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An Adaptive Moving Mesh Method for Two-Dimensional Relativistic Hydrodynamics

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Abstract. This paper extends the adaptive moving mesh method developed by Tang and Tang [36] to two-dimensional (2D) relativistic hydrodynamic (RHD) equations. The algorithm consists of two "independent" parts: the time evolution of the RHD equations and the (static) mesh iteration redistribution. In the first part, the RHD equations are discretized by using a high resolution finite volume scheme on the fixed but nonuniform meshes without the full characteristic decomposition of the governing equations. The second part is an iterative procedure. In each iteration, the mesh points are first redistributed, and then the cell averages of the conservative variables are remapped onto the new mesh in a conservative way. Several numerical examples are given to demonstrate the accuracy and effectiveness of the proposed method.

AMS subject classifications: 35L65, 65M50, 74S10, 76L05, 76N15, 76Y05

Key words: Adaptive moving mesh method, finite volume method, conservative interpolation, relativistic hydrodynamics.

1 Introduction

Relativistic hydrodynamics plays a major role in many fields of modern physics, e.g., astrophysics, nuclear and high-energy physics and, lately, also in condensed matter. A relativistic description of fluid dynamics should be used whenever matter is influenced by large gravitational potentials, where a description in terms of the Einstein field theory of gravity is necessary. It is also necessary in situations where the local velocity of the flow is close to the light speed in vacuum or where the local internal energy density is comparable (or larger) than the local rest mass density of the fluid. Alternatively, relativistic flows are present in numerous astrophysical phenomena from stellar to galactic

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scales, e.g., core collapse super-novae, X-ray binaries, pulsars, coalescing neutron stars and black holes, micro-quasars, active galactic nuclei, super-luminal jets, and gamma-ray bursts etc.

The dynamics of the relativistic systems require solving highly nonlinear equations, rendering the analytic treatment of practical problems extremely difficult. Often studying them numerically is a possible approach. The first attempt to solve the RHD equations was made by Wilson in the 1970s [42,43], using an Eulerian explicit finite difference code with a monotonic transport, depending on the artificial viscosity techniques to handle shock waves. After that, various numerical methods are developed to solve the RHD equations. We refer the readers to the review article by Martí and Müller [27]. Most of those schemes are the generalizations of the shock-capturing Godunov-type methods based on the exact or approximate Riemann solvers. These Riemann solvers either rely on characteristic decompositions of the Jacobian matrix or not. Eulderink and Mellema [15] and Falle and Komissarov [16] developed RHD solvers based on the local linearization, respectively. Balsara [1], Dai and Woodward [8], and Mignone et al. [29] developed two-shock approximation solver for the RHD system. A flux-splitting method was extended to the RHDs in [12]. Schneider et al. [32] and Duncan and Hughes [13] presented the HLL (Harten-Lax-van Leer) method in the context of the RHD equations. An extension of the HLLC (Harten-Lax-van Leer-Contact) approximate Riemann solver for the RHDs was presented by Mignone and Bodo [28]. ENO (essentially non-oscillatory) based methods for the RHD system have been studied by Dolezal and Wong [11] and Del Zanna and Bucciantini [45].

In practice, solutions to the (nonlinear) RHD equations are frequently smooth in large fractions of the physical domain but contain sharp transitions or discontinuities in relatively localized regions. In the smooth regions, relatively coarse numerical zoning may be sufficient to accurately represent the solution, while finer zoning is needed where sharp transitions occur. Because of this, adaptive mesh strategies are needed. Successful implementation of the adaptive approaches can improve the accuracy of the numerical approximation and decrease the computational cost. Adaptive moving mesh methods have been playing an increasingly important role in many branches of scientific and engineering areas. Up to now, there have been many important progresses in the adaptive moving mesh methods for partial differential equations, including grid redistribution approaches based on the variational principle of Winslow [44], Brackbill [2], Brackbill and Saltzman [3], Ren and Wang [31], and Wang and Wang [41]; moving finite element methods of Millers [30], and Davis and Flaherty [9]; moving mesh PDEs methods of Russell et al. [5,33], Li and Petzold [24], and Ceniceros and Hou [6]; and moving mesh methods based on the harmonic mapping of Dvinsky [14], and Li, Tang and Zhang [10,22,23]. Computational costs of moving mesh methods can be efficiently saved with locally varying time steps [34]. Balanced monitoring of flow phenomena in moving mesh method is recently discussed in [39]. We also refer the readers to recent review papers [4,38] and references therein.

The paper is organized as follows. Section 2 introduces the governing equations of