Numer. Math. Theor. Meth. Appl. Vol. **6**, No. 1, pp. 21-46
doi: 10.4208/nmtma.2013.mssvm02 February 2013

Fluid Flow Estimation with Multiscale Ensemble Filters Based on Motion Measurements Under Location Uncertainty

Sébastien Beyou[1], Thomas Corpetti[2], Sai Gorthi[3] and Etienne Mémin[1,*]

[1] *INRIA/FLUMINANCE, 35042 Rennes Cedex, France.*
[2] *CNRS/LETG-Rennes-Costel, Campus Villejean Place du recteur Henri Le Moal CS 24307 35043 Rennes cedex, France.*
[3] *Indian Institute of Space Science and Technology, Valiamala P.O., Thiruvananthapuram – 695 547, Kerala India.*

Received 6 December 2011; Accepted (in revised version) 11 September 2012

Available online 11 January 2013

Abstract. This paper proposes a novel multi-scale fluid flow data assimilation approach, which integrates and complements the advantages of a Bayesian sequential assimilation technique, the Weighted Ensemble Kalman filter (WEnKF) [27]. The data assimilation proposed in this work incorporates measurement brought by an efficient multiscale stochastic formulation of the well-known Lucas-Kanade (LK) estimator. This estimator has the great advantage to provide uncertainties associated to the motion measurements at different scales. The proposed assimilation scheme benefits from this multi-scale uncertainty information and enables to enforce a physically plausible dynamical consistency of the estimated motion fields along the image sequence. Experimental evaluations are presented on synthetic and real fluid flow sequences.

AMS subject classifications: 65M10, 78A48

Key words: Data assimilation, stochastic filter, particle filters, fluid motion estimation.

1. Introduction

The analysis of geophysical fluid flows is of the utmost importance in domains such as oceanography, hydrology or meteorology for applications of forecasting, studies on climate changes or for monitoring hazards or events. The forecasting of such flows requires the precise knowledge of an initial condition which may be only accessible through the measurements of the system's state variables such as pressure, temperature, or fluid flow velocity. These data may be provided through dedicated probes or Lagrangian drifters

*Corresponding author. *Email addresses:* `Sebastien.Beyou@inria.fr` (S. Beyou), `thomas.corpetti@univ-rennes2.fr` (T. Corpetti), `gorthisubrahmanyam@iist.ac.in` (S. Gorthi), `etienne.memin@inria.fr` (E. Mémin)

launch in the ocean or in the atmosphere. However, the coverage of such measurements is usually irregular and sometimes very sparse in underdeveloped regions or across oceans. At the opposite orbital or geostationary satellites provide a huge amount of image data, with a still increasing spatial and temporal resolution. Compared to *in situ* measurements (i.e., measure with local probes located inside the flow), satellite images provide a much more denser observation field. However they unfortunately offer only an indirect access to the physical quantities of interest and give rise consequently to difficult inverse problems to estimate characteristic features of the flow such as velocity fields or vorticity maps.

These kinematical information can be estimated from image sequences through motion estimation techniques. Motion estimation is an old problem in computer vision and a huge number of techniques has been proposed to estimate the motion on the image plane of a 3D scene. The best state-of-the-art approaches perform efficiently for the recovery of rigid scene motions [28, 31]. They are generally built on strong photometric and motion hypothesis which prevent them to be sufficiently accurate for deformation metrology. Several motion estimators dedicated to the measurement of specific phenomenon such as fluid flows have been proposed in the literature (see [16] for a detailed overview). These estimators differ mainly on the smoothness function they are handling: first order penalization [29], second order div-curl regularization [6, 33], data-dependent [3, 7, 24] or power law auto-similarity principles [14, 15]. These methods provide accurate instantaneous displacements as they generally implement additional constraints imposed by the physics, as in [6, 11] and most of them are embedded into a multiscale formalism that enables capturing efficiently the large scales deformations [16]. However, those techniques generally still exhibit difficulties for mid to small scales measurements. Such artifacts may reveal particularly pronounced in regions with poor photometric contrasts where the smoothing prior takes the lead in the solution elaboration. In those regions the data come into play only at the boundary. Bad estimations or instabilities on the boundary vicinity are immediately echoed inside such regions. For large regions this may reveal problematic and constitutes a potential factor of instabilities along time of the estimates.

Dynamical consistency of the velocity measurements can be enforced by embedding the estimation problem within an image based assimilation process. Variational assimilations of image information have been recently considered for the estimation of fluid motion fields [5, 26]. Those optimal control methods, though efficient, constitute batch methods, which requires forward and backward integrations of the dynamical system and the adjoint of the tangent linear dynamics respectively. The latter relies implicitly on a linearization of the dynamics and is adapted in practice for short time horizon. The constitution of this adjoint dynamics may turn out quite tedious in practice for complex dynamical models.

Stochastic filters are also well known alternative techniques for data assimilation. Opposite to the variational data assimilation framework, stochastic filtering has the great advantage to couple noisy data and a stochastic dynamics incorporating the unavoidable uncertainties we have on the system evolution. Such filters are also generally set up in a recursive way and through a Markovian property of the dynamics, they are at least theoretically less dependent on the initial condition. This relative independency with respect to the initial condition and their recursive structure along time are two important advan-