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BIFURCATION ANALYSIS AND COMPUTATION OF DOUBLE TAKENS-BOGDANOV POINT IN Z_2-EQUIVARIABLE NONLINEAR EQUATIONS*

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Abstract *The paper deals with the computation and bifurcation analysis of double Takens-Bogdanov point* (u^0, Λ^0) *(in short,* DTB *point) in the* Z_2*-equivariable nonlinear equation* $f(u, \Lambda) = 0, f : U \times R^4 \to V$ *, where U and V are Banach spaces, parameters* $\Lambda \in R^4$*. At* (u^0, Λ^0) *, the null space of* f_u^0 *has geometric multiplicity* 2 *and algebraic multiplicity* 4*. Firstly a regular extended system for computing* DTB *point is proposed. Secondly, it is proved that there are four branches of singular points bifurcated from* DTB *point: two paths of* STB *points, two paths of* TB*-Hopf points. Finally, the numerical results of one dimensional Brusselator equations are given to show the effectiveness of our theory and method.*

Key words Z_2*-equivariance, Takens-Bogdanov point, Hopf point, extended system.*
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1 Introduction

The singular points in Z_2-equivariable nonlinear equations have been studied in [1]-[6]. The purpose of the paper is to discuss the computation of DTB point in Z_2-equivariable nonlinear equations with four parameters and relative properties.

Consider the Z_2-equivariable nonlinear equation with four parameters

$$f(u, \lambda, \alpha, \beta, \gamma) = 0, \qquad f : U \times R^4 \to V, \tag{1.1}$$

where U and V are Banach spaces, f is a smooth Fredholm operator with index zero satisfying

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Z_2-equivariance. There is an $s \in L(V) \cap L(U)$, such that

$$s \neq I, s^2 = I, f(su, \lambda, \alpha, \beta, \gamma) = sf(u, \lambda, \alpha, \beta, \gamma), \forall (u, \lambda, \alpha, \beta, \gamma) \in U \times R^4.$$

It is well known that the presence of Z_2-equivariance implies the decomposition of U ,V and their dual space $U^{'}, V^{'}$

$$U = U_s \oplus U_a, V = V_s \oplus V_a,$$
$$U^{'} = U_s^{'} \oplus U_a^{'}, V^{'} = V_s^{'} \oplus V_a^{'},$$

where

$$U_s = \{u \in U, su = u\}, U_a = \{u \in U, su = -u\},$$

$$V_s = \{v \in V, sv = v\}, V_a = \{v \in V, sv = -v\},$$

$$U_s^{'} = \{\psi \in U^{'}, \psi s = \psi\}, U_a^{'} = \{\psi \in U^{'}, \psi s = -\psi\},$$

$$V_s^{'} = \{\psi \in V^{'}, \psi s = \psi\}, V_a^{'} = \{\psi \in V^{'}, \psi s = -\psi\}.$$

It is easy to check

$$\psi u = 0, \text{ when } \psi \in U_s^{'}, u \in U_a; \text{ or } \psi \in U_a^{'}, u \in U_s .$$

Fixing γ,(1.1) becomes nonlinear equation with three parameters λ, α, β.

Definition 1.1 $(u_0, \lambda_0, \alpha_0, \beta_0)$ is called a STB1 point of (1.1) if

$$f(u_0, \lambda_0, \alpha_0, \beta_0) = 0, u_0 \in U_s, \tag{1.2a}$$
$$N(f_u^0) = \text{span}(\phi_s, \phi_a), \phi_s \in U_s, \phi_a \in U_a, \tag{1.2b}$$
$$R(f_u^0) = \{v \in V, \psi_s v = \psi_a v = 0\}, \psi_s \in V_s', \psi_a \in V_a', \tag{1.2c}$$
$$\psi_s \phi_s = 0, \psi_a \phi_a \neq 0. \tag{1.2d}$$

Definition 1.2 $(u_0, \lambda_0, \alpha_0, \beta_0)$ is called a STB2 point of (1.1) if

$$f(u_0, \lambda_0, \alpha_0, \beta_0) = 0, u_0 \in U_s, \tag{1.3a}$$
$$N(f_u^0) = \text{span}(\phi_s, \phi_a), \phi_s \in U_s, \phi_a \in U_a, \tag{1.3b}$$
$$R(f_u^0) = \{v \in V, \psi_s v = \psi_a v = 0\}, \psi_s \in V_s', \psi_a \in V_a'. \tag{1.3c}$$
$$\psi_s \phi_s \neq 0, \psi_a \phi_a = 0. \tag{1.3d}$$

Definition 1.3 $(u_0, \lambda_0, \alpha_0, \beta_0)$ is called a TB-Hopf1 point of (1.1) if

$$f(u_0, \lambda_0, \alpha_0, \beta_0) = 0, u_0 \in U_s, \tag{1.4a}$$
$$N(f_u^0) = \text{span}(\phi_s), \phi_s \in U_s, \tag{1.4b}$$
$$R(f_u^0) = \{v \in V, \psi_s v = 0\}, \psi_s \in V_s', \tag{1.4c}$$
$$\psi_s \phi_s = 0, \tag{1.4d}$$
$$N(f_u^0 \pm \omega_0 iI) = \text{span}(a_a + ib_a), \omega_0 > 0, a_a, b_a \in U_a. \tag{1.4e}$$

Definition 1.4 $(u_0, \lambda_0, \alpha_0, \beta_0)$ is called a TB-Hopf2 point of (1.1) if

$$f(u_0, \lambda_0, \alpha_0, \beta_0) = 0, u_0 \in U_s, \tag{1.5a}$$
$$N(f_u^0) = \text{span}(\phi_a), \phi_a \in U_a, \tag{1.5b}$$
$$R(f_u^0) = \{v \in V, \psi_a v = 0\}, \psi_a \in V_a', \tag{1.5c}$$
$$\psi_a \phi_a = 0, \tag{1.5d}$$
$$N(f_u^0 \pm \omega_0 iI) = \text{span}(a_s + ib_s), \omega_0 > 0, a_s, b_s \in U_s. \tag{1.5e}$$

For (1.1) with four parameters, we have

Definition 1.5 $(u_0, \lambda_0, \alpha_0, \beta_0, \gamma_0)$ is called a double TB point of (1.1) if

$$f(u_0, \lambda_0, \alpha_0, \beta_0, \gamma_0) = 0, u_0 \in U_s, \tag{1.6a}$$
$$N(f_u^0) = \text{span}(\phi_s, \phi_a), \phi_s \in U_s, \phi_a \in U_a, \tag{1.6b}$$
$$R(f_u^0) = \{v \in V, \psi_s v = \psi_a v = 0\}, \psi_s \in V_s', \psi_a \in V_a', \tag{1.6c}$$