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An Adaptive Moving Mesh Method for Two-Dimensional Incompressible Viscous Flows

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Abstract. In this paper, we present an adaptive moving mesh technique for solving the incompressible viscous flows using the vorticity stream-function formulation. The moving mesh strategy is based on the approach proposed by Li et al. [*J. Comput. Phys.*, 170 (2001), pp. 562–588] to separate the mesh-moving and evolving PDE at each time step. The Navier-Stokes equations are solved in the vorticity stream-function form by a finite-volume method in space, and the mesh-moving part is realized by solving the Euler-Lagrange equations to minimize a certain variation in conjunction with a more sophisticated monitor function. A conservative interpolation is used to redistribute the numerical solutions on the new meshes. This paper discusses the implementation of the periodic boundary conditions, where the physical domain is allowed to deform with time while the computational domain remains fixed and regular throughout. Numerical results demonstrate the accuracy and effectiveness of the proposed algorithm.

AMS subject classifications: 65M06, 65M50, 76D05

Key words: Moving mesh method, finite volume method, Navier-Stokes equations, vorticity stream-function, incompressible flow.

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1 Introduction

Adaptive moving mesh methods have many important applications in various physical and engineering fields such as solid and fluid dynamics, combustion, heat transfer, material science etc. The physical phenomena in these mentioned areas may develop dynamically singular or nearly singular solutions in fairly localized region. A high fidelity numerical investigation of these physical problems may require extremely fine meshes

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over a small portion of the physical domain to resolve the large solution variations. The use of globally refined uniform meshes becomes computationally wasteful when dealing with systems in two or higher dimensions. In multi-dimensional problems, developing an effective and robust adaptive mesh method becomes almost absolutely necessary. Successful implementation of the adaptive approaches not only produces a high mesh density in regions of large gradient to improve the accuracy of numerical solution, but also decreases the cost of numerical calculation in comparison with the uniform mesh. Currently, there has been much important progress made in adaptive moving mesh methods for partial differential equations, including the grid distribution approach based on the variational principle of Winslow [36], Brackbill et al. [5,7], Ren and Wang [26], and Tang and Tang [33]; the finite element methods of Miller and Miller [24], and Davis and Flaherty [10]; the moving mesh PDEs of Russell et al. [8], Li and Petzold [19], and Ceniceros and Hou [9]; and the moving mesh methods based on harmonic mapping of Dvinsky [14] and Li et al. [11,21]; and others. Computational costs of moving mesh methods can be further reduced with locally varying time steps [31].

There are two main ways to generate an adaptive mesh, namely, local mesh refinements and moving mesh method. In local mesh refinement methods, the adaptive mesh is generated by adding or removing grid points based on the posteriori error of the numerical solution. Local refinement approach requires complicated data structures and technically complex methods to communicate information among different levels of refinement. In the moving mesh methods, the total number of the grid points is kept fixed. The grids are moved continuously in the whole solution domain to cluster grid points in regions where the solution has the larger variations. In the past two decades this numerical technique has been proven very successful for solving time-dependent problems whose solution has large gradient or discontinuities, see, e.g., [2–4,7,11,12,20,28,30,31]. In particular, Almgren et al. [1] introduced a adaptive projection method for the variable density incompressible Navier-Stokes equations on nested grids, while Di et al. [11] developed a moving mesh finite element methods for solving the incompressible Navier-Stokes equations in the primitive variables formulation and devised a divergence-free interpolation which is very essential for incompressible problems. Still, Ding and Shu [13] proposed a stencil adaptive algorithm for finite difference solution of incompressible viscous flows, and Min and Gibou [25] presented an unconditionally stable second-order accurate projection method for the incompressible Navier-Stokes equations on non-graded adaptive Cartesian grids. The latter employed quadtree and octree data structures as an efficient means to represent the grid.

One main difficulty in solving the incompressible viscous flows is the divergence-free constraint of the velocity field. There are two popular approaches to handle the divergence-free constraint in the incompressible Navier-Stokes equations. One is to use projection technique. This technique is commonly used in many incompressible Navier-Stokes solvers [1,25,29]. However, in general, the pressure Poisson solver of projection step will be time consuming on unstructured grids or adaptive grids. The other is to introduce the stream function, see [15,22]. One possible disadvantage of this approach is