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THE STATISTICAL SECOND-ORDER TWO-SCALE METHOD FOR HEAT TRANSFER PERFORMANCES OF RANDOM POROUS MATERIALS WITH INTERIOR SURFACE RADIATION

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Abstract. In this paper, a statistical second-order two-scale (SSOTS) method is presented in a constructive way for predicting heat transfer performances of random porous materials with interior surface radiation. Firstly, the probability distribution model of porous materials with random distribution of a great number of cavities is described. Secondly, the SSOTS formulations for predicting effective heat conduction parameters and the temperature field are given. Then, a statistical prediction algorithm for maximum heat flux density is brought forward. Finally, some numerical results for porous materials with different random distribution models are calculated, and compared with the data by theoretical methods. The results demonstrate that the SSOTS method is valid to predict the heat transfer performances of random porous materials.

Key words. Statistical second-order two-scale method, Interior surface radiation, Random porous materials, Maximum heat flux density.

1. Introduction

Porous materials have been widely used in a variety of engineering and industrial products. Especially, with rapid development of space aircraft, people pay much more attention to porous materials. Inevitably, our attention is focused on the thermal properties of porous materials. So far, some methods to predict physical and mechanical properties of composite materials have been developed, such as the Maxwell-Eucken model [1], the Hashin-Shtrikman bounds [2], effective medium theory [3, 4], the self-consistent method [5] and so on. Although these methods effectively promoted the development of computational material science, the microstructure of real materials was greatly simplified to reduce the theoretical complexity. Furthermore, they are usually used to predict macroscopic heat conductivity parameters without considering the effect of radiation.

In fact, radiative heat transfer plays a significant rule in modern technology. Especially, it is typically the major mode of heat transfer in high-porosity insulations at high temperature environment. In recent years, some worthwhile contributions in predicting thermal radiation properties of periodical porous materials have been achieved. Liu et al.[6] predicted the effective macroscopic properties of heat conduction-radiation problem by homogenization methods. Bakhvalov [7] obtained the formal expansions for heat conduction problem with radiation boundary conditions. Later, Allaire et al.[8] dealt with a linear heat equation with non-linear boundary conditions by two-scale asymptotic expansions method. Meanwhile, Yang et al.[9] presented a second-order two-scale method to solve the heat transfer performances of periodic porous materials with interior surface radiation, and gave the error estimation for the original solution and the asymptotic solution. In theory, [10-13] proved the existence and uniqueness of the heat conduction equation with non-linear radiation boundary conditions.

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Nevertheless, the heat transfer problem of random porous materials with interior surface radiation is not considered so far. Actually, composite materials with random distribution have been widely used in engineering. Such as metal-matrix composites, foamed plastics and polymer blends. For the composites with random distributions, Cui et al.established a statistical second-order two-scale analysis method by introducing a random sample model to predict the physical and mechanical properties of the composite structure [14-17]. Meanwhile, for the physics field problems of composite materials with stationary random distribution, Jikov et al. [18] proved the existences of the homogenization coefficients and the homogenization solution.

However, the previous two-scale asymptotic expansion cannot be employed to the heat transfer problem with interior surface radiation. So, in this paper porous materials with random distribution will be investigated, and a new SSOTS method is developed by a constructive way to predict heat transfer properties, and calculate temperature and heat flux fields in meso-scopic level.

This paper is organized as follows. In the following section, the meso-scopic configurations for porous materials with random distribution are represented. Section 3 is devoted to the formulation of the SSTOS method and the algorithm procedure for the maximum heat flux density. In Section 4 the numerical results for the heat transfer performances of random porous materials are shown. Finally, the conclusions are given.

2. Representation of meso-scopic configurations of porous materials with random distribution

Suppose that the investigated porous materials are made from matrix and random cavities. Refer to Ref. [14, 20]. All the cavities are considered as ellipsoids or the polyhedrons inscribed inside the ellipsoids, which are randomly distributed in the matrix. In this paper all of the ellipsoid cavities are also considered as "same scale", which means all of their long axes satisfy $r_1 < a < r_2$ where r_1 and r_2 are given upper and lower bounds. Then the porous materials with random distribution can be represented as follows:

1) There exists a constant ε satisfying $0 < \varepsilon << L$, where L denotes the macro scale of the investigated structure Ω^ε. Thus, the structure can be regarded as a set of cells with the ε-size, as shown in Fig.1(a).

2) In each cell, the probability distribution of the cavities is identical. Then the investigated structure has periodically random distribution of cavities, and then can be represented by a probability distribution model of the cavities inside a typical cell.

3) Each ellipsoid can be defined by 9 random parameters, including the shape, size, orientation and spatial distribution of ellipsoid cavities: a_1, a_2, a_3, $\alpha_1, \alpha_2, \alpha_3$, x_{01}, x_{02}, x_{03}, where a_1, a_2 and a_3 denote length of three axes; three Euler angles $\alpha_1, \alpha_2, \alpha_3$ of the rotations; x_{01}, x_{02} and x_{03} the coordinates of the center. Let the random vector $\zeta = (a_1, a_2, a_3, \alpha_1, \alpha_2, \alpha_3, x_{01}, x_{02}, x_{03})$. Their probability density functions are denoted by $f_{a_1}(x), f_{a_2}(x), f_{a_3}(x)$,$f_{\alpha_1}(x), f_{\alpha_2}(x), f_{\alpha_3}(x)$,$f_{x_{01}}(x)$, $f_{x_{02}}(x)$, $f_{x_{03}}(x)$, respectively.

4) Suppose that there are K ellipsoids inside a cell εY^s, Y^s represents a normalized cell, then its random sample is defined as ω^s. s=$1, 2, 3...$ denotes the index of samples, then we can define a sample of ellipsoids distribution as follows

$$\omega^s = (\zeta_1^s, \zeta_2^s, \zeta_3^s \cdots, \zeta_{K-1}^s, \zeta_K^s)$$