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CASCADIC MULTIGRID METHOD FOR THE MORTAR ELEMENT METHOD FOR P1 NONCONFORMING ELEMENT *1)

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Abstract

In this paper, we consider the cascadic multigrid method for the mortar P_1 nonconforming element which is used to solve the Poisson equation and prove that the cascadic conjugate gradient method is accurate with optimal complexity.

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Key words: Mortar P_1 nonconforming element, Cascadic multigrid method.

1. Introduction

The mortar finite element method was first introduced by Bernardi, Maday and Patera in [3]. From then on, this method as a special nonconforming domain decomposition technique has aroused many researchers' attention because different types of discretizations can be employed in different parts of the computational domain. We refer to [3] for the general presentation of the mortar element method and [1], [2], [4], [10], [12], [13], [16], [17] and [25] for details.

In the mortar element methods, the computational domain is first decomposed into a polygonal partition. The meshes on different subdomains need not match across subdomain interfaces. The basic idea of this method is to replace the strong continuity condition on the interfaces between different subdomains by a weaker one, i.e., the so called mortar condition. The mortar condition guarantees optimal discretization schemes, this is, the global discretization error is bounded by the sum of the optimal approximation errors on different subdomains.

On the other hand, Bornemann and Deuflhard [6] [7] have proposed the cascadic multigrid method. Compared with usual multigrid methods, this method requires no coarse grid corrections at all that may be viewed as a "one way" multigrid method. Another distinctive feature of this method is performing more iterations on coarser levels so as to obtain less iterations on finer level. Numerical experiments [7] show that this method is very effective. A first candidate of such a cascadic multigrid method was the cascadic conjugate gradient method, in short CCG method, which used the conjugate gradient method as basic iteration method on each level. For the second-order elliptic problem in 2D discretized by the P_1 conforming element, Bornemann and Deuflhard [6] have proved that the CCG method is accurate with optimal computational complexity. The general framework to analyze the cascadic multigrid method has been established by Shi and Xu in [21]. The cascadic multigrid method also has been applied to the elliptic problems in domain with curved boundary by Bi and Li in [5], to the Stokes problems by Braess and Dahmen in [8], to the elliptic problems in domain with re-entrant corners by Shaidurov and Tobiska in [19], to the parabolic problems by Shi and Xu in [22], to the elliptic problems for finite volume methods by Shi, Xu and Man [23], and to the semilinear elliptic problems by Timmermann in [24].

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Recently, Braess, Deuflhard and Lipnikov [9] have proposed and analyzed a subspace cascadic multigrid method for the elliptic problems with strong material jumps in the framework of the mortar mixed method. In the mortar mixed method in [9], the finite element spaces associated with the subdomain grids and the interface between the subdomains are the P_1 conforming finite element space and the piecewise constant functions space.

Marcinkowski [17] has considered the mortar element method for P_1 nonconforming element, obtained the optimal order error estimate in H^1-norm, and proposed an additive Schwarz method to solve the system of linear equations. Xu and Chen [25] have considered the multigrid algorithm for the mortar element method for P_1 nonconforming element and proved that the W-cycle multigrid is optimal, i.e., the convergence rate is independent of the mesh size and mesh level, and constructed a variable V-cycle multigrid preconditioner which results in a preconditioned system with uniformly bounded condition number.

In this paper, for the Poisson problem, we consider the cascadic multigrid method for the mortar P_1 nonconforming element and show that the CCG method is accurate with optimal computational complexity.

The remainder of this paper is organized as follows. In Section 2, we introduce notation and the mortar P_1 nonconforming element. In Section 3, we obtain the optimal order error estimate in L^2-norm. The cascadic multigrid method is considered in Section 4. In Section 5, we present numerical experiments showing the optimality of our theoretical results.

In this paper, C denotes the positive constant independent of the meshsize and the number of the levels which will be stated below and may be different at different occurrence.

2. Mortar P_1 Nonconforming Element

In this section, we provide some notation and preliminaries. We consider the following Poisson problem:

$$-\triangle u = f, \quad \text{in} \quad \Omega, \tag{2.1}$$

$$u = 0, \quad \text{on} \quad \partial\Omega, \tag{2.2}$$

where Ω is a bounded convex polygonal domain and $f \in L^2(\Omega)$.

The variational form of the problem (2.1)-(2.2) is to find $u \in H_0^1(\Omega)$ such that

$$a(u,v) = (f,v), \quad \forall v \in H_0^1(\Omega), \tag{2.3}$$

where

$$a(u,v) = \int_\Omega \nabla u \cdot \nabla v \mathrm{d}x, \quad (f,v) = \int_\Omega f v \mathrm{d}x, \quad \forall u,v \in H_0^1(\Omega). \tag{2.4}$$

In this paper, we will need to assume the H^2-regularity on the problem (2.1)-(2.2), i.e., for any $f \in L^2(\Omega)$, the problem (2.1)-(2.2) has a unique solution $u \in H^2(\Omega) \cap H_0^1(\Omega)$ satisfying $\|u\|_{H^2(\Omega)} \le C\|f\|_{L^2(\Omega)}$. The definitions of integral and fractional Sobolev spaces and associated norms are the same as those in [14].

In this paper, we consider a geometrically conforming version of the mortar element method, i.e., Ω is divided into non-overlapping polygonal subdomains Ω_i, $\overline{\Omega} = \cup_{i=1}^N \overline{\Omega}_i$, where $\overline{\Omega}_i \cap \overline{\Omega}_j$ is an empty set or an edge or a vertex for $i \neq j$.

Each subdomain Ω_i is triangulated to produce a regular mesh $\mathcal{T}_h^i$ with mesh parameter h_i, where h_i is the largest diameter of the elements in $\mathcal{T}_h^i$. The triangulations of subdomains generally do not align at the subdomain interfaces. Let Γ_{ij} denote the open straight line segment which is common to $\overline{\Omega}_i$ and $\overline{\Omega}_j$ and Γ denote the union of all interfaces between the subdomains, i.e., $\Gamma = \cup\partial\Omega_i \backslash \partial\Omega$. We assume that the endpoints of each interface segment in Γ are vertices of $\mathcal{T}_h^i$ and $\mathcal{T}_h^j$. Let $\mathcal{T}_h$ denote the global mesh $\cup_i \mathcal{T}_h^i$ with $h = \max_{1\le i\le N} h_i$.