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DEVELOPMENT, ANALYSIS AND NUMERICAL TESTS OF A COMPOSITIONAL RESERVOIR SIMULATOR

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Abstract. The governing equations of a compositional model for three-phase multicomponent fluid flow in multi-dimensional petroleum reservoirs are cast in terms of a pressure equation and a set of component mass balance equations in this paper. The procedure is based on a pore volume constraint for component partial molar volumes, which is different from earlier techniques utilizing an equation of state for phase fluid volumes or saturations. The present technique simplifies the pressure equation, which is written in terms of various pressures such as phase, weighted fluid, global, and pseudo-global pressures. The different formulations resulting from these pressures are numerically solved; the numerical computations use a scheme based on the mixed finite element method for the pressure equation and the finite volume method for the component mass balance equations. A qualitative analysis of these formulations is also carried out. The analysis yields that the differential system of these formulations is of mixed parabolic-hyperbolic type, typical for fluid flow equations in petroleum reservoirs. Numerical experiments based on the benchmark problem of the third comparative solution project organized by the society of petroleum engineers are presented.

Key Words. compositional model, reservoir simulation, mixed finite elements, finite volume, thermodynamic equilibrium, numerical experiments.

1. Introduction

A compositional model for three-phase multicomponent fluid flow in petroleum reservoirs has been recently analyzed in [10]. This model incorporates compressibility, compositional effects, and mass interchange between phases. It consists of Darcy's law for volumetric flow velocities, mass balance for hydrocarbon components, thermodynamic equilibrium for mass interchange between phases, and an equation of state for phase saturations. It models important enhanced oil recovery procedures such as condensing gas drive and miscible gas injection. To understand complex chemical and physical phenomena of fluid flow in petroleum reservoirs, it has become increasingly important to study such a realistic model.

In this paper this compositional model is further studied. Instead of the equation of state for phase saturations, a pore volume constraint for component partial molar volumes is exploited for this model. Specifically, the governing equations of this model are cast in terms of a pressure equation and a set of component mass balance equations, and the procedure is based on this pore volume constraint. The advantages of using this constraint over the state equation for phase saturations

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are that the saturations can be zero but the number of overall moles of each component appearing in this constraint is often positive, that the present development is practically well suited for multiphase flow since this constraint does not involve phases, and that the formulation of the pressure equation is simpler.

As in [10], the pressure equation is written in terms of various pressures such as phase, weighted fluid, global, and pseudo-global pressures. The emphasis here is to carry out numerical computations for the different formulations resulting from these pressures. The numerical computations use a scheme based on the mixed finite element method for the pressure equation and the finite volume method for the component mass balance equations. This scheme is suitable for numerical simulation of multiphase flow through geometrically complex geological petroleum reservoirs [11, 12]. Numerical experiments based on the benchmark problem of the third comparative solution project organized by the society of petroleum engineers [13] are presented.

A qualitative analysis of the differential system of these formulations is given. This system is of mixed parabolic-hyperbolic type, typical for fluid flow equations in petroleum reservoirs. We show that the pressure equation is a standard parabolic problem and the component mass balance equations are advection-dominated problems in the presence of capillary diffusive forces; they are purely hyperbolic in the absence of these diffusive forces. For simplicity, we neglect hydraulic dispersion and molecular diffusion effects in this paper. The mathematical structure of a one-dimensional two-phase multicomponent compositional model without capillary pressure effects was analyzed in [21] by a different approach.

The rest of the paper is organized as follows. In the next section, we review the governing equations for a compositional model. Then in the third section, we derive some lemmas from thermodynamic equilibrium conditions, which are used in the development of the pressure equation. In the fourth section, we derive the pressure and component mass balance equations. In the fifth section, we give a qualitative analysis of the derived differential system, and in the sixth section, we develop our numerical scheme. Finally, in the seventh section we report numerical experiments.

2. Governing Equations for Compositional Flow

There are books that develop the equations for compositional flow in petroleum reservoirs (e.g., [8, 15]). In this section, we briefly review these equations. The compositional flow involves mass interchange between phases and compressibility. In a model for this type of flow, a finite number of hydrocarbon components is used to represent the composition of reservoir fluids. These components associate as phases in the reservoirs. In this paper, we describe a compositional model under the assumptions that the flow process is isothermal (i.e., the constant temperature), the components form at most three phases (e.g., gas, oil, and water), and there is no mass interchange between the water phase and the hydrocarbon phases (i.e., the oil and gas phases).

Because of mass interchange between phases, mass is not conserved within each phase; the total mass of each component is conserved:

$$
\begin{aligned}
&\partial_t(\phi n_w) + \nabla \cdot (\xi_w u_w) = q_w, \\
&\partial_t(\phi n_i) + \nabla \cdot (x_{ig}\xi_g u_g + x_{io}\xi_o u_o) = q_i, \qquad i = 2, \ldots, N_c,
\end{aligned}
\tag{2.1}
$$

where ∂_t denotes time differentiation, ϕ is the porosity of the reservoir, g, o, and w refer to gas, oil, and water phases, i is the component index, $N_c - 1$ is the number of hydrocarbon components, n_w and n_i denote the number of overall moles per