLINE METHODS

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Abstract. In this work, an innovative numerical algorithm accompanied by considerable accuracy is presented to reduce the computational cost of subsurface flow modeling. This method combines a modified multi-scale finite volume method (MMsFV) and streamline method based on the sequential approach. First, the modified multi-scale finite volume method, which includes a physical adaptation on the localization assumption, is employed to obtain a conservative velocity field with a similar cost to traditional upscaling methods. Then, the swift streamline method is utilized to solve the transport equation using the computed, conservative velocity field. The physical modification on the multi-scale framework imposes the nature of the actual flow moving from the multi-dimensional into 1-D local problems, which are constructed for calculating the boundary conditions in localization procedures. This physical modification is known as the modified variable boundary conditions (VBC) approach. The more accurate boundary conditions are generated for calculating basis and correction functions applied in multi-scale finite volume method. Here, the formulation and algorithm of the proposed and combined method, called the Modified Multi-scale Finite Volume Streamline (MMsFVSL) method, are presented for 2-D problems. Several test cases, including both incompressible single-phase and two-phase flow are investigated in which the obtained results show that the MMsFVSL method has a good accuracy with a high speed-up factor to reduce the total CPU time in the simulation process. Consequently, the MMsFVSL method offers a significantly efficient simulation algorithm capable of direct simulation for high resolution geological models.

Key words. Subsurface flow modeling, Multi-scale finite volume method, Streamline method, Modified localization assumption, Variable Boundary Conditions.

1. Introduction

Today's modern reservoir images and advanced geological models are used to describe reservoir rock properties. These models usually contain 10^6 to 10^9 grid blocks, and often cannot be used directly in a reservoir simulation because of restrictions of the required time and memory. In addition, small-scale heterogeneities can have a dominant effect on the behavior of fluid flow in porous media at much larger scales. Thus, accurate simulation of porous media flow relies on being able to use highly detailed descriptions of the permeability data. Many efficient solvers have been developed to reduce the computational cost, yet extensive studies are focused on how to generate new, rapid, and authentic methodologies. In the past decade the applicable series of these novel methodologies named by Multi-scale methods are developed that introduced an accurate and coarsened problem to simulate flow in porous media in a reasonable time. The multi-scale methods are implemented in two different finite element (see e.g. [3, 4, 5]) and finite volume (see e.g. [6, 7, 8, 9]) approaches. Multi-scale methods attempt to describe physical phenomena on coarse grids while accounting for the influence of fine-scale structures. Then, multi-scale methods, which are extensively used for reservoir simulation, use both coarse and fine grid information during subsurface flow modeling. The main

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idea of Multi-scale methods is that solving many local problems is more efficient than solving one large, global system [10]. In these methods, first the coarse-scale system is derived from fine scale information, then the unknown is calculated on a coarse grid with a cost comparable to traditional upscaling methods, finally the fine-scale unknown is approximated using the coarse-scale grids.

The big families of multi-scale methods are a locally conservative multi-scale formulation based on the finite volume approach (MsFV) that was developed by Jenny et al. [6] for incompressible flow. In this method, they used two sets of basis functions to calculate the coarse-scale transmissibilities and reconstruct the velocity field to achieve a conservative velocity field. Then, Jenny et al. [7, 8] developed an adaptive sequential (based on IMPES) and a fully implicit MsFV algorithm for multi-phase flow simulation, respectively. The MsFV method was also developed to simulate the compressible multi-phase flow by Lunati et al. in [9] which they introduced another set of correction functions to capture the fine-scale source terms and capillary pressure. The MsFV formulation was extended to simulate immiscible and compressible three phase flow in the presence of gravity and capillary forces by Lee et al. [11]. An iterative MsFV method in which the multi-scale solutions are converged to the corresponding fine-scale reference solutions by updating the local boundary conditions using global information was expressed by Hajibeygi et al. [10]. Hajibeygi and Jenny [12] also developed a MsFV method to solve the parabolic problems arising with the compressible multi-phase flow in porous media. Finally, Jenny and Lunati [13] used the MsFV method to incorporate complex wells.

Solving the pressure equation in a reservoir simulation requires a higher computer-processing time than the transport equation. Nevertheless, solving the transport equation using a rapid method can play a major role to reduce the total run time in reservoir flow modeling. In recent years, the streamline method has developed to solve the transport equation more efficiently for subsurface modeling than the conventional methods, such as finite differences or finite volumes [14, 15, 16, 17]. Specifically, the low memory requirement and CPU time in the streamline method is owing to this fact that the transport equation is decoupled into multiple 1-D transport equations along traced streamlines in terms of a new coordinate, so-called Time-of-Flight (ToF). Therefore, the strict stability constraint (CFL Condition) of the underlying fine grid to solve the transport equation will be removed [16]. Streamline simulators were developed for simplified physics in [14, 18], and the advanced version of a streamline simulator has been extended for more complex physics like compressible three-phase and compositional flow in [19]. In this paper, a novel numerical scheme is developed for both incompressible single and immiscible two-phase flow modeling. The main idea is to apply the modified multi-scale finite volume method to discretize the pressure equation, obtain high resolution conservative velocities, and then the rapid streamline method is used to discretize fluid transport. The proposed method was inspired by the concept used in the combination of the mixed Multi-scale Finite Element (MsFE) method with rapid methods such as streamlines and implicit upstream (including reordering cells) that were developed by Aarnes et al. [20, 21]. They utilized this combination to solve the two-phase flow in strongly heterogeneous 2-D and 3-D problems, and demonstrated that this combination is a robust and practicable substitute to traditional upscaling methods. Gautier et al. [22] almost used this idea by introducing a nested gridding method, and combining with the streamline method for simulation flow modeling on the fine-scale. Vegard et al. [23, 24] also used the computational efficiency of the