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THE ADAPTIVE IMMERSED INTERFACE FINITE ELEMENT METHOD FOR ELASTICITY INTERFACE PROBLEMS*

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Abstract

In this paper, we propose adaptive finite element methods with error control for solving elasticity problems with discontinuous coefficients. The meshes in the methods do not need to fit the interfaces. We establish a residual-based a posteriori error estimate which is λ-independent multiplicative constants; the Lamé constant λ steers the incompressibility. The error estimators are then implemented and tested with promising numerical results which will show the competitive behavior of the adaptive algorithm.

Mathematics subject classification: 49J20, 65N30.
Key words: Adaptive finite element method, Elasticity interface problems.

1. Introduction

The interface problems which involve partial differential equations having discontinuous coefficients across certain interfaces are often encountered in fluid dynamics, electromagnetics, and materials science. Especially, the elasticity problems of multiple phase elastic materials separated by phase interfaces often arise in materials science. Two important examples of such problems occur in the microstructural evolution of precipitates in an elastic matrix due to the diffusion of concentration and in the morphological instability due to stress-driven surface diffusion in solid thin films, cf. e.g., [1–3] and the references therein. The understanding of these physical processes is crucial to improve material stability properties, and in turn to develop new and advanced materials that have many applications in automobile manufacture, aircraft industries, and modern communication technologies.

However, solving such elasticity problems are often very difficult due to complicated geometries, multiple components that appear in these problems. Moreover, the low global regularity and the irregular geometry of the interface, the standard numerical methods which are efficient for smooth solutions usually lead to loss in accuracy across the interface. For these reasons, there has been a great interest recently, in materials science, scientific computing, and applied mathematics communities, in developing efficient and accurate numerical techniques for elasticity problems with interfaces.

In this paper, we propose the elasticity problems with interfaces in which the physical parameters are discontinuous across an interface. Let Ω be a bounded domain in R^3 which is divided into two subdomains Ω_1, Ω_2 by some surface $\Gamma = \bar{\Omega}_1 \cap \bar{\Omega}_2$. The problem we will consider

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is the following

$$
\begin{aligned}
& -\nabla \cdot \sigma(\mathbf{u}) = \mathbf{f} \qquad \text{in } \Omega_1 \cup \Omega_2, && (1.1)\\
& [\mathbf{u}]_\Gamma = 0, && (1.2)\\
& [\sigma(\mathbf{u})\mathbf{n}]_\Gamma = 0, && (1.3)\\
& \mathbf{u} = 0 \qquad \text{on } \Gamma_D, && (1.4)\\
& \sigma(\mathbf{u})\mathbf{n} = \mathbf{g} \qquad \text{on } \Gamma_N. && (1.5)
\end{aligned}
$$

Here $\sigma(\mathbf{u})$ is the stress tensor, $\mathbf{f} \in L^2(\Omega)^3$ is the given body force and $\mathbf{g} \in L^2(\Gamma_N)^3$ is the surface load. $\mathbf{u}$ is the displacement field, $[\mathbf{v}]_\Gamma$ stands for the jump of a quantity $\mathbf{v}$ across the interface Γ and $\mathbf{n}$ denotes the unit outer normal to the boundary of one subdomain, say $\partial\Omega_1$. The Lipschitz boundary $\partial\Omega$ consists of a Neumann part Γ_N with positive surface measure and a Dirichlet part Γ_D.

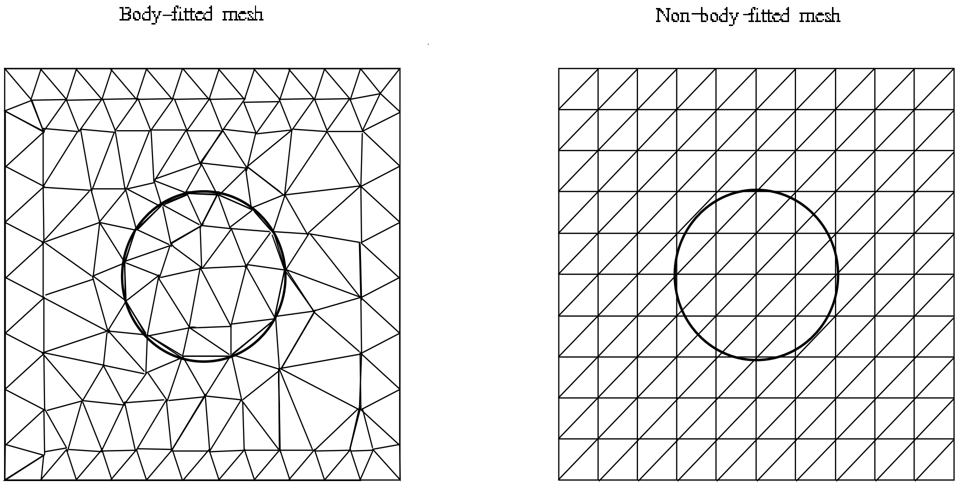

Fig. 1.1. The body-fitted and non-body-fitted mesh in 2D.

We assume that the material is isotropic. So, the stress-strain relation is given by

$$\sigma(\mathbf{u}) = 2\mu\varepsilon(\mathbf{u}) + \lambda tr(\varepsilon(\mathbf{u}))I,$$

where $\varepsilon(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ is the linear strain and I is the 3×3 identity matrix.

$$\mu = \frac{E}{2(1+\nu)} \qquad and \qquad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \tag{1.6}$$

are the Lamé coefficients, E is Youngs modulus, and ν is Poissons ratio. ν is dimensionless and typically ranges from 0.2 to 0.49, and is around 0.3 for most materials. So μ and λ are positive and across the interface Γ they are discontinuous. For simplicity, we assume that $\mu = \mu_i$ and $\lambda = \lambda_i$ in Ω_i for positive constants μ_i, λ_i, $i = 1, 2$.

For elliptic interface problems, it is known that optimal or nearly optimal convergence rate can be achieved if bodyfitted finite element meshes are used, see e.g. [4, 5]. In a body-fitted mesh, the sides (2D) or the edges (3D) intersect with the interface only through the vertices, see Fig. 1.1. Unfortunately, it is usually a nontrivial and time-consuming task to construct good