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A DIRECT SEARCH FRAME-BASED ADAPTIVE BARZILAI-BORWEIN METHOD*

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Abstract

This paper proposes a direct search frame-based adaptive Barzilai-Borwein method for unconstrained minimization. The method is based on the framework of frame-based algorithms proposed by Coope and Price, but we use the strategy of ABB method and the rotational minimal positive basis to reduce the computation work at each iteration. Under some mild assumptions, the convergence of this approach will be established. Through five hundreds and twenty numerical tests using the CUTEr test problem library, we show that the proposed method is promising.

Mathematics subject classification: 90C56, 90C30, 65K05.
Key words: Direct search, Rotational minimal positive basis, Adaptive Barzilai-Borwein method.

1. Introduction

We consider the unconstrained optimization problem

$$\min f(x), \quad x \in \mathbb{R}^n,$$

where the function $f : \mathbb{R}^n \to \mathbb{R}$ is assumed to be continuously differentiable on $\mathbb{R}^n$ and the derivative information is unavailable or unreliable. Direct search methods are subset of derivative-free methods, which are the most important and challenging areas in computational science and engineering.

In the 1950s, Box and Wilson [5] introduced direct search method related to coordinate search, while Hooke and Jeeves [10] first used the term of direct search method. In the 1990s, Torczon [18, 19] established the convergence theory firstly, which triggered the interest of the numerical optimization community. According to the work of Torczon, *et al.* [2] proposed a general framework for direct search method. In particular, some classical and modern direct search methods were introduced by Kolda, *et al.* [14].

In the 2000s, Coope and Price [6,7] study a class of direct search unconstrained optimization algorithms employing fragments of grids called frames, and they prove convergence under some mild conditions. In 2004, Coope and Price [8] presented a direct search frame-based conjugate gradients algorithm (MAPRP for short). The algorithm performs finite direct search conjugate

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gradient steps, and then resets. At each reset, the algorithm get the estimate of the first and second gradients according to the fixed maximal positive basis, then obtain next search direction by applying the modified PRP formula. Finally, a parabolic lines search is designed to locate a line local minimizer. Numerical results show that the algorithm is effective. The application of Coope-Price's direct search framework could be seen in [15], which employed Coope-Price's framework and recently developed descent conjugate gradient methods.

In 1988, Barzilai and Borwein [11] proposed BB method, which used the negative gradient direction as search direction and calculated the step length according to the secant equation. Two different secant equation deduced the large step length α^{LBB} and the small length α^{SBB}. BB method achieved better performance and cheaper computation than the steepest descent method in numerical experiments. Because of the simplicity and efficiency, BB method triggered a lot of research on the gradient method in recent decades, see, e.g. [3, 20, 21]. And it seems that up to now the good method is the ABB method, which is proposed by Zhou, Gao and Dai [3]. At every iteration, ABB method choose a large step size or a small step size adaptively. Extensive numerical experiments indicate that ABB method surpass the PRP method for many unconstrained optimization problems.

Motivated by the efficiency of the ABB method, we propose a new direct search method, which combines the frame-based strategies and the ABB method. Because the ABB method only needs the first gradient information, our method employs the minimal positive basis. In each iteration, the minimal positive basis just need to compute the $n+1$ function value, while the maximal positive basis require evaluate the $2n$ function value. So the computation work of the new direct search method is about half of the MAPRP for approximate gradient. In addition, benefit from the characteristics of ABB method, we only require calculate step length by α^{LBB} and α^{SBB}, without the need for lines search. Further more, we rotate the minimal positive basis according to the local topography of objective function, which make our method more effective in practice. The convergence is proved under some mild conditions. Some numerical results show that our direct search method is promising.

This paper is organized as follows. In Section 2, we present some basic notions for frame, and describe our direct search method. In Section 3, we prove the convergence of the proposed method. In Section 4, numerical results show the efficiency of method derived in this paper compared to MAPRP [8] and Nelder-Mead [1]. Concluding remarks are given in Section 5. The default norm used in this paper is Euclidean.

2. The New Direct Search Method

In order to introduce our method, we give some concepts about positive basis, which can be found in [1].

Definition 2.1. *A positive span of a set of vectors* $\{v_1, \cdots, v_s\}$ *in* $\mathbb{R}^n$ *is the convex cone*

$$\Big\{v = a_1 v_1 + \cdots + a_s v_s, v \in \mathbb{R}^n, a_i \geq 0, i = 1, \cdots, s\Big\}.$$

A positive spanning set in $\mathbb{R}^n$ *is a set of vectors whose positive span is* $\mathbb{R}^n$.

Definition 2.2. *A positive basis* $\mathcal{V}$ *in* $\mathbb{R}^n$ *is a set of vectors with the following two properties:*

- *(i) every vector in* $\mathbb{R}^n$ *is a linear combination of the members of* $\mathcal{V}$*, where all coefficients of the linear combination are non-negative; and*