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New Exact Structures for the Nonlinear Lattice Equation by the Auxiliary Fractional Shape

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Abstract. In this paper, the auxiliary rational shape is successfully applied for obtaining new exact travelling solutions of the nonlinear lattice equation. These solutions include rational solutions, with periodic and doubly periodic wave profiles. Many new exact traveling wave solutions are successfully obtained.

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1 Introduction

In recent decades, much progress has been made to study the systems of nonlinear partial differential equations. Because of its importance to the field of engineering and many physical systems, a great deal of methods and numerical calculations were appeared [1–6] to give different structures to the solutions. Besides traditional methods such as auto-Backlund transformation, Lie Groups, inverse scattering transformation and Miura's transformation, a vast variety of the direct methods for obtaining explicit travelling solitary wave solutions have been found [7–14].

The availability of symbolic computation packages can be facilitate many direct approaches to establish solutions to non-linear wave equations [15–23]. Various extension forms of the sine-cosine and tanh methods proposed by Malfliet and Wazwaz have been applied to solve a large class of nonlinear equations [24–27]. More importantly, another mathematical treatment is established and used in the analysis of these nonlinear problems, such as Jacobian elliptic function expansion method, the variational iteration method, pseudo spectral method, and many others powerful methods [28–39].

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The governing equation that studies the propagation of solitons is formulated through an nonlinear lattice. We expect that the presented method could lead to construct successfully many other solutions for a large variety of other nonlinear evolution equations. The prime motive of this work is to derive an effective auxiliary technique for a class of nonlinear model equations.

2 Mathematical formulation of the problem

The governing equation written in several independent variables is presented as

$$R(u,u_t,u_x,u_y,u_z,u_{xy},u_{yz},u_{xz},u_{xx},...)=0, \tag{2.1}$$

where the subscripts denote differentiation, while $u(t,x,y,z,...)$ is an unknown function to be determined. Equation (2.1) is a nonlinear partial differential equation that is not integrable in general. Sometime it is difficult to find a complete set of solutions. If the solutions exist, there are many methods which can be used to handled these nonlinear equations.

The following transformation is used for the new wave variable as

$$\xi=\sum_{i=0}^{p}\alpha_i\chi_i+\delta, \tag{2.2}$$

χ_i are distinct variables, and when $p=1$, $\xi=\alpha_0\chi_0+\alpha_1\chi_1+\delta$, the quantities α_0,α_1 are called the wave pulsation ω and the wave number k respectively if χ_0,χ_1 are the variables t and x respectively. In the discrete case for the position x and with continuous variable for the time t, ξ becomes with some modifications $\xi_n=nd+ct+\delta$ and n is the discrete variable. d and δ are arbitrary constants and c is the velocity.

We give the traveling wave reduction transformation for Eq. (2.1) as

$$u(\chi_0,\chi_1,...)=U(\xi), \tag{2.3}$$

and the chain rule

$$\frac{\partial}{\partial\chi_i}(\cdot)=\alpha_i\frac{\mathrm{d}}{\mathrm{d}\xi}(\cdot),\qquad \frac{\partial^2}{\partial\chi_i\partial\chi_j}(\cdot)=\alpha_i\alpha_j\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}(\cdot),\cdots. \tag{2.4}$$

Upon using (2.3) and (2.4), the nonlinear problem (2.1) becomes an ordinary differential equation (ODE) like

$$Q(U,U_\xi,U_{\xi\xi},U_{\xi\xi\xi},U_{\xi\xi\xi\xi},...)=0. \tag{2.5}$$