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Higher-Order Compact Scheme for the Incompressible Navier-Stokes Equations in Spherical Geometry

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Abstract. A higher-order compact scheme on the nine point 2-D stencil is developed for the steady stream-function vorticity form of the incompressible Navier-Stokes (N-S) equations in spherical polar coordinates, which was used earlier only for the cartesian and cylindrical geometries. The steady, incompressible, viscous and axially symmetric flow past a sphere is used as a model problem. The non-linearity in the N-S equations is handled in a comprehensive manner avoiding complications in calculations. The scheme is combined with the multigrid method to enhance the convergence rate. The solutions are obtained over a non-uniform grid generated using the transformation $r=e^{\xi}$ while maintaining a uniform grid in the computational plane. The superiority of the higher order compact scheme is clearly illustrated in comparison with upwind scheme and defect correction technique at high Reynolds numbers by taking a large domain. This is a pioneering effort, because for the first time, the fourth order accurate solutions for the problem of viscous flow past a sphere are presented here. The drag coefficient and surface pressures are calculated and compared with available experimental and theoretical results. It is observed that these values simulated over coarser grids using the present scheme are more accurate when compared to other conventional schemes. It has also been observed that the flow separation initially occurred at $Re=21$.

AMS subject classifications: 76D05, 35Q35, 65N06

Key words: Fourth order compact scheme, Navier-stokes equations, spherical polar coordinates, drag coefficient.

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1 Introduction

The complexity involved in solving N-S equations by numerical approximations differs for various geometries such as cartesian, cylindrical and spherical polar coordinates, especially while handling non-linearity of the N-S equations. The present paper is concerned with solving the steady two-dimensional Navier-Stokes equations in spherical polar coordinates using higher order compact scheme (HOCS) on the nine point 2-D stencil as shown in Fig. 1.

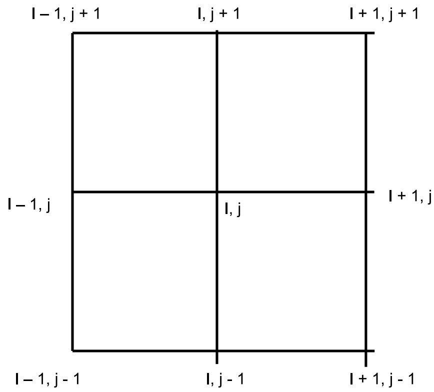

Figure 1: Nine point 2-D stencil.

The study of steady incompressible N-S equations using finite difference methods vary considerably in terms of accuracy and efficiency. The central difference approximations to all the derivatives of the N-S equations yields second order accuracy but the resulting solutions may exhibit non-physical oscillations. The combination of central differences to second order derivatives and first order upwind differences to nonlinear terms (here after denoted as CDS-UPS) as described by Ghia et al. [1], Juncu and Mihail [2] and Sekhar et al. [3] yields a stable scheme but is of first order accurate and the resulting solutions exhibit the effects of artificial viscosity. Also, at high *Re*, approximation of convective terms using CDS-UPS scheme may not capture the flow phenomena accurately due to the dominance of inertial forces. To capture the flow phenomena, at least second order accuracy is required. The second order upwind differences to nonlinear terms are no better than the first-order ones for large values of *Re* and also require ghost points. The second order accuracy can be achieved by employing defect correction technique (DC) for CDS-UPS scheme [1,2]. The traditional higher order finite difference methods [4] contains ghost points and requires special treatment near the boundaries. If the domain is large, the above first and second order accurate methods may not converge with coarser grids and grid independence can be achieved only with very high finer grids which consumes more CPU time and memory [3]. An exception has been found in the high order finite difference schemes of compact type, which are computationally stable, efficient and yield highly accurate numerical solutions [5,6]. Jiten et al. [7] developed