INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING, *SERIES B*
Volume 3, Number 4, Pages 460–485

AN H^1-GALERKIN MIXED FINITE ELEMENT METHOD FOR THE EXTENDED FISHER-KOLMOGOROV EQUATION

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Abstract. An H^1-Galerkin mixed finite element method is applied to the extended Fisher-Kolmogorov equation by employing a splitting technique. The method described in this paper may also be considered as a Petrov-Galerkin method with cubic spline space as trial space and piecewise linear space as test space, since second derivative of a cubic spline is a linear spline. Optimal order error estimates are obtained without any restriction on the mesh. Fully discrete scheme is also discussed and optimal order estimates are obtained. The results are validated with numerical examples.

Key words. Extended FisherKolmogorov (EFK) equation, Second-order splitting, H^1 - Galerkin method, Auxiliary projection, Semi and Fully discrete schemes, *A priori* bounds, Optimal order error estimates, Cubic B-splines.

1. Introduction.

In this paper, we discuss an H^1-Galerkin mixed finite element cubic spline approximation method for the following extended Fisher-Kolmogorov equation :

$$u_t + \gamma u_{xxxx} - u_{xx} + f(u) = 0, \quad 0 < t < T,\ \gamma > 0,\ x \in I = (0,1); \tag{1.1}$$

subject to the initial and boundary conditions

$$u(0,t) = 0,\ u(1,t) = 0,\ u_{xx}(0,t) = 0,\ u_{xx}(1,t) = 0; \tag{1.2}$$

$$u(x,0) = g(x); \tag{1.3}$$

where $f(u) = u^3 - u$. When $\gamma = 0$ in (1.1), it becomes the standard Fisher-Kolmogorov equation.

The above extended Fisher-Kolmogorov equation has got significance, as many problems in physics related to phase transition and other bistable phenomena are mathematically modelled as equation (1.1). For the case $\gamma > 0$, it was first proposed by Dee and Van Saarloos [17] as a higher order model equation for physical systems that are bistable. Further, the extended Fisher-Kolmogorov equation (1.1) has a lot of applications in the theory of instability in nematic liquid crystals and traveling waves in reaction-diffusion systems as discussed in W. Zimmerman [39] and D.G. Aronson [9] respectively. Marginal stability for (1.1) is discussed in Van Saarloos [35, 36]. Chaotic characteristics are studied for (1.1) in P. Coullet *et al.* [11]. Steady state equation corresponding to (1.1) is analyzed by shooting methods in L. A. Peletier *et al.* [28, 29]. Periodic solutions of the EFK equation are discussed in L. A. Peletier *et al.* [30] and Stepan Tersian *et al.* [33]. Existence of a global attractor for (1.1) is proved in H^k spaces for all $k > 0$ in Hong Luo [19].

As far as computational studies are concerned, there are some results in the literature related to the numerical approximations to (1.1) - (1.3). In Danumjaya *et al.* [14], a second order splitting combined with orthogonal spline collocation

Received by the editors November 2, 2012.
2000 *Mathematics Subject Classification.* 65N30, 65N15, 65M60, 65M15, 35G20.

method for (1.1) is formulated, analyzed and the error bounds are obtained for semi discrete scheme. Further, using C^1-conforming finite element method, optimal error estimates are established in Danumjaya *et al.* [15] for both the semi-discrete and fully-discrete cases for the extended Fisher-Kolmogorov equation in two space dimension. A Crank-Nicolson type finite difference scheme to approximate the extended Fisher-Kolmogorov equation is presented and the existence and uniqueness of the solution are discussed in Tlili *et al.* [34]. Further, existence, uniqueness and convergence of CrankNicolson type finite difference solutions are discussed for the extended FisherKolmogorov (EFK) equation in two space dimension in Noomen Khiari *et al.* [25]. In this article, an attempt has been made to discuss H^1-Galerkin mixed finite element method with cubic B-splines for Extended Fisher-Kolmogorov equation (1.1) - (1.3).

Related to fourth order evolution equations, in Pani and Chung [6], a C^1-conforming finite element method is analyzed for the Rosenau equation. Numerical studies of one dimensional and multidimensional Cahn-Hilliard equation are discussed by Elliott *et al.* [12, 13], Danumjaya *et al.* [16] and Qiang and Nicolaides [31]. High-order finite element methods for the Kuramoto-Sivashinsky equation are presented in Akrivis [3]. Recently in Akrivis *et al.* [4], fully discrete schemes for a general class of dispersively modified KuramotoSivashinsky equations are presented in which linearly implicit schemes and spectral methods are used for the temporal and spatial discretizations respectively. Existence and numerical approximations of periodic solutions of semilinear fourth-order differential equations related to either extended Fisher-Kolmogorov equation or Swift-Hohenberg equation are discussed in Julia Chaparova [21]. For biharmonic problem, mixed methods are discussed in A. Quarteroni [32]. Further, a quadrature Galerkin method is analysed for biharmonic problem in R. Aitbayev [2].

In general, the LBB (*Ladyženskaja-Babuška-Brezzi*) stability condition is required for the mixed finite element method, which restricts the choice of finite element spaces. In Pani, A. K., [5], an H^1-Galerkin mixed finite element method is applied and error estimates are obtained for a parabolic partial differential equation. Further, the parabolic equation is split into two partial differential equations leading to a first order system after introducing intermediate function where C^0 elements are used relaxing C^1 smoothness without requiring LBB condition on the finite element spaces. In recent years, substantial progress has been made in H^1-Galerkin mixed finite element method. In Amiya K. Pani and Graeme Fairweather [7, 8], H^1-Galerkin mixed finite element method is employed for evloution equation and also for parabolic partial integro-differential equations seperately. Subsequntly, this method is applied to reaction-diffusion equations Arul Veda Manickam *et al.* [10], hyperbolic equations Pani, A.K., *et al.* [27], nonlinear parabolic problems Neela Nataraj *et al.* [24] and regularized long wave equation Guo *et al.* [18]. For more applications of this mixed formulation for pseudo-hyperbolic and for a class of heat transport equations, see Yang Liu *et al.* [37] and Zhaojie Zhou [38]. Most recently, there are superconvergence results on H^1-Galerkin mixed finite element method for parabolic problems Madhusmita *et al.* [22] and for second-order elliptic equations Madhusmita *et al.* [23].

The aim of this paper is to analyze the H^1-Galerkin mixed finite element method for the above extended Fisher-Kolmogorov equation after splitting (1.1) with an