INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, Series B
Volume 3, Number 1, Pages 1–19

SOME TOPICS FROM CONTINUUM MECHANICS RELATED TO BRAIN NEURO-MECHANICS

ALAN WINEMAN

Abstract. A number of topics from continuum mechanics are presented that are useful in developing mathematical models of brain neuromechanics. Part I reviews basic concepts used to describe deformation or distortion of brain tissue, strains, stresses, their connection and material stiffness. Part II presents concepts from viscoelasticity used to describe the time dependent response of brain tissue such as stress relaxation, creep, response to sinusoidal loading, energy dissipation, characteristic response times and their alteration due to non-mechanical influences. Part III describes recent modeling approaches that can be used to describe microstructural changes in materials due to large deformation, disease or other non-mechanical sources.

Key words. continuum mechanics, microstructural changes, large deformations.

1. Introduction

Continuum mechanics encompasses many topics that can contribute to the investigation of brain injuries resulting from concussions in sports and improvised explosive devices in military theaters, as well as brain diseases such as hydrocephalus. This article presents an overview of three such topics that should be useful in research in brain neuro-mechanics. The subject matter was selected based on two criteria. First, it was apparent from the current literature that research in brain neuro-mechanics could benefit from deeper insights into the implications of many fundamental concepts of continuum mechanics. The article aims to improve that insight. Second, there are a number of recent research directions within continuum mechanics that are potentially useful in the study of brain neuromechanics. The article provides an overview of two of these.

Section 2 contains a review of the notions of stress, strain and constitutive equations. It points out several issues that are generally not considered in the formulation of mechanical models, but could influence the mechanical response. Section 3 presents the essentials of viscoelasticity, namely stress relaxation, creep, linearity and the response to sinusoidal oscillations. Two important consequences of time dependent response are pointed out. Section 4 introduces a mechanical model that can account for chemical changes in materials such as brain tissue that are composed of networks of macromolecules. Among these may be chemical changes associated with disease, age or medication as well as large deformation due to swelling. Concluding comments are made in Section 5.

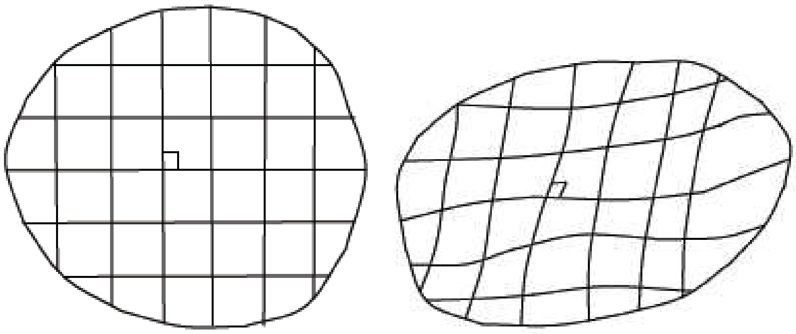
2. Basic Notions from Continuum Mechanics

Thorough presentations of the material of this section can be found in the classic reference by Fung [1] and the recent book by Cowin and Doty [2].

2.1. Reference Configuration, Material Elements, Strains. In developing a mathematical model for its mechanical response, the brain is idealized as a solid body composed of a continuous distribution of material particles. At a given instant,

Received by the editors October 3, 2011.

2000 *Mathematics Subject Classification.* 74A20, 74D10, 74L15.



(a) (b)

FIGURE 1. (a) Body, orthogonal grid and differential block in reference configuration; (b) deformed body, grid and differential block in a later configuration.

a material particle is associated with a spatial point. The distribution of material particles is visualized by the set of such points, i.e. the spatial configuration they occupy. The configurations of the brain can vary with time. For this reason, one is chosen as a reference and is used as a base state for defining material properties and determining changes.

Figure 1a shows a reference configuration with a grid of orthogonal straight lines. The points of the grid coincide with material particles. As time varies, the material particles can occupy different spatial points and the grid undergoes a distortion. Straight lines change length and become curved and the right angles increase or decrease. A deformed grid in shown in Figure 1b. In continuum mechanics, the material particles are regarded as differential cubes whose edges are line segments of the grid. As a result of the local distortion of the grid, each differential cube becomes a differential parallelepiped whose edges have new lengths and whose surfaces are no longer orthogonal. This introduces the two basic modes of deformation:

Normal Strains due to Changes in Length

Figure 2a shows a differential cube of material in the reference configuration whose three edges have the same length $dA_x = dA_y = dA_z = dA$. Figure 2b shows the differential cube in a later configuration when its edges have changed length and it has become a differential rectangular parallelepiped with edges da_x, da_y and da_z. The normal strain is defined as the change in length per unit length and is usually denoted by the symbol ϵ. The block has three normal strains, one associated with each edge,

$$\epsilon_x = \frac{da_x - dA_x}{dA_x},\ \epsilon_y = \frac{da_y - dA_y}{dA_y},\ \epsilon_z = \frac{da_z - dA_z}{dA_z} \tag{1}$$

Shear Strains due to Changes in Angle

Figure 2c shows the differential cube in a later configuration when the top surface has displaced distance da with respect to the bottom surface thereby