Multiple-Body Collision Algorithms for Computational Simulation of High-Speed Air-Delivered Systems

Robert E. Harris¹,*, Peter A. Liever¹, Edward A. Luke² and Jonathan G. Dudley³

¹ Aerospace & Defense Division, CFD Research Corporation, Huntsville, AL 35806, USA.
² Computer Science Department, Mississippi State University, Mississippi State, MS 39762, USA.
³ Air Force Research Laboratory, Eglin AFB, FL 32542-6810, USA.

Received 23 November 2013; Accepted (in revised version) 29 September 2014

Communicated by Rho Shin Myong

Abstract. Currently, there exists a lack of confidence in the computational simulation of multiple body high-speed air delivered systems. Of particular interest is the ability to accurately predict the dispersion pattern of these systems under various deployment configurations. Classical engineering-level methods may not be able to predict these patterns with adequate confidence due primarily to accuracy errors attributable to reduced order modeling. In the current work, a new collision modeling capability has been developed to enable multiple-body proximate-flight simulation in the Loci/CHEM framework. This approach maintains high-fidelity aerodynamics and incorporates six degrees of freedom modeling with collision response, and is well-suited for simulation of a large number of projectiles. The proposed simulation system is intended to capture the strong interaction phase early in the projectile deployment, with subsequent transfer of projectile positions and flight states to the more economical engineering-level methods. Collisions between rigid bodies are modeled using an impulse-based approach with either an iterative propagation method or a simultaneous method. The latter is shown to be more accurate and robust for cases involving multiple simultaneous collisions as it eliminates the need to sort and resolve the collisions sequentially. The implementation of both the collision detection methodology and impact mechanics are described in detail with validation studies to demonstrate the efficiency and accuracy of the developed technologies. The studies chronologically detail the findings for simulating simple impacts and collisions between multiple bodies with aerodynamic interference effects.

*Corresponding author. Email addresses: reh@cfdrc.com (R. E. Harris), pal@cfdrc.com (P. A. Liever), luke@cse.msstate.edu (E. A. Luke), jonathan.dudley@eglin.af.mil (J. G. Dudley)

http://www.global-sci.com/564
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AMS subject classifications: 76

Key words: Submunition, projectile, ballistic, CFD, 6-DoF, solid body collision, contact force, hypersonic, aerodynamic, interference effects.

Nomenclature

\( a, b \) Center of gravity locations for colliding bodies
\( [A] \) Matrix of mass and inertial properties for colliding system
\( \{B\} \) Vector of motion properties for colliding system
\( e \) Coefficient of restitution for collision
\( J \) Inertia tensor expressed in inertial reference frame
\( L \) Length of projectile dart
\( m \) Mass
\( M \) Total number of bodies in contact during simultaneous collision
\( M_C \) Total number of contact points during simultaneous collision
\( \hat{n} \) Unit normal vector at contact point
\( N \) Number of bodies in active contact during simultaneous collision
\( N_C \) Number of active contact points during simultaneous collision
\( p \) Collision contact point
\( \{P\} \) Vector of collision impulses for colliding system
\( \vec{r} \) Distance vector from mass center to contact point
\( V_{REL} \) Normal relative velocity at contact point
\( V_0 \) Initial velocity of colliding sphere
\( \vec{V} \) Velocity of mass center
\( \vec{\omega} \) Angular velocity
\( z_0 \) Distance from dart CG to ground for static dart drop case
\( \alpha_0 \) Angle between dart axis and horizontal for static dart drop case
\( \epsilon \) Separation distance between two colliding spheres

1 Introduction

A variety of weapon systems consist of carrier vehicles that dispense submunitions over a target area such as a mine field. The submunitions are typically unguided darts that rely on aerodynamic forces for achieving stable projectile flight with high kinetic energy
and even dispersion over the target area. Fig. 1 shows examples of such delivery systems that range from small numbers to thousands of submunitions. Strong interference effects between the submunitions and carrier vehicle flowfield are observed which result in unpredictable motion in the interaction of the cloud of projectiles.

It is highly desirable that the darts disperse in a manner that is predictable, insensitive to variations in the release conditions, and result in a uniform dispersion pattern. At transonic and supersonic speeds aerodynamic interactions between the shock wave system of the parent vehicle and the darts can greatly alter the dart trajectories. In addition, the darts may initially be carried behind the nose section or within a bay of the parent vehicle. Interaction of the darts with the resulting weapons bay sheallayer will induce large scale destabilizing forces on the light-weight projectiles. Aerodynamic configurations involving the release of large numbers of bodies present the computational community with unique challenges in simulation and analysis. These challenges include an unsteady flow field, time-dependent behavior of bodies in relative motion, the requirement to model multiple bodies moving under aerodynamic forces over a wide range of ambient conditions and the requirement to accommodate the possibility of collisions between the individual darts and between the darts and parent vehicle.

Computational fluid dynamics (CFD) simulations of clouds of projectiles have been performed with overset grid tools such as the OVERFLOW-2 code [1, 2], which featured rudimentary implementations of collision effects. These simulations provided initial useful insight into the complex aerodynamic and collision driven interference between the tightly packed projectiles. However, scalability to large-scale collision simulations on modern DOD computer clusters has only been pursued for simulations with up to approximately 400 projectiles over short time spans with roadblocks encountered in further scalability for memory and parallelism needed to extend the simulations to larger numbers on projectiles. To improve computational efficiency, and to make this approach practical for designing weapon system operations, improvements are needed in the scalability
of the coupled CFD/6-DoF/Collision model simulations for performance on massively parallel computing clusters. This paper describes the development of a highly scalable simulation tool for coupled CFD/6-DoF/Collision modeling with the potential to provide a dramatic increase in the simulation scalability on these systems. This makes it possible to model the highly complex interference between submunitions in the early stages of the dispensing process. The main focus of this effort is to incorporate a robust and accurate solid body collision modeling methodology into a highly scalable coupled 6-DoF/overset CFD simulation framework to enable simulation of the simultaneous collision and aerodynamic interference among large numbers of projectiles. The simulation tool under development is based on the Loci/CHEM coupled 6-DoF/CFD flow solver that is developed with support from various sources from government, academia, and industry [3]. The CHEM solver is built upon the Loci rule based framework that features inherent parallelism and automatic decomposition for very large simulations on supercomputer systems [4, 5]. Loci accomplishes this using a run-time logical deduction engine, which provides mechanisms for intra-application resources management. The adopted framework automatically parallelizes and distributes the simulation on the computer resources, even for complex overset grid 6-DoF simulations with chemical reaction effects. Integrating a body collision and contact force module into the existing 6-DoF simulation environment enables the complete analysis of the aerodynamic and collision interference field for groups of projectiles. The extreme scalability of the Loci framework provides a unique enabling technology to extend the multi-body interference simulation to very large numbers of bodies. The ultimate aim of this effort is to develop a computational toolset that enables the simulation of extreme cases such as those shown in Fig. 1 with thousands of projectiles in close proximity.

This paper is organized as follows. In Section 2 we present an overview of the Loci framework and the Loci/CHEM flow solver. The approaches employed for rigid body collision detection and impulse-based collision resolution are detailed in Section 3. Numerical results including accuracy studies of collision resolution capabilities for single and multiple-body simultaneous impacts are presented in Section 4. Additionally, Section IV includes scalability studies for: 1) multiple submunition darts in proximate flight; 2) results of static dart drop simulations with qualitative comparisons to test data; and 3) demonstrations of the complete coupled 6-DoF/CFD simulation system with collision mechanics for pitch-induced collisions between clusters of submunition darts flying in proximity at supersonic speeds. Finally, we conclude with a discussion of ongoing work in Section 5.

2 Overview of Loci framework and Loci/CHEM flow solver

The Loci framework [6–8] was originally developed in the late 1990s with funding from the National Science Foundation with the goal of simplifying the development of complex numerical models that can take advantage of massively parallel high end computing
systems. The framework provides a rule-based programming model whereby an application is described in terms of a collection of simple computational kernels. The Loci framework can assemble these kernels and optimize their scheduling on parallel high performance architectures. In addition, the framework is able to detect common programming errors by verifying that the algorithm conforms to a simple logical model. As a result, the Loci framework makes an excellent platform for the development and integration of a wide range of computational models. The framework supports the development of run-time loadable modules that allow Loci applications to be extended to support new physics and models with ease. Additionally, the verification capabilities of the Loci framework can provide assurances that the composition of these models satisfies rules of internal logical consistency.

The Loci/CHEM code [3, 9] was originally developed as a technology demonstrator for the Loci framework and has become a mature software for complex multi-physics simulations. The CHEM solver is a density-based Navier-Stokes solver employing high-resolution approximate Riemann solvers implemented for multi-component mixing and chemically reacting flows, and implicit time integration. These approaches make the CHEM solver very well suited for compressible flow simulation. In addition, the core algorithms have been extended to accurately model flows at low speeds through the use of preconditioning techniques. The code has a variety of turbulence models including RANS and hybrid RANS/LES turbulence model treatments that include high-speed compressibility corrections [10]. Loci/CHEM numerical models have been demonstrated to be at least second order accurate in space and time through rigorous verification using the Method of Manufactured Solutions (MMS) [11]. Loci/CHEM supports adaptive mesh refinement [12], simulations of complex equations-of-state including cryogenic fluids [3], multiphase simulations of dispersed liquid and solid particulates using both Lagrangian and Eulerian approaches [13], conjugate heat transfer through solids [14], fluid structure interaction modeling [15], and non-gray radiative heat transfer associated with both, gas and particulate phases [16].

The Loci/CHEM solver infrastructure supports overset meshes for efficient simulation of flows over complex geometry and moving boundary problems including prescribed or unconstrained 6-DoF motion. In this methodology, a grid is generated about each body independently without concern for other components. Then cells are removed from each grid such that the simulation space is covered. In the technique employed in Loci/CHEM, the medial surface (the surface of equal distance between two objects) is used to determine the geometry for hole-cutting. Interpolation is provided for cells in the immediate vicinity of the medial surface using a cloud of points interpolation method. This interpolation method finds the smallest containing tetrahedra that can be formed from donor cells in the neighborhood of the interpolating point utilizing a linear interpolation function. As part of the medial surface hole cutting procedure, interpolating cells may be found to lie inside one of the simulated bodies and in this case the cell is marked for special “orphan” interpolation. For points marked as orphan, neighboring cell data is used to extrapolate flow conditions using a no-flow slip-wall condition at the boundary.
surface. In addition, any flux computed between an active cell and an orphan cell is adjusted to have an identically zero mass flux guaranteeing a no-flow condition. Therefore, when objects are in close proximity, the methodology can automatically cut the independent mesh to provide a unified mesh for numerical solution, without the need for any user input related to the hole-cutting.

3 Impulse-based modeling of collision mechanics

In the current work, we seek to accurately model the flow and collision response for systems of many projectiles in close proximity that are being dispensed at high speed. In recent years, numerous research efforts have been undertaken to facilitate modeling and simulation of problems involving multiple bodies in proximate flight. Keller [17] and Stronge [18] outlined a general procedure for modeling rigid body collisions including rough surface effects using Coulomb’s law of friction. This procedure was later implemented by Wei [19] in the FLOW-3D solver to complement existing coupled 6-DoF/CFD capabilities. While this approach attempts to include all relevant physics, it can be computationally expensive due to an iterative approach that can necessitate solution of a non-linear system of equations at each iteration. In addition, numerical implementation of the procedure can be very time consuming.

In recent work tailored to multiple-body proximate flight simulation for large numbers of bodies, researchers have presented more straightforward approaches that neglect frictional forces. The advantage is that a closed-form expression for the collision impulse is available for single point collisions. This approach was recently implemented by Meakin [20] as an enhancement to the coupled 6-DoF/CFD capabilities of the OVERFLOW-2 code. Meakin [21] later extended this approach to handle multiple simultaneous collisions by migration to an iterative approach deemed the propagation method. In this approach, simultaneous collisions between multiple bodies are addressed through sequential resolution of individual collisions in the order of largest-to-smallest relative normal approach velocity. Meakin showed that while this approach prevents inter-body penetration, conserves energy (in the case of perfectly elastic collisions), and resolves single contact incidents correctly, the results can be heavily dependent on the order in which the collisions are resolved [21]. Indeed, there are countless simple scenarios for which the method produces non-physical results. For example, consider the case of a single sphere colliding with two stationary spheres as shown in Fig. 2. All spheres have the same size and mass, and are of uniform density. Sequential resolution of the collisions using the propagation method will result in sphere 2 having greater momentum than sphere 3, or vice-versa, while they should be equal due to symmetry. This is an example of a case in which the collisions at several points must be resolved simultaneously in order to obtain the physically correct result.

There are methods for simultaneous collision resolution described in the literature [22], which generally necessitate solution of a so-called linear complementarity prob-
lem (LCP) [23]. In 2005, Ermolin and Kazakov [24] presented an elegant approach for resolving simultaneous collisions that has some similarities to the propagation method. Sub-systems of active collisions, which ignore resting or sliding contact, are solved in an iterative sequence until all bodies in contact are either retreating or in resting or sliding contact. In this manner, collisions that are occurring simultaneously are not forced to occur in any specific order. This method is simple and efficient, requiring only the solution of small sparse linear systems. Since simultaneous collisions will inevitably need to be properly resolved in order to correctly predict the behavior of hundreds of projectiles in proximate flight, the method of Ermolin and Kazakov is appropriate for this application. We have implemented both this approach and the propagation approach described by Meakin [21]. Each approach will be contrasted in terms of efficiency, accuracy and robustness for selected test cases.

Rigid body collision modeling is generally based on the assumption that all colliding bodies experience negligible deformation during collision and the collisions result in instantaneous changes in velocity. The collision resolution methods presented in the literature are predominantly based on computing the collision impulses and using these impulses to update the linear and angular velocities of the bodies after the collisions. The propagation and simultaneous methods of collision resolution are both impulse-based approaches.

### 3.1 Propagation method of collision resolution

The mathematical formulation for the propagation method of collision resolution is derived for a collision between two bodies. Collisions between multiple bodies are modeled using an iterative application of this method on consecutive collisions until all bodies are either in resting or sliding contact. Consider two colliding bodies, \( a \) and \( b \), as shown in the schematic in Fig. 3, where the bodies are in contact with each other at the contact point \( p \), and a unit normal vector to the common tangent plane between the bodies at contact point, which points into body \( a \), is given by \( \hat{n} \). The masses of bodies \( a \) and \( b \) are \( m_a \) and \( m_b \), respectively, and the inertia tensors expressed in the inertial frame for bodies \( a \) and \( b \), are \( J_a \) and \( J_b \), respectively. These are readily obtained by applying a transformation from
Figure 3: Schematic showing two bodies in contact with each other at a single point.

the body-centric frame to the inertial frame. At the instant of contact, the translational velocities at the body mass centers are \( \vec{V}_a \) and \( \vec{V}_b \), and the rotational velocities are \( \vec{\omega}_a \) and \( \vec{\omega}_b \). The velocities of each body at the contact point \( p \) are given by

\[
\vec{V}^p_a = \vec{V}_a + \vec{\omega}_a \times \vec{r}_a, \quad \vec{V}^p_b = \vec{V}_b + \vec{\omega}_b \times \vec{r}_b, \tag{3.1}
\]

where \( \vec{r}_a \) and \( \vec{r}_b \) are distance vectors from the mass centers of body \( a \) and \( b \) to the contact point, respectively, and \( \vec{V}^p_a \) and \( \vec{V}^p_b \) are the velocities of bodies \( a \) and \( b \), respectively, at the contact point. Since frictional forces are neglected in this formulation, the only component of the velocity that affects the collision dynamics is that normal to the common tangent plane at the contact point. The normal relative velocity at the contact point immediately before impact is given by

\[
V_{REL} = (\vec{V}^p_a - \vec{V}^p_b) \cdot \hat{n}. \tag{3.2}
\]

Since \( \hat{n} \) always points from body \( b \) toward body \( a \), the sign of \( V_{REL} \) indicates the status of the collision. Namely, \( V_{REL} < 0 \) indicates that the bodies are advancing toward each other and penetration is eminent, while \( V_{REL} > 0 \) indicates that the bodies are retreating, and \( V_{REL} = 0 \) indicates that the bodies are either in resting contact or sliding tangent to the contact plane. Rigid body collisions result in a reaction impulse that changes the translational and rotational velocities of the colliding bodies. Following the formulation of Meakin [21], we arrive at the following closed-form expression for the reaction impulse magnitude

\[
P = -\frac{(1+e)V_{REL}}{m^{-1}_a + m^{-1}_b + \left\{ f^{-1}_a (\vec{r}_a \times \hat{n}) \times \vec{r}_a \right\} \cdot \hat{n} + \left\{ f^{-1}_b (\vec{r}_b \times \hat{n}) \times \vec{r}_b \right\} \cdot \hat{n}}, \tag{3.3}
\]

where \( e \) is the coefficient of restitution for the collision. A value of \( e = 1 \) indicates a perfectly elastic collision, while a value of \( e = 0 \) indicates a perfectly inelastic (or plastic) collision, and values of \( 0 < e < 1 \) indicate partially plastic collisions. Once the reaction impulse magnitude \( P \) is determined, the post-impact translational and rotational velocities of the colliding bodies are computed as

\[
\begin{align*}
\vec{V}_a' &= \vec{V}_a + m^{-1}_a P \hat{n}, & \vec{\omega}_a' &= \vec{\omega}_a + f^{-1}_a \vec{r}_a \times P \hat{n}, \tag{3.4a} \\
\vec{V}_b' &= \vec{V}_b - m^{-1}_b P \hat{n}, & \vec{\omega}_b' &= \vec{\omega}_b - f^{-1}_b \vec{r}_b \times P \hat{n}. \tag{3.4b}
\end{align*}
\]
Once a list of contact points is determined from the collision detection method, the collisions are sorted in order from largest-to-smallest (negative) relative normal velocity. The above approach is then applied iteratively in this order until all bodies are either in resting contact or sliding tangentially at the contact point. As mentioned earlier, a disadvantage of this approach is that it may result in collision dynamics that exhibit order dependencies.

3.2 Simultaneous method of collision resolution

In the simultaneous collision approach described by Ermolin and Kazakov [24] the collisions are separated into two phases. The first phase is a perfectly inelastic collision while the second phase is one of restoration in which the coefficient of restitution for each collision is taken into account. Order dependencies are avoided by iteratively solving subsystems of active equations until all contact points are either in resting or sliding contact. This approach is applicable to simultaneous collisions between multiple bodies as well as multiple collisions between individual bodies, such as those encountered during both line and planar collisions. It should be noted that the formulation presented in Ermolin and Kazakov [24] contains errors in the mathematical notation. An unpublished document (in Russian) [25] that contains the correct mathematical formulation was obtained through personal communication with the first author of Ermolin and Kazakov [24], and the formulation presented below is consistent with that document.

Consider a system of \( M \) bodies that are in contact at \( M_C \) distinct contact points at a particular instance in time. Using the computed normal relative velocity at each contact point, denote contact points at which bodies are approaching each other as active contact points, or active collisions. Segregating the contact points in this manner yields a subset of \( N \) bodies that are in contact at \( N_C \) distinct contact points. Denote the pairs of colliding bodies at active contact points as \((a_i, b_j)\) and unit normal vectors pointing from \( a_i \) to \( b_j \) as \( \hat{n}_i \) for \( i = 1, \cdots, N_C \). The body masses are denoted as \( m_i \) and the inertia tensors expressed in the inertial frame are denoted as \( J_i \), for \( i = 1, \cdots, N \). Unlike the propagation method, the simultaneous method presented here considers the entire system of active contact points as a simultaneous system of equations \([A]\{P\} = \{B\}\), the solution of which yields the collision impulses \( P_i \) for \( i = 1, \cdots, N_C \). The components of \([A]\) and \([B]\) are given by

\[
A_{ij} = A^1_{ij} - A^2_{ij} \quad i = 1, \cdots, N_C; \quad j = 1, \cdots, N_C, \tag{3.5a}
\]

\[
A^1_{ij} = \text{sgn}(a_i, i) \left[ m_i^{-1} \hat{n}_i + \left\{ J_i^{-1} (\vec{r}_j - \vec{r}_a) \times \hat{n}_j \right\} \times (\vec{r}_i - \vec{r}_a) \right] \cdot \hat{n}_i, \quad i = 1, \cdots, N_C; \quad j = 1, \cdots, N_C, \tag{3.5b}
\]

\[
A^2_{ij} = \text{sgn}(b_j, i) \left[ m_i^{-1} \hat{n}_i + \left\{ J_j^{-1} (\vec{r}_j - \vec{r}_b) \times \hat{n}_j \right\} \times (\vec{r}_i - \vec{r}_b) \right] \cdot \hat{n}_i, \quad i = 1, \cdots, N_C; \quad j = 1, \cdots, N_C, \tag{3.5c}
\]

\[
B_i = \left\{ \vec{V}_{b_j} + \vec{\omega}_{b_j} \times (\vec{r}_i - \vec{r}_b) \right\} - \left\{ \vec{V}_{a_i} + \vec{\omega}_{a_i} \times (\vec{r}_i - \vec{r}_a) \right\} \cdot \hat{n}_i, \quad i = 1, \cdots, N_C, \tag{3.5d}
\]
where $\vec{r}_j$ is the location of contact point $j$, $\vec{r}_{a_i}$ is the location of the center-of-gravity for the first body at contact point $i$, $\vec{V}_{a_i}$ is the velocity of the first body at contact point $i$, $\vec{\omega}_{a_i}$ is the angular velocity of the first body at contact point $i$, and

$$\text{sgn}(i,j) = \begin{cases} -1, & i = a_j, \\ 1, & i = b_j, \\ 0, & \text{otherwise}. \end{cases}$$  (3.6)

Quantities for the second body at a particular contact point are similarly denoted with a $b$ subscript. The ordering of the bodies is not important but it is important to note that the normal vector $\hat{n}_i$ points from the first body to the second body. Solution of the system of equations $[A]\{P\} = \{B\}$ yields the impulses for a perfectly inelastic set of collisions. A restoration step is then applied to account for the coefficients of restitution $e_i$, which can differ between contact points. The final collision impulses are then given by

$$P'_i = (1 + e_i)P_i; \quad e_i = e_i(a_i,b_i).$$  (3.7)

The impulses are known for the set of active contact points so both the kinematic velocities and angular velocities of the associated bodies may be computed by

$$\vec{V}'_i = \vec{V}_i + m_i^{-1} \sum_{j=1}^{N_c} \text{sgn}(i,j) P_j \hat{n}_j, \quad i = 1,\ldots,N,$$  (3.8a)

$$\vec{\omega}'_i = \vec{\omega}_i + J_i^{-1} \sum_{j=1}^{N_c} \text{sgn}(i,j) P_j (\vec{r}_j - \vec{r}_i) \times \hat{n}_j, \quad i = 1,\ldots,N.$$  (3.8b)

Recall that only the $N_C$ active contact points out of the total set of $M_C$ contact points at a particular instance in time are considered in the above formulation. Once the above system of equations has been solved and the body motions have been updated via Eq. (3.8), the entire set of $M_C$ contact points is again examined for bodies that are approaching each other. A new set of $N_C$ active contact points is then defined and the above procedure is applied iteratively until all bodies are either in resting contact or sliding tangentially at the contact points.

### 3.3 Collision detection

While discussion has thus far focused solely on the collision resolution methodology, the accurate detection of the collision events themselves is a prerequisite for accurate collision resolution. Implementation of a numerical collision detection algorithm has been carried out to detect collisions among multiple bodies in proximate flight during a 6-DoF simulation. In order to detect imminent collisions between bodies, nodes on solid surfaces are traversed and a check is made to detect if these nodes are in close proximity to any other solid surface face. This step is optimized by using a search radius which is
computed by the maximum edge length plus two times the maximum velocity times the
time-step. A kd-tree-based spatial search structure is then used to identify any faces that
are within the search radius of any given mesh node. For each node-face pair, a closing
velocity is computed and it is determined if the node will intersect the face in the next
time-step. If an intersection between the face and node is predicted, the time of colli-
sion is then computed from the closing velocity, and if this time is within the collision
time-step, the node is marked as colliding with the surface. As a means to prevent inter-
body penetration, a collision time-step may be selected that is some factor larger than
the simulation time step. This is necessary because a typical simulation time step may
not be small enough to resolve collisions between bodies that are moving rapidly due to
six-degree-of-freedom motions. If a larger collision time step is used, the collisions are
actually applied when the objects are about to collide, but before the actual intersection
of the surfaces takes place. If a node collides with several faces, then the earliest collision
time takes precedence. This approach may result in the location of multiple distinct con-
tact points between individual bodies in the cases of linear or planar contact, but both the
propagation and simultaneous methods of collision resolution described above can readily
handle this situation.

4 Results

Results for several different types of test cases are presented in the following section.
Initially, the ease-of-use and superior simulation scalability benefits of the Loci/CHEM
overset 6-DoF/CFD simulation architecture are demonstrated for modeling the interference
effects between large numbers of bodies. In particular, the overset grid hole-cutting
logic is put to the test for its ability to automatically handle a large number of projectiles
in proximity and its scalability for simulation of multiple bodies on a large number of
processor. This initial testing is designed to avoid collisions and focus on the overset
technology aspects. Secondly, the newly developed collision detection and resolution ca-
pabilities are demonstrated, with a focus on the accuracy of resolving simultaneous colli-
sions among multiple bodies. These cases are carried out in a kinematics only mode, with
the flow simulation capabilities. Finally, everything is put together for a final demonstra-
tion involving multiple bodies in proximate flight with the 6-DoF/CFD and collision
capabilities enabled.

4.1 Submunition dart projectiles in proximate flight

One of the intended applications of the technology under development is the simula-
tion of a dart delivery system required to fly at supersonic and hypersonic conditions
and evenly dispense darts over a prescribed target area in order to efficiently neutral-
ize landmine fields. To demonstrate the effectiveness and benefits of the Loci/CHEM
overset capabilities for such an application, a realistic dart geometry [26,27] was used for
demonstration cases with large numbers of darts. Fig. 4 shows the dart geometry that
is modeled in the current study. The projectile center body has a total length of \( L \) and a diameter of \( 0.086L \). The tail fins have a length of \( 0.172L \), a span of \( 0.034L \) and a thickness of \( 0.0034L \). The center body is modeled as a body of revolution, as shown in Fig. 5(a), whereas the actual dart shown in Fig. 4 features various flat machined surface segments.

A computational domain for a single dart is constructed with a cylindrical outer domain boundary as shown in Fig. 5. An unstructured hybrid volume grid model is generated using the Advancing Front with Local Reconnection (AFLR3) software developed at Mississippi State University [28]. The normal spacing at the wall was chosen such that \( y^+ < 1 \) on most of the dart surface. Prism cells are generated in the viscous layer region near the wall out to a boundary layer thickness distance calculated internally by AFLR3 based on a Reynolds number of \( 10^6 \) followed by a smooth transition to the tetrahedral region. The resulting volume grid is shown in Fig. 6 in various cuts showing a crinkle cut grid. The boundary layer is well resolved with smooth transitions and outgrowth from complex corner features of the topology. The resulting volume grid contains 640,000 cells.
A box shaped outer computational domain is constructed with an evenly spaced structured Cartesian grid. The dart domain is then placed inside this outer background domain. The volume grid domains for the background and the single-dart models are generated independently and saved in the platform independent VOG format. A multi-dart model is then generated using the vogmerge Loci utility. A combined overset grid model is then generated by embedding and translating/rotating/scaling the dart domain multiple times into the background domain. The result is a single VOG-format grid model containing multiple domains. The cylindrical outer domain surfaces of each dart are used as the default hole-cutting surfaces. Specifying the overset component boundaries in the input file automatically invokes the hole cutting process and no further user intervention is required except submitting the simulation.

We now employ the above overset methodology and consider a case with nine darts in close proximity for testing the moving body CFD/6-DoF capabilities. The transformation of the single dart domain multiple times into the background domain is accomplished using the Loci domain merging functionality by simple duplicating, translating, and rotating the dart through a simple scripted input. The mass and inertial properties of the darts are taken from a previously published Navy project [29]. Each dart has a mass of 55 grams. Unsteady Navier-Stokes simulations of the dart dispersion with 6-DoF activated are commenced from an initial solution at a freestream Mach number of \( M = 3.0 \) for conditions at 10,000ft altitude. The simulation is carried out using 2nd order accurate spatial and temporal schemes, Spalart-Allmaras turbulence modeling, and the HLLE inviscid flux with Venkatakrishnan flux limiting. Fig. 7 shows surface pressures on the darts at the initial position and pressure contours in various planes which clearly show considerable aerodynamic interference between the bodies for this case. The interaction of the shock wave patterns and the resulting complex surface forces are extremely complex and vary considerably between trajectory points. The wave interference appears to be captured well even on the coarse background grid. When accounting for body collisions resulting in considerably more complex relative body orientations, the resolution of the highly three-dimensional interference patterns between bodies may become intractable on a static grid. The resulting forces on the projectiles may be sensitive

![Figure 7: Mach=3.0 simulation surface pressures and pressure contours in various planar cuts for initial position and at \( t = 0.01 \text{s} \).](image-url)
to adequate local surface pressure resolution and solution adaptive mesh refinement is obviously a highly desirable feature.

Fig. 8 shows the trajectory evolution of the dart dispersion due to the aerodynamic interference affects while Fig. 9 shows the time history of the forces experienced by several of the darts. The side forces due to the interference vary distinctly with the position of the
darts in the stack and reach levels of 80% of the axial supersonic drag forces. This results in rapid dispersion of the darts, where the dart orientation remains very much aligned with the flow due to the forward location of the dart CGs. Variations and oscillations in the side forces resulting from the interference of the time-dependent impingement of the shock waves are captured well.

Fig. 10 shows the transverse dart velocity, as well as the longitudinal and radial dart motion after release for all of the darts on the same plot. This essentially displays a trajectory envelope that encompasses all darts and clearly illustrates the spread of darts as a function of time. It is evident that strong interference pressures and moving shock systems are producing a flowfield that is dominated by complex unsteady aerodynamic interactions.

The 6-DoF simulations are performed on a multi-processor compute cluster, offering the opportunity to perform analysis on the scalability of the simulation. In production use, the overset capabilities implemented have demonstrated scalability to thousands of processors in applications run by NASA MSFC [5]. However, production uses of the code are usually not under controlled circumstances and to get a better idea of the scalability of the overset algorithm implemented in Loci/CHEM, we now consider a multi-body problem under controlled circumstances. In this case we consider the parallel simulation of 9 darts using the 6-DoF model for 500 time-steps on between 24 and 192 processors. This simulation is performed on a cluster comprised of 2 CPU nodes where each CPU is a 6 core Intel Westmere processor giving 12 cores per node. The mesh for this case contains 8.23 million cells, 0.64M for each of the nine darts and 2.47M for the background mesh.
The parallel cost given by the processor time product is now examined, where the ratio of parallel costs provides an estimate of parallel efficiency. We scale the problem from 24 processors to 192 processors. At 24 processors there are 343,000 cells per processor while at 192 processors there are just 42,000 cells per processor. The runtime and scalability analysis for the simulation is shown in Table 1. From this table we can see that running on 192 processors is performed with 76% of the efficiency of the 24 processor run, showing that the overset mesh capabilities are able to scale to a large number of processors with a relatively modest efficiency degradation showing that the code is highly scalable. As an additional test, this same case is carried out without the 6-DoF model on 96 processors. In this case, the overset hole-cutting and donor searches are only performed once, so this provides an indication of the cost of unsteady overset computations. The 96 processor static case executed in 8,548 seconds, which puts the cost of the overset processing at a modest 20% of the computation time for the 6-DoF case. It should be noted that the scalability study was performed with the 6-DoF analysis module implemented in the current Loci/CHEM release version which had not been parallelized (all bodies are combined in a single 6-DoF problem). The current effort is the first application of Loci/CHEM where 6-DoF for a large number of bodies is addressed. The work performed in our collision model implementation into the 6-DoF framework has already advanced the parallelism and scalability of this initial 6-DoF model implementation. Now individual 6-DoF models are solved in a parallel implementation for each body with considerable gains to be expected for solving the motion of large numbers of bodies.

### 4.2 Demonstration of collision modeling capabilities in Loci/CHEM

Two different collision resolution approaches have been successfully implemented during this effort. We employ the propagation method as described by Meakin [22] and compare to results obtained using the method of Ermolin and Kazakov [24] for resolution of simultaneous collisions. For single contact collisions, both methods produce identical results and therefore only results for the propagation method are presented. Analytical collision detection is used for all subsequent cases involving simple spherical geometries. Two spheres are assumed to be in contact when the distance between their centers-of-gravity is less than or equal to the sum of their radii plus a small tolerance on the order

<table>
<thead>
<tr>
<th>Processors (p)</th>
<th>Run Time (sec) (T_p)</th>
<th>Cells/Processor</th>
<th>Parallel Cost (p*T_p)</th>
<th>Cost Efficiency (cost ratio)</th>
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</thead>
<tbody>
<tr>
<td>24</td>
<td>37,289</td>
<td>343K</td>
<td>895K</td>
<td>100%</td>
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<td>6,126</td>
<td>43K</td>
<td>1176K</td>
<td>76%</td>
</tr>
</tbody>
</table>

Table 1: Parallel scalability analysis of the 9 dart problem.
of the maximum body velocity multiplied by the time step size. This tolerance is chosen based on the obvious restriction that a collision cannot be detected to within a smaller tolerance than the maximum distance through which the bodies have moved in a given time step. In practice, the precision of the collision detection algorithm strongly influences the accuracy of the collision resolution. This is mainly due to the numerical determination of the contact point locations and the fact that the normal vectors are defined with respect to faceted surface geometries.

Consider a pair of spheres, $A$ and $B$, of radii $r_A$ and $r_B$, and mass $m_A$ and $m_B$, respectively. Sphere $A$ is initially moving in the positive $x$-direction with velocity $V_0$ toward sphere $B$, which is at rest. Sphere $A$ is travelling parallel to the $x$-axis, and sphere $B$ is offset from sphere $A$ by distance $\varepsilon$ in the $y$-direction, as shown in Fig. 11. Assuming the spheres are sufficiently smooth such that frictional forces can be neglected, the post-impact motion can be determined analytically for this configuration.

The analytical post-impact velocity components of sphere $A$ are

$$u_A = V_0 - 2\frac{m_B}{(m_A + m_B)} \left( \frac{(r_A + r_B)^2 - \varepsilon^2}{(r_A + r_B)^2} \right) V_0, \quad v_A = -2\varepsilon \frac{m_B}{(m_A + m_B)} \sqrt{\frac{(r_A + r_B)^2 - \varepsilon^2}{(r_A + r_B)^2}} V_0,$$  \hspace{1cm} (4.1)

and those of sphere $B$ are

$$u_B = 2\frac{m_A}{(m_A + m_B)} \left( \frac{(r_A + r_B)^2 - \varepsilon^2}{(r_A + r_B)^2} \right) V_0, \quad v_B = 2\varepsilon \frac{m_A}{(m_A + m_B)} \sqrt{\frac{(r_A + r_B)^2 - \varepsilon^2}{(r_A + r_B)^2}} V_0.$$  \hspace{1cm} (4.2)

We have carried out two different parametric studies to verify the accuracy of the current collision resolution capability for impact between two spheres. In the first study, the masses and radii of the spheres were fixed to $m_A = m_B = 1$ kg and $r_A = r_B = 0.025$ m, respectively, and the eccentricity $\varepsilon$ was allowed to vary between 0 and 0.0125 m. In the second study, the eccentricity and radii of the spheres was fixed to $\varepsilon = 0.0125$ m and $r_A = r_B = 0.025$ m, respectively, while the masses of the spheres were $m_A = 1$ kg and $1.0 \leq m_B \leq 2.0$ kg. The simulations were carried out using a time step of $2.5e-5$ s in order to produce results that were independent of time step size. The computed post-impact velocity components for these cases are presented in Table 2 and Table 3 along with the
Table 2: Post-impact trajectory verification results for sphere-sphere collisions. Constraints: \( r_A = r_B = 0.025 \) m; \( m_A = m_B = 1 \) kg.

<table>
<thead>
<tr>
<th>( \varepsilon ) (m)</th>
<th>( u_A ) (m/s)</th>
<th>( v_A ) (m/s)</th>
<th>( u_B ) (m/s)</th>
<th>( v_B ) (m/s)</th>
<th>Max. Error (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>1.000000</td>
<td>0.000000</td>
<td>0</td>
</tr>
<tr>
<td>0.00625</td>
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<td>0.124039</td>
<td>5.38e-5</td>
</tr>
<tr>
<td>0.01250</td>
<td>0.062530</td>
<td>-0.242115</td>
<td>0.937470</td>
<td>0.242115</td>
<td>1.90e-5</td>
</tr>
</tbody>
</table>

Table 3: Post-impact trajectory verification results for sphere-sphere collisions. Constraints: \( r_A = r_B = 0.025 \) m; \( m_A = 1 \) kg; \( \varepsilon = 0.0125 \) m.

<table>
<thead>
<tr>
<th>( m_B ) (kg)</th>
<th>( u_A ) (m/s)</th>
<th>( v_A ) (m/s)</th>
<th>( u_B ) (m/s)</th>
<th>( v_B ) (m/s)</th>
<th>Max. Error (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.062530</td>
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<td>0.937470</td>
<td>0.242115</td>
<td>5.38e-5</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.124964</td>
<td>-0.290538</td>
<td>0.749976</td>
<td>0.193692</td>
<td>6.45e-5</td>
</tr>
<tr>
<td>2.0</td>
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<td>-0.322820</td>
<td>0.624980</td>
<td>0.161410</td>
<td>7.17e-5</td>
</tr>
</tbody>
</table>

Figure 12: Time history of sphere center locations. \( r_A = r_B = 0.025 \) m; \( m_A = m_B = 1 \) kg; \( \varepsilon = 0.0125 \) m.

maximum absolute difference between the computed and analytical post-impact velocity components given by Eqs. (4.1)-(4.2). Fig. 12 shows the computed sphere trajectories for one of the above cases. Differences between the computed and analytical trajectory are indistinguishable from the plot.

Next we employed both the propagation and simultaneous method for two different cases involving multiple simultaneous perfectly elastic collisions between spheres. Referring to Fig. 13, the first case involves a sphere impacting a group of three spheres at 3 m/s while the second case involves a sphere impacting a group of fifteen spheres at 1 m/s. While the grids are not relevant since the flow solution is disabled for these cases the collision detection is analytical, this demonstrates that the overset hole-cutting methodology in Loci/CHEM is capable of handling multiple bodies in close proximity, and even in contact. In both cases presented here, the moving sphere originates at \( \mathbf{r}_{cg} = (0.1, 0, 0) \) m and the sphere at the leading edge of the group is at \( \mathbf{r}_{cg} = (0.4, 0, 0) \). The remainder of the grid system essentially forms an equilateral triangle of spheres in resting contact.
Figure 13: Initial computational grids and position of spheres for multiple simultaneous collision cases; (left) impact with 3 spheres; (right) impact with 15 spheres.

The sphere positions at several different time instances for the first case using the propagation and simultaneous method of collision are shown in Fig. 14 and Fig. 15, respectively. It is apparent from the results that the propagation method has produced a non-physical result. Due to the order in which the collisions are applied, the top right-most sphere achieves greater post-impact momentum than the bottom right-most sphere resulting in an unexpected loss of symmetry in the group displacement. It is clear from Fig. 15 that the simultaneous method of collision resolution does not suffer from this problem. In this case, the top and bottom right-most spheres both achieve the same post-impact momentum and symmetry is preserved. While the propagation method and simultaneous method both conserve linear momentum and kinetic energy for this case, only the simultaneous method of collision resolution was able to realize the physically correct solution. In fact, the propagation method predicts completely different post-impact trajectories than those predicted by the simultaneous method.

The second case exhibits similar behavior as observed in Fig. 16 and Fig. 17. The momentum from the first sphere is transferred through the system of spheres and again the top right-most sphere achieves greater momentum and loss of symmetry when compared to results obtained using the propagation method. The simultaneous method has again transferred momentum and energy through the system of spheres in a symmetric fashion imparting equal amounts to the top and bottom right-most spheres.

To further demonstrate the robustness of the 6-DoF simulation capabilities and over-set hole-cutting process when many bodies are in close proximity, a utility has been developed to generate an initial grid and CHEM input file for cases with large numbers of duplicate bodies. The only required input is the near-body grid for the body under consideration. As an initial test case we consider a random distribution of spheres in close proximity. A uniform distribution of spheres separated by one sphere diameter is generated. Next, a random perturbation between 0 and half the separation distance is applied to the position of each sphere with a random initial velocity between 0 and 1 m/s assigned to each sphere. Example configurations of $2^3, 3^3, 4^3, 5^3, 6^3,$ and $7^3$ spheres.
Figure 14: Sphere positions at various time instances for single sphere impacting group of three spheres using the propagation method of collision resolution.

Figure 15: Sphere positions at various time instances for single sphere impacting group of three spheres using the simultaneous method of collision resolution.
Figure 16: Sphere positions at several different time instances for single sphere impacting group of fifteen spheres using propagation method.

Figure 17: Sphere positions at several different time instances for single sphere impacting group of fifteen spheres using simultaneous method.
in close proximity are shown in Fig. 18. A cutting-plane through the initial grid for the $7^3$ sphere case is shown in Fig. 19. The grid for this case is comprised of 6.1M cells with 8K cells belonging to each sphere grid and the background Cartesian grid consists of 3.3M cells.

With the collision resolution methods verified to be working properly, we now consider a static dart drop case to verify the proper functionality of the numerical collision detection capabilities under development. The same dart geometry and mesh presented in Section 3.1.1 is again utilized here. The case under consideration was recently investigated by Meakin [21] using OVERFLOW-2 and involves a dart dropped from rest onto a flat surface. This static drop case allows for the kinematics and collision dynamics to be observed independently of aerodynamic effects. Experimental photographs are supplied which qualitatively compare the dart position at several different distinct collision events.
before the dart reaches an equilibrium state. A schematic illustrating the dart orientation and relevant parameters is shown in Fig. 20.

The dart length is $L$ and the center-of-gravity is located $0.33L$ aft of the nose. The dart is held at $z_0 = 4.78L$ and declined at $a_0 = -7$ degrees (nose down) with the In-Plane Fin (IPF) initially pointing straight up. The value of the coefficient of restitution is taken to be $0.235$. As reported by Meakin [21], the dart nose is comprised of a different material than the shaft and fins which leads to an optimal coefficient of restitution that is different from that prescribed for this case. This causes the computed collision dynamics shown in Fig. 21 to be too plastic, leading to excessively weak reaction impulses. The current
results shown in Fig. 21 (right) are presented alongside results from OVERFLOW-2 [21] (middle) and high-speed experimental photographs [21] (left). Although there are some slight differences, the qualitative comparison between the current results and those from both OVERFLOW-2 and the experiment are quite good. Fig. 21(a) shows the initial impact of the nose with the ground, with the subsequent 2nd impact of the inboard and outboard fins with the ground shown in Fig. 21(b). The maximum computed rebound altitude is shown in Fig. 21(c) and is comparable to that obtained using OVERFLOW-2. This case serves as a good test of the current collision resolution capabilities because the results are heavily dependent on the ability to correctly predict the post-impact rolling-moments. Additionally, since the dart surface is faceted, a slight counter-clockwise roll is observed due to the finite precision in the surface definition and normal vector calculation. This causes the dart to rotate slowly about its longitudinal axis, while the nose traces an arc along the ground plane. This behavior was predicted by Meakin [21] using OVERFLOW-2, and it is observed in the current study using Loci/CHEM as well.

For more complete picture of the transient behavior, the dart positions at 0.002 second intervals beginning 0.3802 seconds into the simulation are shown in Fig. 22 and Fig. 23. Fig. 22 shows the dart position from shortly before the first impact with the ground plane, through the subsequent collision between the inboard and outboard fins with the ground plane, and up until the point at which the maximum rebound altitude is observed. Fig. 23 shows the dart position from the point at which the maximum rebound altitude is observed, until the simulation was terminated. Additionally, the time histories of the linear momentum, angular momentum, and total energy (translational + rotational) of the dart are plotted in Fig. 24.

### 4.3 Pitch-induced collisions between submunition darts in close proximity

As a demonstration of the complete Loci/CHEM simulation system including CFD/6-DoF and collision resolution for multiple bodies, we now revisit the case of Mach 3.0 flight of a 9-dart submunition cluster presented above in Section IV.A. Previously, this case was used to demonstrate the parallel scalability of the coupled CFD/6-DoF capabilities of Loci/CHEM, as well as to assess the importance of the interference pressures between multiple bodies flying at supersonic speeds. Since we are predominantly interested in the effect of inter-body collisions on the resulting dart trajectories, an initial downward pitch rate of 2,500 rpm is applied to the center dart. While this is an admittedly contrived way to encourage collisions between the darts, such behavior could certainly be possible if an explosive detonation was used to expel the darts from a carrier vehicle at high speeds. Such a large downward pitch rate was necessary to overcome the large interference pressures and ensure that collisions occur before the darts are driven apart due to the high supersonic flight speed. To aid in subsequent discussion, the center, top, right, and upper right darts are referred to as dart02, dart08, dart03, and dart05, respectively. Fig. 25 shows the darts colored by pressure at several different time instances during this simulation. It is apparent that the nose-up pitch of the center dart
Figure 22: Transient dart drop test results using Loci/CHEM. From top left to bottom right are snapshots at 0.002 second intervals starting from 0.3802 seconds into the simulation until the maximum rebound altitude is observed.

Figure 23: Transient dart drop test results using Loci/CHEM. From top left to bottom right are snapshots at 0.002 second intervals starting from the maximum rebound altitude.
results in multiple simultaneous collisions between the bottom dart and the lower right dart, producing a flowfield that is significantly different than the free-flying 6-DoF results presented in Section 5.1.

The time history of the aerodynamic forces for several representative darts is plotted in Fig. 26. It is immediately apparent that the force profile even for darts that do not experience collisions is significantly altered from the free-flying 6-DoF case. The center dart
experiences the highest force changes on the order of 150N, but even the side darts that do not experience collisions still encounter force changes on the order of 50N. The non-colliding darts experience changes in the aerodynamic forces due mainly to adjustments in the interference pressures brought about by the pitching center dart, which is affected by multiple collisions during the flight.

Fig. 27 shows the transverse dart velocity, as well as the longitudinal and radial dart motion after release for all of the darts on the same plot. This essentially displays a trajectory envelope that encompasses all darts and clearly illustrates the spread of darts as a function of time. In addition to the strong interference pressures and moving shock systems that were observed in the free-flying 6-DoF case, the presence of inter-body collisions are now significantly affecting the projectile trajectories. This is particularly evident in the first few milliseconds of the transverse dart velocity profiles shown in Fig. 27.

This demonstration of the full CFD/6-DoF/Collision simulation system in Loci/CHEM, while contrived, illustrates that the collision module is able to detect and resolve collisions between the darts flying at supersonic velocities, and produce reasonable post-impact dart trajectories. Furthermore, it is apparent that the collision detection capability is able to properly detect collisions in a variety of different scenarios, including high-speed fin-fin collisions.
Figure 27: Time histories for proximate flight for all darts in 9-dart cluster configuration: (top left) post-release longitudinal dart movement envelope; (top right) post-release radial dart movement envelope; and (bottom) post-release transverse ($y-z$) dart velocity envelope.

5 Summary and conclusions

The methodology for accurate simulation of rigid body impact among multiple bodies in proximate flight has been successfully implemented and demonstrated in the Loci/CHEM coupled 6-DoF/CFD framework. Both a sequential propagation and a simultaneous approach for collision resolution have been successfully implemented in Loci/CHEM. The latter was shown to be more accurate and robust for cases involving multiple simultaneous collisions. It was shown that this approach eliminates the need to sort and resolve the collisions sequentially, thereby avoiding non-physical solutions. The suitability and superior scalability performance of the Loci/CHEM simulation tools for coupled CFD/6-DoF simulations of a large number of moving bodies have been demonstrated. Additionally, demonstrations of the complete Loci/CHEM simulation system including CFD with 6-DoF rigid body dynamics and collision resolution have been successfully carried out for pitch-induced collisions among clusters of projectile darts flying in close proximity at supersonic speeds. Current ongoing and planned future work in this area includes implementation of improved collision detection methodologies, implementation of fully coupled solution adaptive mesh refinement capabilities to resolve the complex aerodynamic interference effects occurring for bodies in close proximity, and establishing a sound and efficient process to extract probabilistic descriptions of projectile end states from deterministic sets of simulations. The end result will be the character-
ization of complex multiple-body proximate flight including body collisions and aerodynamic interference effects which uses probabilistic models derived from performing a limited number of high-fidelity CFD simulations.

Acknowledgments

This study is being supported by the U.S. Air Force AFRL Munitions Directorate under SBIR contract No. FA8651-12-C-0074. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the U.S. Air Force.

References