INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 8, Number 1, Pages 156–173

INEXACT SOLVERS FOR SADDLE-POINT SYSTEM ARISING FROM DOMAIN DECOMPOSITION OF LINEAR ELASTICITY PROBLEMS IN THREE DIMENSIONS

XINGDING CHEN AND QIYA HU

(Communicated by Jun Zou)

Abstract. In this paper, we propose a domain decomposition method with Lagrange multipliers for three-dimensional linear elasticity, based on geometrically non-conforming subdomain partitions. Some appropriate multiplier spaces are presented to deal with the geometrically non-conforming partitions, resulting in a discrete saddle-point system. An augmented technique is introduced, such that the resulting new saddle-point system can be solved by the existing iterative methods. Two simple inexact preconditioners are constructed for the saddle-point system, one for the displacement variable, and the other for the Schur complement associated with the multiplier variable. It is shown that the global preconditioned system has a nearly optimal condition number, which is independent of the large variations of the material parameters across the local interfaces.

Key Words. Domain decomposition, geometrically non-conforming, Lagrange multiplier, saddle-point system, preconditioners, condition number.

1. Introduction

In recent years, there has been a fast growing interest in the domain decomposition methods (DDMs) with Lagrange multipliers, which were studied early in [6], [7], and [22]. Such DDMs have many advantages over the traditional DDMs in applications (cf. [1], [5], [21]). In this paper, we will develop a domain decomposition method with Lagrange multipliers to solve compressible elasticity problems in three dimensions. We consider certain geometrically non-conforming subdomain partitions with meshes that are nonmatching across the subdomain interfaces.

The Lagrange multiplier DDM has been developed as a non-conforming discretization method, such that the resulting approximation possesses the optimal accuracy, see [4], [20], [26]. For this purpose, the jumps of the solutions across the subdomain interfaces would be orthogonal to a certain Lagrange multiplier space, which should be appropriately chosen. This weak continuity condition leads to a saddle-point system for the displacement variable and the multiplier variable. It is known that the displacement variable corresponds to a singular problem on each floating subdomain. There exist many techniques to deal with such singularity, for example, the FETI-type methods [7, 8, 9, 19], regularized method [12] and

Received by the editors March 2, 2009 and, in revised form, May 26, 2010.

2000 *Mathematics Subject Classification.* 65F10, 65N30, 65N55.

This research was supported by The Key Project of Natural Science Foundation of China G11031006, and National Basic Research Program of China G2011309702 and Natural Science Foundation of China G10771178.

augmented method [15]. After handling the singularity, we can eliminate the displacement variable to build an interface equation, or solve the saddle-point system directly by some preconditioned iterative methods.

A domain decomposition with Lagrange multipliers for solving linear elasticity problems in two dimensions was introduced in [19], in which inexact solvers were considered. A recent work on mortar discretization with geometrically nonconforming partitions for solving linear elasticity problems is a FETI-DP algorithm designed in [18]. To resolve the singularity associated with the displacement variable, a certain set of primal constraints was selected in [18] from the subdomain faces by some rules. After building the saddle-point system, Schur complement system was first got by eliminating the interior displacement variables in every subdomain, then an interface equation of the Lagrange multiplier was obtained by eliminating the primal constraint unknowns. Similar to other FETI-DP algorithms, a Neumann-Dirichlet preconditioner was constructed for the interface equation.

In the present paper, we study DDM with Lagrange multipliers for solving three-dimensional linear elasticity problems with jump coefficients. As in [15] (for Laplace equations), we propose a special augmented method to handle the singularity of the floating subdomains without introducing any additional constraints. But, we here introduce a different augmented term from the one considered in [15], since the original augmented term seems inefficient to elasticity problems. Since no interface equation needs to be built in the method, inexact solvers can be applied to both the primal operator and the Schur complement operator. For our method, we design a small coarse problem with the degree of freedoms equaling six times the number of the floating subdomains. We notice that the elasticity operator is spectrally equivalent to Laplace operator in every subdomain, then any existing preconditioner for the vector Laplace operator can be used directly as an inexact solver for the underlying operator. We show that the global preconditioned system has a nearly optimal condition number, which is independent of the large variations of the material parameters across the local interfaces.

The outline of the reminder of the paper is as follows. We introduce a new augmented saddle-point problem in section 2. In section 3, we construct two preconditioners for the saddle-point system and give a convergence of the preconditioned system. The main results of the paper will be shown in section 4. In section 5, we describe a class of cheap local solvers. Finally, we report some numerical results in section 6.

2. Linear elasticity and domain decomposition

In this section, we introduce a variational problem arising from the displacement formulation of compressible linear elasticity, and describe a discretization based on geometrically non-conforming domain decompositions.

2.1. The model problem.

2.1. The model problem. The unknown in the equations of linear elasticity is the displacement of a linear elastic material under the actions of external and internal forces. We denote the elastic body by $\Omega \subset \mathbb{R}^3$, and its boundary by $\partial\Omega$. We assume that one part of the boundary Γ_0, is clamped, i.e. with homogeneous Dirichlet boundary conditions, and that the rest, $\Gamma_1 := \partial\Omega\backslash\Gamma_0$, is subject to a surface force $\mathbf{g}$, i.e. a natural boundary condition. We can also introduce an internal volume force $\mathbf{f}$, e.g. gravity. The differential formulation is as follows (i=1,