

Numerical Methods for Two-Fluid Dispersive Fast MHD Phenomena

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Abstract. The finite volume wave propagation method and the finite element Runge-Kutta discontinuous Galerkin (RKDG) method are studied for applications to balance laws describing plasma fluids. The plasma fluid equations explored are dispersive and not dissipative. The physical dispersion introduced through the source terms leads to the wide variety of plasma waves. The dispersive nature of the plasma fluid equations explored separates the work in this paper from previous publications. The linearized Euler equations with dispersive source terms are used as a model equation system to compare the wave propagation and RKDG methods. The numerical methods are then studied for applications of the full two-fluid plasma equations. The two-fluid equations describe the self-consistent evolution of electron and ion fluids in the presence of electromagnetic fields. It is found that the wave propagation method, when run at a CFL number of 1, is more accurate for equation systems that do not have disparate characteristic speeds. However, if the oscillation frequency is large compared to the frequency of information propagation, source splitting in the wave propagation method may cause phase errors. The Runge-Kutta discontinuous Galerkin method provides more accurate results for problems near steady-state as well as problems with disparate characteristic speeds when using higher spatial orders.

AMS subject classifications: 35Q35, 35Q61, 65L60, 65M08, 82D10

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1 Introduction

There are a number of equation systems that are either hyperbolic or contain hyperbolic parts. Homogeneous, hyperbolic equation systems are written as conservation laws of the form [1,2]

$$\frac{\partial \mathbf{Q}}{\partial t} + \nabla \cdot \mathbf{F} = 0, \quad (1.1)$$

where $\mathbf{Q} \in \mathbb{R}^m$ represents the m conserved variables and $\mathbf{F} \in \mathbb{R}^{m \times d}$ represents fluxes in d spatial directions. For all unit vectors $\boldsymbol{\omega} \in \mathbb{R}^d$ the flux Jacobian, $\partial(\mathbf{F} \cdot \boldsymbol{\omega}) / \partial \mathbf{Q}$, has real eigenvalues and a complete set of right eigenvectors. Some homogeneous, hyperbolic equation systems include the Euler equations and magnetohydrodynamic (MHD) equations.

Inhomogeneous, hyperbolic equation systems are described by balance laws of the form

$$\frac{\partial \mathbf{Q}}{\partial t} + \nabla \cdot \mathbf{F} = \mathbf{S}, \quad (1.2)$$

where $\mathbf{S} \in \mathbb{R}^m$ represents the source terms. The source Jacobian for Eq. (1.2) is $\partial \mathbf{S} / \partial \mathbf{Q}$. The presence of real eigenvalues in the source Jacobian results in an equation system that contains diffusive sources. The Navier-Stokes equations and the 10-moment fluid equations [3] are examples of inhomogeneous, hyperbolic equation systems containing diffusive source terms.

For inhomogeneous, hyperbolic equation systems described by Eq. (1.2), the presence of imaginary eigenvalues in the source Jacobian results in an equation system that contains dispersive sources. The two-fluid plasma model is a system of inhomogeneous, hyperbolic equations containing dispersive source terms. The dispersive source terms arise from the physical properties of the plasma medium. Dispersive source terms present a unique challenge for numerical algorithms because low-order, explicit-time-stepping schemes can be unstable when applied to the wave equation leading to numerical dispersion [4]. The physical dispersion can be difficult for numerical schemes to capture and can be difficult to distinguish from the numerical dispersion or "noise". In this paper, numerical methods for solving inhomogeneous, hyperbolic equations containing dispersive source terms are investigated for accuracy and computational effort.

Hyperbolic conservation laws can have discontinuous solutions even if the initial conditions are smooth, and this makes the approximation of the solution difficult. First order upwind methods are needed to effectively capture such discontinuities. However, first order methods are highly diffusive in smooth regions. Second order extensions can be constructed which both resolve the discontinuities and provide better accuracy in smooth regions. Smooth nonlinear solutions can achieve second order accuracy when using Godunov's method with second order corrections [5] even though the method is formally first order accurate, e.g., in Section 15.6 of [2]. [5] provides proof of second order accuracy for smooth problems including the case with source terms (Section 7 of [5]). For