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MULTILEVEL NON-CONFORMING FINITE ELEMENT METHODS FOR COUPLED FLUID-STRUCTURE INTERACTIONS

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Abstract. Computational mathematics is constantly evolving to develop novel techniques for solving coupled processes that arise in multi-disciplinary applications. Often such analysis may be accomplished by efficient techniques which involve partitioning the global domain (on which the coupled process evolves) into several sub-domains on each of which local problems are solved. The solution to the global problem is then constructed by suitably piecing together solutions obtained locally from independently modeled sub-domains. In this paper we develop a multilevel computational approach for coupled fluid-structure interaction problems. The method relies on computing coupled solutions over different sub-domains with different multigrid levels. Numerical results for the reliability of the schemes introduced are also presented.

Key words. finite element methods, fluid-structure interaction, Arbitrary Lagrangian-Eulerian formulation, non-conforming, multilevel.

1. Introduction

The past few decades have seen significant advances in the development of computational methods to obtain efficient solutions to complex coupled systems that consist of interactions between functionally distinct components. Coupled with advances in finite element methods, these methods have provided new algorithms for large scale simulations [13, 14]. Often in such methods, the interface continuity between solutions in independently modeled sub-domains is enforced weakly via Lagrange multipliers that are defined on the interface. The *mortar finite element method* is one example of such a technique (see e.g. [8, 4, 5, 6, 13, 15, 16, 7, 17] and references therein) where precise choices are described for the two fields (the interior solution variable and the interface Lagrange multiplier) to ensure *stability.* One can also employ more general three-field methods, where one field represents the solution variable on the interface and is modeled independently from the interior solution variables on either side of the interface. Here, two Lagrange multipliers will be required in order to enforce continuity between each interior variable and the interface variable. In either case, Lagrange multiplier methods allow for optimal rates of convergence along the interface between distinct components of a coupled system.

In recent years flexible multilevel multigrid methods have been introduced [20, 19, 12, 1, 2, 3], whose solvers are based on the iterative solution of several problems over smaller domains. These techniques allow solutions to be computed at the element level and also help us to achieve proper accuracy, load balancing and computational efficiency. Such novel techniques provide motivation for us to develop fast and efficient algorithms to solve complex fluid-structure interaction (FSI) problems [18, 3].

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It is well known that the one of the most difficult parts of numerically approximating the fluid-structure coupling arises from the fact that the structural equations are usually formulated with material (Lagrangian) coordinates, while the fluid equations are typically written using spatial (Eulerian) coordinates. This is important since the nodes on the fluid mesh are attached to the surface of the structure and hence should move with the displacement of the structure. Therefore, a straightforward approach to the solution of the coupled fluid-structure dynamic equations requires moving at each time step at least the portions of the fluid grid that are close to the moving structure. This can be appropriate for small displacements of the structure but may lead to severe grid distortions when the structure undergoes large motion. Several different approaches have emerged as an alternative to partial re-gridding in transient computations, one of which is the Arbitrary Lagrangian Eulerian (ALE) formulation [9, 11, 18].

When constructing a numerical method for time-dependent coupled systems that involve a moving boundary, differences in scale between the solutions in each sub-domain should be incorporated into the approximation. For example, in a fluid-structure interaction where the geometry of the problem evolves due to the deformation of an elastic structure, the magnitude of the strain rate of the solid body may be much smaller than the velocity of molecules in the fluid region. Non-conforming finite element methods offer a promising framework for this situation since the scale of the computational grid and degree of polynomial approximation can be refined in each sub-domain independently. In this setting, each sub-domain is independently partitioned by regular families of meshes, where the intersection of any two distinct elements is either a vertex, an edge, or an empty set, and a restriction on the ratio between edges and diameters of the elements prevents them from becoming arbitrarily thin. This approach will avoid the necessity of creating transition elements between the sub-domains, which often lead to solution inaccuracy due to severe distortions, especially in cases where an initial numerical grid is allowed to move in response to deformation of the original domain. Our objective will be to develop a non-conforming finite element methodology to couple a Lagrangian model describing a structure interacting with a fluid that is described by the ALE strategy in order to simulate a full unsteady physical phenomenon.

The outline of the paper is as follows. Section 2 introduces a model fluid-structure interaction problem and presents a brief background on the methods that are employed to accomplish coupling. The mathematical formulation of the non-conforming technique is illustrated on a one-dimensional problem for simplicity. Numerical experiments for the one-dimensional model problem are presented that indicate the robustness of the method introduced. Section 3 presents the extension of the problem to higher dimensions and presents the solution methodology as well as a numerical validation through a model problem involving a beam and fluid interaction. Finally, section 4 presents discussions and future research.

2. A One-Dimensional Model Problem and Governing Equations

For simplicity, let us now describe the mathematical formulation and solution methodology for a fully coupled system of equations governing the interaction between a fluid and a structure in a one-dimensional setting. We will present both the continuous problem and a discrete approximation of the model that incorporates an ALE formulation, allowing the numerical grid in the fluid region to move along with the interface between the two sub-domains. Such models can help to provide insight into fluid-structure interaction effects for a totally or partially submerged