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Partial Shape Matching Without Point-Wise Correspondence

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Abstract. Partial similarity of shapes is a challenging problem arising in many important applications in computer vision, shape analysis, and graphics, e.g. when one has to deal with partial information and acquisition artifacts. The problem is especially hard when the underlying shapes are non-rigid and are given up to a deformation. Partial matching is usually approached by computing local descriptors on a pair of shapes and then establishing a point-wise non-bijective correspondence between the two, taking into account possibly different parts. In this paper, we introduce an alternative correspondence-less approach to matching fragments to an entire shape undergoing a non-rigid deformation. We use region-wise local descriptors and optimize over the integration domains on which the integral descriptors of the two parts match. The problem is regularized using the Mumford-Shah functional. We show an efficient discretization based on the Ambrosio-Tortorelli approximation generalized to triangular point clouds and meshes, and present experiments demonstrating the success of the proposed method.

AMS subject classifications: 65D18, 68U05

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1. Introduction

Many shape analysis applications arising in computer and vision and graphics require matching of partially similar shapes [33]. Such problems typically arise in two scenarios. On the one hand, partial similarity may be the right description of the similarity relationship between two shapes (for example, consider a centaur shape: the centaur is partially

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similar to a human because they share the human-like upper body, and at the same time, partially similar to a horse because the share the horse-like lower body [29]). On the other hand, many real-world data are degraded by acquisition imperfections and noise (missing views, holes, etc) that are especially acute when acquiring 3D scenes using a range sensing device such as Microsoft Kinect, resulting in the need to work with partially given objects. Such cases are common, for example, in face recognition, where the facial surface may be partially occluded by hair [14], or in computational archeology where one often has to deal with missing pieces [28].

In rigid shape analysis, introducing weights that reject corresponding points that are too distant or whose normals are misaligned into the popular iterative closest point (ICP) algorithm [7, 20] are able to deal with partial shape alignment. However, it is impossible to guarantee how large and regular the resulting corresponding parts will be. In order to cope with this problem, a Mumford-Shah [19, 39]-like regularization allowing to explicitly control the size of the rejected part and its regularity was used in [10], resulting in a robust ICP algorithm allowing to match rigid shapes with significant dissimilar parts.

This approach is, in fact, a particular setting of the framework introduced by Bronstein *et al.* [11], in which non-rigid partial similarity is formulated as a multi-criterion optimization problem, wherein one tries to find the corresponding parts in two shapes by simultaneously maximizing significance and similarity criteria. The framework allows plugging in different similarity (e.g., some intrinsic metric distortion [11, 13, 26, 37] for non-rigid shapes, or Hausdorff-like distance [10] for rigid shapes) and significance (e.g. part area [10, 11] or statistical occurrence of local shape descriptors [16]) criteria to address different settings of the problem. The solution proposed in the methods above requires the knowledge of correspondence between the shapes, and in the absence of a given correspondence, can be solved by alternating between weighted correspondence finding and maximization of part area. Such an alternating optimization scheme is computationally very expensive.

A different class of methods not relying on correspondence are bags of features [46] approaches popular in image analysis recently adopted in 3D shape analysis [12, 38, 40, 49]. The main idea is to represent the shape as a collection of some local feature descriptors [17, 21, 25, 30, 32, 36, 38, 41, 45, 48, 51, 52] and quantize them in some vocabulary of "geometric words" in order to compute a histogram representing the occurrence of different geometric words in the shape (this method follows the "bag of words" approach in text retrieval, hence the name). If the geometric vocabulary is large enough and the shapes have significant common parts, it is possible to compare partially similar shapes. However, if the similar parts are small, this method usually does not work. Furthermore, since the bag of features representation looses the spatial information, it does not allow to identify the parts that are similar in two shapes.

In this paper, we present an approach for correspondence-less partial matching of non-rigid 3D point clouds and shapes that is, in a sense, a combination of the two aforementioned methods but because our method doesn't require to calculate point-wise correspondence it is less computationally expensive. Our work is inspired by the recent work on partial matching of images [22]. The main idea is to find similar parts by comparing part-