J. At. Mol. Sci.
doi: 10.4208/jams.121610.012211a

Vol. **2**, No. 4, pp. 317-324
November 2011

Relativistic paramagnetism of a weakly interacting Fermi gas

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Received 16 December 2010; Accepted (in revised version) 22 January 2011
Published Online 28 June 2011

Abstract. By considering strict relativistic energy of single particle and using the methods of quantum statisticsąAthe relativistic paramagnetism of a weakly interacting Fermi gas in a weak magnetic filed is studied, and the relativistic most probable paramagnetic susceptibility as well as the average magnetic susceptibility of the system are solved. On the basis, the influences of relativistic effect on the most probable paramagnetic susceptibility of the system are discussed, and the relativistic critical value of particle number is given. It is shown that, comparing with nonrelativistic situationąAwhen, the relativistic most probable magnetic susceptibility and the relativistic critical value of particle number have not changed. When ąAthe relativistic effects make the system display paramagnetism easily and susceptibility increase,but the relativistic effects also amplified the impact of the interaction on the magnetic susceptibility.

PACS: 05.30.-d, 51.60.+a

Key words: Fermi gas, the relativistic effect, paramagnetism

1 Introduction

In recent years, the discovering of neutrino mass interests one in theoretical research of the relativistic Fermi gas. The rest mass of a neutrino is one of the million less than that of an electron. For a system which consists of these small particles, the relativistic effect on the statistic properties of the system needs to be considered even if under the condition of low temperatures. However, the most studies are thermodynamic properties of Fermi systems [1–5], the studies on magnetism of Fermi systems for finite number of particles are much few, and the paramagnetic researches of considering the relativistic effects have not been

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reported. Xiong *et al.* [6] and Dong [7] have respectively investigated Pauli's paramagnetism of an ideal and a weakly interacting Fermi gas with finite number of particles trapped in a weakly magnetic field, and have given the respective critical value of magnetic field. Men *et al.* [8, 9] respectively investigated relativistic thermodynamic properties and stability of a weakly interacting Fermi gas in a weak magnetic filed. For the Fermi system, these researches are important to deep understand magnetic filed and interparticle interactions.

In the present paper, by considering the relativistic effects and using the probability distribution function, we will continually study the paramagnetism of a weakly interacting Fermi system for finite number of particles, and give the relativistic most probable susceptibility as well as the average magnetic susceptibility of system. At the same time, we also give the relativistic critical values of particle number of the system.

2 The relativistic most probable magnetic susceptibility of the system

We study an imperfect gas of spin-1/2 fermions, with volume V and particle number N, confined in an homogenous weak magnetic field $B=B_Z$. It has been derived that, if s-wave considered only, the energy eigenvalue of the system can be written as [10]

$$E=\sum_p (n_p^+ + n_p^-)\varepsilon_p + \frac{\alpha}{V}N^+N^- - (N^+ - N^-)\mu B, \tag{1}$$

where μ is μ_B, the magnetic moment of fermions, $\alpha = 4\pi a\hbar^2/m$ is the parameter of interactions, a is s-wave scattering length, $n_p^+(n_p^-)$ is the number of particles in the states of momentum p with spin-up(spin-down), $N^+(N^-)$ is the total number of particles with spin-up(spin-down). Considering the relativistic effect, the energy of a single particle is expressed as

$$\varepsilon_p = mc^2\sqrt{1+\frac{p^2}{m^2c^2}} - mc^2. \tag{2}$$

Based on the means which deal with nonideal Bose gases proposed by Huang, Yang *et al.* [11, 12], we introduce $A_0(\xi)$ and consider it as the relativistic free energy of the system of ξ spinless and noninteracting fermions without external potential in volume V

$$A_0(\xi) = -\frac{1}{\beta}\ln\sum_{\{\xi_p\}}\exp\left(-\beta\sum_p \xi_p\varepsilon_p\right), \tag{3}$$

followed by the method of Ref. [6], through calculation, the probability distribution function of N^+ consideration the relativistic effect can be given as

$$G(\beta,N^+) = A_n\exp\int_{\overline{N}^+}^{N^+}\delta(\beta,N^+)dN^+ = A_n\exp\left(-\left(\frac{\beta\sigma}{N} - \frac{2a\lambda^2}{V}\right)\left(N^+ - \overline{N}^+\right)^2\right), \tag{4}$$