Optimal Error Estimation of the Modified Ghost Fluid Method

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Abstract. The modified ghost fluid method (MGFM) has been shown to be robust and efficient when being applied to multi-medium compressible flows. In this paper, we rigorously analyze the optimal error estimation of the MGFM when it is applied to the multi-fluid Riemann problem. By analyzing the properties of the MGFM and the approximate Riemann problem solver (ARPS), we show that the interfacial status provided by the MGFM can achieve "third-order accuracy" in the sense of comparing to the exact solution of the Riemann problem, regardless of the solution type. In addition, our analysis further reveals that the ARPS based on a doubled shock structure in the MGFM is suitable for almost any conditions for predicting the interfacial status, and that the "natural" approach of "third-order accuracy" is practically less useful. Various examples are presented to validate the conclusions made.

AMS subject classifications: 35L45, 65C20, 76T10

Key words: Modified ghost fluid method, Riemann problem, approximate Riemann problem solver.

1 Introduction

In recent years, with the continuous improvement of numerical simulation and the matureness of various algorithms, many complex flow issues, which were not able to be explored in depth in the past, have reentered the horizons of scientific researchers. Some high resolution schemes for compressible flows, such as the total variation diminishing (TVD) schemes [1,2] and the essentially non-oscillatory (ENO) schemes [3–5], can work very successfully for pure medium compressible flows. However, when we employ such schemes to simulate multi-medium compressible flows, unexpected difficulties occur due

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to nonphysical oscillations generated in the vicinities of the material interfaces. To suppress the oscillations, various techniques have been developed, see, e.g., [6–9,11–16,19].

Among the above mentioned methods, the ghost fluid method (GFM) [11] and other GFM-based techniques [8,9,12,19] provide simple and flexible ways for handling multi-medium flows. The easy extension to multi-dimensions and maintenance of a sharp interface are the advantages of the GFM-based techniques. The key point of these GFM-based techniques is to properly define the properties of the ghost fluids, which is also the primary difference among the various versions of the methods. The original GFM [11] uses the local real fluid velocity and pressure to define the corresponding ghost fluid status, and the density of the ghost fluid is obtained via isobaric fixing [10]. It has been shown, however, that such a definition of ghost fluid status is not efficient when applied to gas-water flows [12].

In fact, the pressure or the velocity across the material interface can have a sudden jump when there is a strong wave interacting with the interface. Whether in the original GFM or its later gas-water version [12], the definition of ghost fluid status is not strictly sufficient to take into account the effects of wave interaction and material properties. To overcome such shortcomings, a modified GFM (MGFM) was developed in [8], where a Riemann problem was defined along the normal direction of the interface and solved using approximate Riemann problem solver (ARPS) to predict the interfacial status. The predicted interfacial status was then utilized to define the ghost fluid status. The MGFM has been shown to be robust and less problem related and successfully applied to various gas-gas, gas-water and fluid-structure coupling problems [8,9,20–22]. In addition, it has been proved that the interfacial status captured by the MGFM approximates the exact solution to "second-order accuracy" for the gas-gas Riemann problem [18].

However, we find that the above analytical conclusions are not optimal. In this paper, a further analysis is carried out for the MGFM in the absence of vacuum or cavitation. We shall show that the interfacial status captured by the MGFM can achieve "third-order accuracy" in the sense of comparing to the exact solution for any multi-fluid Riemann problem. Moreover, we shall find that the implicit ARPS based on a doubled shock structure is much stable and suitable for almost any initial conditions without restrictions in predicting the interfacial status, while a "natural" approach, which is also a "third-order" approximation, is proved to be less useful. It should be noted that the "accuracy" discussed in this paper means how accurate the boundary conditions are implicitly imposed at the material interface and how accurate the interface states are approximated by the GFM technique, which is in contrast with the accuracy of the numerical scheme or the errors between the exact solution and the numerical solution.

The paper is organized as follows. In Section 2 we introduce the Euler equations and equations of state (EOS) followed by a brief description of the level set equation. In Section 3 the solution structure of a multi-fluid Riemann problem is presented. In this section, the estimates about the interfacial status of the Riemann problem are derived and discussed. These estimates will serve as the basis for accuracy analysis of the MGFM. In Section 4, the multi-fluid Riemann problem is split into two pure fluid Riemann prob-