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Comparison of Two Advection-Diffusion Methods for Tephra Transport in Volcanic Eruptions

Kae Tsunematsu[1], Bastien Chopard[2,*], Jean-Luc Falcone[2] and Costanza Bonadonna[1]

[1] *Department of Mineralogy, University of Geneva, 13 Rue des Maraichers, 1205 Geneve, Switzerland.*
[2] *Department of Computer Science, University of Geneva, 7 Route de Drize, 1227 Carouge, Switzerland.*

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Abstract. In order to model the dispersal of volcanic particles in the atmosphere and their deposition on the ground, one has to simulate an advection-diffusion-sedimentation process on a large spatial area. Here we compare a Lattice Boltzmann and a Cellular Automata approach. Our results show that for high Peclet regimes, the cellular automata model produce results that are as accurate as the lattice Boltzmann model and is computationally more effective.

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1 Introduction

When a volcano erupts explosively, tephra are ejected from the crater within a mixture of gas. Tephra are fragments of magma that travel through the atmosphere and eventually sediment on the ground. Numerical models of tephra transport are important for hazard assessments.

Tephra transport is modeled as an advection-diffusion-sedimentation process. In the air, particles are advected by wind and sediment according to the particle terminal velocity. At the same time, they diffuse as a product of turbulence and small particles ($<125\mu$m) aggregate as a result of particle-particle interaction and atmospheric condition. Field observations show that aggregation significantly affect tephra deposit [3].

*Corresponding author. *Email addresses:* Kae.Tsunematsu@unige.ch (K. Tsunematsu), Bastien.Chopard@unige.ch (B. Chopard), Jean-Luc.Falcone@unige.ch (J. Falcone), Costanza.Bonadonna@unige.ch (C. Bonadonna)

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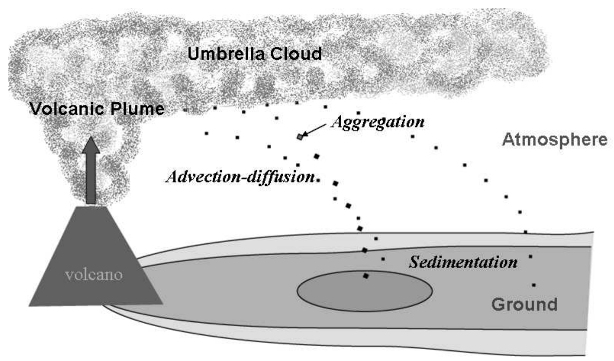

Figure 1: Advection-diffusion-sedimentation process of volcanic tephra transport.

The lattice Boltzmann method (LB) is widely used for advection-diffusion process [4, 7, 8, 11]. However its numerical stability and accuracy depend on calculation parameters. In [11] the stability of the LB advection-diffusion is shown to depend on the Peclet number.

An alternative to the LB approach is the multiparticle Cellular Automata (CA) model designed to describe the transport of passive scalar point particles in a given velocity field $\vec{u}(\vec{r},t)$. This model has been successfully used in [6,9] to describe snow or sand transport and validated on several non-trivial examples of snow accumulation by wind and sand erosion around submarine pipelines. In [12] the application of this CA model to tephra transport is also reported.

In spite of these results and the unconditional stability of the CA scheme, no analytical derivation was ever proposed to describe its behavior in terms of a differential equation, and no studies were conducted to compare its computational efficiency with a LB approach.

In what follows we show that the CA transport model actually obeys an anisotropic advection-diffusion equation. However, this unwanted anisotropy only introduces a small error for system with high Peclet numbers, as is the case in tephra transport. In addition, we show that the unconditional stability of the CA model allows us to choose a coarser discretization than with the LB model. As a result, computations of tephra transport with the CA model can be faster and less memory consuming than with the LB model, yet for a comparable accuracy.

2 The multiparticle cellular automata transport model

The snow and sand transport model proposed in [6,9] is a stochastic multiparticle cellular automata (CA). Each cell contains an arbitrary number of point particles which move to a nearest neighbor cell according to a given advecting field $\vec{u}(\vec{r},t)$. The particles keep their discrete nature all along the process. If needed additional interactions (such as aggregation) between grains that meet on the same lattice site can be added. As far as