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Comparison of Fifth-Order WENO Scheme and Finite Volume WENO-Gas-Kinetic Scheme for Inviscid and Viscous Flow Simulation

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Abstract. The development of high-order schemes has been mostly concentrated on the limiters and high-order reconstruction techniques. In this paper, the effect of the flux functions on the performance of high-order schemes will be studied. Based on the same WENO reconstruction, two schemes with different flux functions, i.e., the fifth-order WENO method and the WENO-Gas-kinetic scheme (WENO-GKS), will be compared. The fifth-order finite difference WENO-SW scheme is a characteristic variable reconstruction based method which uses the Steger-Warming flux splitting for inviscid terms, the sixth-order central difference for viscous terms, and three stages Runge-Kutta time stepping for the time integration. On the other hand, the finite volume WENO-GKS is a conservative variable reconstruction based method with the same WENO reconstruction. But, it evaluates a time dependent gas distribution function along a cell interface, and updates the flow variables inside each control volume by integrating the flux function along the boundary of the control volume in both space and time. In order to validate the robustness and accuracy of the schemes, both methods are tested under a wide range of flow conditions: vortex propagation, Mach 3 step problem, and the cavity flow at Reynolds number 3200. Our study shows that both WENO-SW and WENO-GKS yield quantitatively similar results and agree with each other very well provided a sufficient grid resolution is used. With the reduction of mesh points, the WENO-GKS behaves to have less numerical dissipation and present more accurate solutions than those from the WENO-SW in all test cases. For the Navier-Stokes equations, since the WENO-GKS couples inviscid and viscous terms in a single flux evaluation, and the WENO-SW uses an operator splitting technique, it appears that the WENO-SW is more sensitive to the WENO reconstruction and boundary treatment. In terms of efficiency, the finite volume WENO-GKS is about 4 times slower than the finite difference WENO-SW in two dimensional simulations. The current study clearly shows that besides high-order reconstruction, an accurate gas evolution model or flux function in a high-order scheme is also important in the capturing of

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physical solutions. In a physical flow, the transport, stress deformation, heat conduction, and viscous heating are all coupled in a single gas evolution process. Therefore, it is preferred to develop such a scheme with multi-dimensionality, and unified treatment of inviscid and dissipative terms. A high-order scheme does prefer a high-order gas evolution model. Even with the rapid advances of high-order reconstruction techniques, the first-order dynamics of the Riemann solution becomes the bottleneck for the further development of high-order schemes. In order to avoid the weakness of the low order flux function, the development of high-order schemes relies heavily on the weak solution of the original governing equations for the update of additional degree of freedom, such as the non-conservative gradients of flow variables, which cannot be physically valid in discontinuous regions.

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Key words: WENO scheme, gas-kinetic scheme, Euler equations, Navier-Stokes equations, high-order methods.

1 Introduction

Computational Fluid Dynamics has made great progress in 1970s and 1980s due to the development of the concept of the nonlinear limiter and the characteristic wave decomposition of the Euler equations. The 2nd-order schemes are mostly used in practical engineering applications at current stage. On the other hand, with the increasing of computational power and the requirement of accurate solutions for more challenging problems, such as compressible turbulent flow and aero-acoustics, the development of reliable high-order methods has attracted much attention [4]. A direct extension of the concept of nonlinear limiter to high-order is the reconstruction schemes of essentially non-oscillatory (ENO) and weighted essentially non-oscillatory (WENO) methods [7,10,14,17]. There are two versions of WENO schemes: finite difference and finite volume. For the rectangular mesh, the main advantage of the finite difference framework is that multi-dimensional calculations do not increase the complexity of the algorithm and the computational cost is much lower than the finite volume version. As tested, a finite volume WENO scheme is usually 4 times more expensive in 2D than a finite difference WENO method due to many flux calculations at the Gaussian points of a cell interface. Therefore, in simple geometry cases, the finite difference WENO scheme is the top choice. The most widely used WENO scheme is the fifth order WENO method [10]. In comparison with other high-order methods, such as discontinuous Galerkin method (DG), the WENO scheme is much more robust and reliable.

The main steps of finite difference WENO scheme are the WENO reconstruction for the fluxes at the cell interface and the Runge-Kutta time stepping to update the flow variables. In the WENO reconstruction, a stencil-weighted technique is used to avoid cross-shock interpolation so as to reduce spurious oscillations. Instead of choosing the smoothest stencil out of many candidates to get the *rth* order accuracy in the ENO reconstruction, the WENO uses a convex combination of all candidates by assigning a