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FINITE ELEMENT ANALYSIS OF SEMICONDUCTOR DEVICE EQUATIONS WITH HEAT EFFECT

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Abstract. In this paper, the system of the semiconductor device equations with heat effect is considered. An approximation to the system that makes use of a mixed finite element method for the electrostatic potential equation combined with a finite element method for the densities equations and the temperature equation is proposed. Existence and uniqueness of the approximate solution are proved. A convergence analysis is also given.

Key words. Semiconductor device with heat effect, finite element scheme, existence and uniqueness, convergence analysis.

1. Introduction

We consider in this paper the following drift-diffusion model with heat effect arising in semiconductor physics [1, 2, 3, 4, 5]:

$$(1)\quad \begin{aligned} &\text{(a)}\ \ n_t - \nabla\cdot J_n = R + g && \text{in } Q_T\\ &\text{(b)}\ \ p_t - \nabla\cdot J_p = R + g && \text{in } Q_T\\ &\text{(c)}\ \ -\nabla\cdot(\sigma(\theta)\nabla\psi) = p - n + f && \text{in } Q_T\\ &\text{(d)}\ \ \theta_t - \nabla\cdot(k(\theta)\nabla\theta) = H && \text{in } Q_T \end{aligned}$$

where n and p are the densities of electrons and holes, respectively; ψ is the electrical potential; θ is the temperature; $n_t = \partial n/\partial t$, $p_t = \partial p/\partial t$, $\theta_t = \partial\theta/\partial t$; $J_n = D_1\nabla n - n\mu_1\sigma(\theta)\nabla\psi$, $J_p = D_2\nabla p + p\mu_2\sigma(\theta)\nabla\psi$, $D_i = D_i(x,\psi,T(\theta))\,(i = 1,2)$; μ_1 and μ_2 are the nonnegative real numbers; k is the thermal conductivity of the semiconductor device; $R = R(x,n,p,\nabla\psi,T(\theta))$; $g = g(x,n,p,\nabla\psi,T(\theta))$; $H = H(x,n,p,\nabla\psi,T(\theta)) = -\nabla\psi\cdot(J_n - J_p)$; Ω is a bounded open subset of $\mathbb{R}^2$; $Q_T = \Omega\times J$ and $J = [0,T]$.

The homogeneous (for simplicity) mixed boundary conditions are supplemented to system (1):

$$(2)\quad \begin{cases} n = 0,\ p = 0,\ \psi = 0,\ \theta = 0 & \text{on } \Gamma_D\times J\\ J_n\cdot\nu = J_p\cdot\nu = \dfrac{\partial\psi}{\partial\nu} = \dfrac{\partial\theta}{\partial\nu} = 0 & \text{on } \Gamma_N\times J \end{cases}$$

where Γ the boundary of Ω is split into $\Gamma_D\cup\Gamma_N$, $\Gamma_D\cap\Gamma_N = \varnothing$ and $|\Gamma_D| > 0$; ν is the unit outer normal vector to Γ.

The following initial conditions are applied:

$$(3)\quad n(x,0) = n_0(x),\ p(x,0) = p_0(x),\ \theta(x,0) = \theta_0(x) \quad \text{in } \Omega.$$

There are a lot of works on mathematical analysis of the basic equations of semiconductor devices with heat effect, cf. [1, 2, 4, 5, 6, 7, 8, 9, 10]. However, to our knowledge, it seems that there is no publication on numerical analysis of

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the system, which is the goal of this paper. The approximate procedure of the system by a Galerkin method that makes use of a mixed finite element method for the electrostatic potential equation combined with a finite element method for the densities equations and the temperature equation is proposed. We then analyze the existence, uniqueness and convergence of the approximate solution.

2. Preliminaries and description of approximations

Throughout this paper, usual definitions, notations, and norms of Sobolev spaces as in [11] are used. Let $(\cdot,\cdot)$ be the inner products in $L^2(\Omega)$ or $[L^2(\Omega)]^2$. denote by $H^{1+\alpha}(\Omega)$ (α is a real number with $0<\alpha<1$) the non-integral Sobolev space on Ω with norm

$$\|\varphi\|_{H^{1+\alpha}(\Omega)} = \big\{\|\varphi\|_{H^1}^2 + \sum_{|s|=1}\int_\Omega\int_\Omega \frac{|\partial^s\varphi(x)-\partial^s\varphi(y)|^2}{|x-y|^{2(1+\alpha)}}dxdy\big\}^{1/2},$$

where $|x|$ denotes the Euclidean norm of $I\!R^2$. Introduce the spaces as follows: $W = L^2(\Omega)$, $S=\{z\in H^1(\Omega),\ z\,|_{\Gamma_D}=0\}$, $H(\text{div};\Omega)=\{\mathbf{v}\in[L^2(\Omega)]^2,\ \nabla\cdot\mathbf{v}\in L^2(\Omega)\}$ and $\mathbf{V}=\{\mathbf{v}\in H(\text{div};\Omega);\ \mathbf{v}\cdot\nu\,|_{\Gamma_N}=0\}$ with norm

$$\|\mathbf{v}\|_{\mathbf{V}} = (\|\mathbf{v}\|_{L^2(\Omega)}^2 + \|\nabla\cdot\mathbf{v}\|_{L^2(\Omega)}^2)^{1/2}.$$

We denote also by $\|\cdot\|_{L^q(X)}$ the norm of the space $L^q(J;X)$, where $q=2,\infty$ and X is a Sobolev space on Ω.

For convenience, we make the following assumptions on the data: There exists a uniform constant L such that, for $x\in\Omega,\ t\in(0,T]$,

$$(4)\quad \begin{aligned} &\text{(a)}\quad |D_i(\psi,T(\theta))-D_i(\psi_1,T(\theta_1))|\le L\{|\psi-\psi_1|+|\theta-\theta_1|\},\\ &\qquad\qquad 0<D_*\le D_i\le D^*<+\infty,\ (i=1,2),\\ &\text{(b)}\quad |R(n,p,\nabla\psi,T(\theta))-R(n_1,p_1,\nabla\psi_1,T(\theta_1))|\\ &\qquad\qquad \le L\{|n-n_1|+|p-p_1|+|\nabla\psi-\nabla\psi_1|+|\theta-\theta_1|\},\\ &\text{(c)}\quad |g(n,p,\nabla\psi,T(\theta))-g(n_1,p_1,\nabla\psi_1,T(\theta_1))|\\ &\qquad\qquad \le L\{|n-n_1|+|p-p_1|+|\nabla\psi-\nabla\psi_1|+|\theta-\theta_1|\},\\ &\text{(d)}\quad 0<\sigma_*\le\sigma\le\sigma^*<+\infty,\quad 0<k_*\le k\le k^*<+\infty. \end{aligned}$$

From [4, 5], we know that

Lemma 1. *The solution (n,p,ψ,θ) of problem (1)-(2)-(3) satisfies the estimate*

$$(5)\quad \|n\|_{L^\infty(L^\infty)}+\|p\|_{L^\infty(L^\infty)}+\|\psi\|_{L^\infty(L^\infty)}\le C<+\infty.$$

Furthermore, we need the following regularity assumptions:

$$(6)\quad \begin{aligned} &\text{(a)}\quad \|n\|_{L^\infty(H^{1+\alpha})}+\|p\|_{L^\infty(H^{1+\alpha})}+\|\theta\|_{L^\infty(H^{1+\alpha})}\le C_0,\\ &\text{(b)}\quad \|n_t\|_{L^2(L^2)}+\|p_t\|_{L^2(L^2)}+\|\theta_t\|_{L^2(L^2)}\le C_0. \end{aligned}$$

where $0<\alpha<1$ and C_0 are some fixed constants.

Set $\mathbf{u}=-\sigma(\theta)\nabla\psi$ or $\nabla\psi=-a(\theta)\mathbf{u}$ where $a(\theta)=1/\sigma(\theta)$. Then, (1) can be read:

$$(7)\quad \begin{aligned} &\text{(a)}\quad n_t-\nabla\cdot(D_1\nabla n+n\mu_1\mathbf{u})=R+g, && \text{in } Q_T\\ &\text{(b)}\quad p_t-\nabla\cdot(D_2\nabla p-p\mu_2\mathbf{u})=R+g, && \text{in } Q_T\\ &\text{(c)}\quad a(\theta)\mathbf{u}+\nabla\psi=0, && \text{in } Q_T\\ &\text{(d)}\quad \nabla\cdot\mathbf{u}=p-n+f, && \text{in } Q_T\\ &\text{(e)}\quad \theta_t-\nabla\cdot(k(\theta)\nabla\theta)\\ &\qquad\qquad =a(\theta)\mathbf{u}[D_1\nabla n-D_2\nabla p]+a(\theta)[n\mu_1+p\mu_2]|\mathbf{u}|^2, && \text{in } Q_T. \end{aligned}$$