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A HYBRID FIXED POINT RESULT FOR LIPSCHITZ HOMOMORPHISMS ON QUASI-BANACH ALGEBRAS

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Abstract. We shall generalize the results of [9] about characterization of isomorphisms on quasi-Banach algebras by providing integral type conditions. Also, we shall give some new results in this way and finally, give a result about hybrid fixed point of two homomorphisms on quasi-Banach algebras.

Key words: *homomorphism, hybrid fixed point, integral-type condition, p-norm, quasi-Banach algebra*

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1 Introduction

The stability problem of functional equations originated from a question of Ulam[12] concerning the stability of group homomorphisms: Let $(G_1,*)$ be a group and $(G_2,\diamond,d)$ be a metric group. Given $\varepsilon>0$, does there exist $\delta(\varepsilon)>0$ such that if a mapping $h:G_1\to G_2$ satisfies the inequality

$$d(h(x*y),h(x)\diamond h(y))<\delta$$

for all $x,y\in G_1$, then there is a homomorphism $H:G_1\to G_2$ with $d(h(x),H(x))<\varepsilon$ for all $x\in G_1$? If the answer is affirmative, we would say that the equation of the homomorphism $H(x*y)=H(x)\diamond H(y)$ is stable. The concept of stability for a functional equation arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. Thus, the stability question of functional equations is that how do the solutions of the inequality differ from those of the given functional equation?

Hyers[7] gave a first affirmative answer to the question of Ulam for Banach spaces. Let X and Y be Banach spaces. Assume that $f:X\to Y$ satisfies

$$\|f(x+y)-f(x)-f(y)\|\le\varepsilon$$

for all $x, y \in X$ and some $\varepsilon \geq 0$. Then, there exists a unique additive mapping $T : X \to Y$ such that $\|f(x) - T(x)\| \leq \varepsilon$ for all $x \in X$.

Let X and Y be Banach spaces and $f : X \to Y$ a mapping such that $f(tx)$ is continuous in $t \in \mathbf{R}$ for each fixed $x \in X$. Th. M. Rassias[10] introduced the following inequality: Assume that there exist constants $\theta \geq 0$ and $p \in [0, 1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \theta(\|x\|^p + \|y\|^p)$$

for all $x, y \in X$. He proved that there exists a unique $\mathbf{R}$-linear mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \leq \frac{2\theta}{2 - 2^p}\|x\|^p$$

for all $x \in X$. The above inequality has provided a lot of influence in the development of what is now known as Hyers-Ulam-Rassias stability of functional equations and there are a lot of works in this field. In 2007, Park, Cho and Han[8] proved the Hyers-Ulam-Rassias stability of functional inequalities associated with Jordan-Von Neumann type additive functional equations. Then, Park and An characterized isomorphisms in quasi-Banach algebras in this way.

On the other hand, Hybrid fixed point theory is an important topic and there are some papers in this field (see for example [3]-[6]). In this paper, we shall generalize the results of [9] about characterization of isomorphisms on quasi-Banach algebras by providing integral type conditions. Also, we shall give some new results in this way and finally give a result about hybrid fixed point of two homomorphisms on quasi-Banach algebras. Here, we recall some basic facts concerning quasi-Banach spaces and some preliminary results.

Definition 1.1[2],[11]. Let X be a real linear space. A quasi-norm is a real-valued function on X satisfying the following conditions:

(1) $||x|| \geq 0$ for all $x \in X$ and $||x|| = 0$ if and only if $x = 0$.

(2) $||\lambda x|| = |\lambda| \cdot ||x||$ for all $\lambda \in \mathbf{R}$ and all $x \in X$.

(3) There is a constant $K \geq 1$ such that $||x+y|| \leq K(||x|| + ||y||)$ for all $x, y \in X$.

The pair $(X, ||\cdot||)$ is called a *quasi-normed* space if $||\cdot||$ is a quasi-normed on X.

A quasi-Banach space is a complete quasi-normed space.

Definition 1.2[1]. Let $(A, ||\cdot||)$ be a quasi-normed space. The quasi-normed space $(A, ||\cdot||)$ is called a *quasi-normed algebra* if A is an algebra and there is a constant $K > 0$ such that $||xy|| \leq K||x|| \cdot ||y||$ for all $x, y \in A$.

A quasi-Banach algebra is a complete quasi-normed algebra.

Definition 1.3[9]. A $\mathbf{C}$-linear mapping $H : A \to B$ is called a homomorphism on quasi-normed algebras if $H(xy) = H(x)H(y)$ for all $x, y \in A$. If in addition, the mapping $H : A \to B$ is bijective, then the mapping $H : A \to B$ is called an isomorphism on quasi-normed algebras.

Definition 1.4. Let X be a set such that a product can be defined on X, that is, $x\cdot y\in X$ for any $x,y\in X$. We say that the mappings $f,g:X\to X$ have a hybrid fixed point whenever there exists $x_0\in X$ such that $f(x_0)g(x_0)=x_0$.

Finally, Park and An proved the following results about characterization of isomorphisms on quasi-Banach algebras[9].

Theorem 1.1. *Let $r\neq 1$ and θ be nonnegative real numbers and $f:A\to B$ a bijective mapping such that*

$$||\mu f(x)+f(y)+2f(z)||_B\leq ||2f(\frac{\mu x+y}{2}+z)||_B, \tag{1.1}$$

$$||f(xy)-f(x)f(y)||_B\leq \theta(||x||_A^{2r}+||y||_A^{2r}) \tag{1.2}$$

for $\mu=1,i$ and all $x,y,z\in A$. If $f(tx)$ is continuous in $t\in\mathbf{R}$ for each fixed $x\in A$, then the bijective mapping $f:A\to B$ is an isomorphism on quasi-Banach algebras.

Theorem 1.2. *Let $r\neq 1$ and θ be nonnegative real numbers, and $f:A\to B$ a bijective mapping satisfying* (1.1) *and*

$$||f(xy)-f(x)f(y)||_B\leq \theta||x||_A^{r}\cdot||y||_A^{r} \tag{1.3}$$

for all $x,y\in A$. If $f(tx)$ is continuous in $t\in\mathbf{R}$ for each fixed $x\in A$, then the bijective mapping $f:A\to B$ is an isomorphism on quasi-Banach algebras.

2 Characterization of Isomorphisms

Throughout this paper we assume that A and B are quasi-Banach algebras with quasi-norm $||\cdot||_A$ and $||\cdot||_B$, respectively. First, we generalize the results of Park and An. In this way, we suppose that $\psi:[0,\infty)\to[0,\infty)$ is a Lebesgue integrable mapping such that

$$\int_0^{\varepsilon}\psi(t)\mathrm{d}t>0$$

for all $\varepsilon>0$ and $\varphi:[0,\infty)\to[0,\infty)$ is a continuous mapping such that $\varphi(0)=0$.

Theorem 2.1. *Let $r\neq 1$ be a nonnegative real number, and $f:A\to B$ a $\mathbf{C}$-linear bijective mapping such that*

$$\int_0^{\lambda||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t\leq\varphi(\int_0^{\lambda(||x||_A^{2r}+||y||_A^{2r})}\psi(t)\mathrm{d}t)$$

for all $x,y\in A$ and $\lambda>0$. Then, f is an isomorphism on quasi-Banach algebras.

Proof. First, assume that $r<1$. Then for all $x,y\in A$,

$$\begin{aligned}
0 &\leq \int_0^{||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t=\int_0^{\lim_{n\to\infty}\frac{1}{4^n}||f(4^nxy)-f(2^nx)f(2^ny)||_B}\psi(t)\mathrm{d}t\\
&= \lim_{n\to\infty}\int_0^{\frac{1}{4^n}||f(4^nxy)-f(2^nx)f(2^ny)||_B}\psi(t)\mathrm{d}t\leq\lim_{n\to\infty}\varphi(\int_0^{\frac{1}{4^n}(||2^nx||_A^{2r}+||2^ny||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \lim_{n\to\infty}\varphi(\int_0^{\frac{4^{nr}}{4^n}(||x||_A^{2r}+||y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(\lim_{n\to\infty}\int_0^{\frac{4^{nr}}{4^n}(||x||_A^{2r}+||y||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \varphi(\int_0^{\lim_{n\to\infty}\frac{4^{nr}}{4^n}(||x||_A^{2r}+||y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(0)=0,
\end{aligned}$$

Thus, $f(xy)=f(x)f(y)$ for all $x,y\in A$.

Now assume that $r>1$. Then we have

$$\begin{aligned}
0 &\leq \int_0^{||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t=\int_0^{\lim_{n\to\infty}4^n||f(\frac{xy}{4^n})-f(\frac{x}{2^n})f(\frac{y}{2^n})||_B}\psi(t)\mathrm{d}t\\
&= \lim_{n\to\infty}\int_0^{4^n||f(\frac{xy}{4^n})-f(\frac{x}{2^n})f(\frac{y}{2^n})||_B}\psi(t)\mathrm{d}t\leq\lim_{n\to\infty}\varphi(\int_0^{4^n(||\frac{x}{2^n}||_A^{2r}+||\frac{y}{2^n}||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \lim_{n\to\infty}\varphi(\int_0^{\frac{4^n}{4^{nr}}(||x||_A^{2r}+||y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(\lim_{n\to\infty}\int_0^{\frac{4^n}{4^{nr}}(||x||_A^{2r}+||y||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \varphi(\int_0^{\lim_{n\to\infty}\frac{4^n}{4^{nr}}(||x||_A^{2r}+||y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(0)=0,
\end{aligned}$$

for all $x,y\in A$. Thus, $f(xy)=f(x)f(y)$ for all $x,y\in A$. Therefore $f:A\to B$ is an isomorphism on quasi-Banach algebras.

By using a similar proof, we have the following result which is a generalization of Theorem 1.2.

Theorem 2.2. *Let $r\neq 1$ be a nonnegative real number, and $f:A\to B$ a $\mathbb{C}$-linear bijective mapping such that*

$$\int_0^{\lambda||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t\leq\varphi(\int_0^{\lambda(||x||_A^r\cdot||y||_A^r)}\psi(t)\mathrm{d}t)$$

for all $x,y\in A$ and $\lambda>0$. Then, f is an isomorphism on quasi-Banach algebras.

Now, we state some another new results about characterization of isomorphisms on quasi-Banach algebras.

Theorem 2.3. *Let $r\neq 1$ be a nonnegative real number, and $f:A\to B$ a $\mathbf{C}$-linear bijective mapping such that*

$$\int_0^{\lambda||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t\leq\varphi(\int_0^{\lambda(||x-y||_A^{2r})}\psi(t)\mathrm{d}t)$$

for all $x,y\in A$ and $\lambda>0$. Then, f is an isomorphism on quasi-Banach algebras.

Proof. First, assume that $r<1$. Then,

$$\begin{aligned}
0 &\le \int_0^{||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t=\int_0^{\lim_{n\to\infty}\frac{1}{4^n}||f(4^nxy)-f(2^nx)f(2^ny)||_B}\psi(t)\mathrm{d}t\\
&= \lim_{n\to\infty}\int_0^{\frac{1}{4^n}||f(4^nxy)-f(2^nx)f(2^ny)||_B}\psi(t)\mathrm{d}t\le\lim_{n\to\infty}\varphi(\int_0^{\frac{1}{4^n}(||2^nx-2^ny||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \lim_{n\to\infty}\varphi(\int_0^{\frac{4^{nr}}{4^n}(||x-y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(\lim_{n\to\infty}\int_0^{\frac{4^{nr}}{4^n}(||x-y||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \varphi(\int_0^{\lim_{n\to\infty}\frac{4^{nr}}{4^n}(||x-y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(0)=0
\end{aligned}$$

for all $x,y\in A$. Thus, $f(xy)=f(x)f(y)$ for all $x,y\in A$.

Now, assume that $r>1$. Then, we have

$$\begin{aligned}
0 &\le \int_0^{||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t=\int_0^{\lim_{n\to\infty}4^n||f(\frac{xy}{4^n})-f(\frac{x}{2^n})f(\frac{y}{2^n})||_B}\psi(t)\mathrm{d}t\\
&= \lim_{n\to\infty}\int_0^{4^n||f(\frac{xy}{4^n})-f(\frac{x}{2^n})f(\frac{y}{2^n})||_B}\psi(t)\mathrm{d}t\le\lim_{n\to\infty}\varphi(\int_0^{4^n(||\frac{x}{2^n}-\frac{y}{2^n}||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \lim_{n\to\infty}\varphi(\int_0^{\frac{4^n}{4^{nr}}(||x-y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(\lim_{n\to\infty}\int_0^{\frac{4^n}{4^{nr}}(||x-y||_A^{2r})}\psi(t)\mathrm{d}t)\\
&= \varphi(\int_0^{\lim_{n\to\infty}\frac{4^n}{4^{nr}}(||x-y||_A^{2r})}\psi(t)\mathrm{d}t)=\varphi(0)=0
\end{aligned}$$

for all $x,y\in A$. Thus, $f(xy)=f(x)f(y)$ for all $x,y\in A$. Therefore, $f:A\to B$ is an isomorphism on quasi-Banach algebras.

Corollary 2.4. *Let $r\neq1$ and θ be nonnegative real numbers and $f:A\to B$ a $\mathbb{C}$-linear bijective mapping satisfying*

$$||f(xy)-f(x)f(y)||_B\le\theta(||x-y||_A^{2r})$$

for all $x,y\in A$. Then, f is an isomorphism on quasi-Banach algebras.

Now by using similar proofs, we can state the following results.

Theorem 2.5. *Let $r\neq1$ be a nonnegative real number, and $f:A\to B$ a* **C***-linear bijective mapping such that*

$$\int_0^{\lambda||f(xy)-f(x)f(y)||_B}\psi(t)\mathrm{d}t\le\varphi(\int_0^{\lambda(||x-f(x)||_A^{2r}+||y-f(y)||_A^{2r})}\psi(t)\mathrm{d}t)$$

for all $x,y\in A$ and $\lambda>0$. Then, f is an isomorphism on quasi-Banach algebras.

Corollary 2.6. *Let $r\neq1$ and θ be nonnegative real numbers and $f:A\to B$ a $\mathbb{C}$-linear bijective mapping satisfying*

$$||f(xy)-f(x)f(y)||_B\le\theta(||x-f(x)||_A^{2r}+||y-f(y)||_A^{2r}),$$

for all $x, y \in A$. Then, f is an isomorphism on quasi-Banach algebras.

Theorem 2.7. *Let $r \neq 1$ be a nonnegative real number, and $f : A \to B$ a $\mathbf{C}$-linear bijective mapping such that*

$$\int_0^{\lambda||f(xy)-f(x)f(y)||_B} \psi(t)\mathrm{d}t \leq \varphi(\int_0^{\lambda(||x-f(x)||_A^r \cdot ||y-f(y)||_A^r)} \psi(t)\mathrm{d}t)$$

for all $x, y \in A$ and $\lambda > 0$. Then, f is an isomorphism on quasi-Banach algebras.

Corollary 2.8. *Let $r \neq 1$ and θ be nonnegative real numbers and $f : A \to B$ a $\mathbb{C}$-linear bijective mapping satisfying*

$$||f(xy) - f(x)f(y)||_B \leq \theta(||x - f(x)||_A^r \cdot ||y - f(y)||_A^r)$$

for all $x, y \in A$. Then, f is an isomorphism on quasi-Banach algebras.

3 A Hybrid Fixed Point Result

In this section, suppose that $(A, ||\cdot||)$ is a quasi-Banach algebra such that

$$||x + y|| \leq K(||x|| + ||y||)$$

and

$$||xy|| \leq K'(||x|| \cdot ||y||)$$

for all $x, y \in A$, where the constants K and K' satisfy $K \cdot K' \geq 1$.

Theorem 3.1. *Let $(A, ||\cdot||)$ be a quasi-Banach algebra and E a complete subset of A satisfying the following properties:*

(i) *$x \cdot y \in E$ for all $x, y \in E$;*

(ii) *$\frac{x}{m} \in E$ for all $x \in E$ and all $m \geq 1$;*

(iii) *$mf(\frac{x}{m^2}), mg(\frac{x}{m^2}) \in E$ for all $x \in E$ and all $m \geq 1$.*

Suppose that $f, g : E \to E$ are two bounded Lipschitz homomorphisms with Lipschitz constants L_1 and L_2 respectively. Then, f and g have a hybrid fixed point.

Proof. Since f and g are bounded, there exist $M_1, M_2 \in [0, \infty)$ such that

$$\sup_{x\in E} ||f(x)|| = M_1, \ \sup_{x\in E} ||g(x)|| = M_2.$$

Define the metric D on $E \times E$ by

$$D((x, y), (u, v)) = ||x - u|| + ||y - v||.$$

Then, $(E \times A, D)$ is is a complete metric space. Choose $k > 1$ such that

$$k > 2(K^2K'M_1L_1 + K^2K'M_2L_2).$$

Define $H:E\times E\longrightarrow E\times E$ by $H(x,y)=(kf(\frac{xy}{k^2}),kg(\frac{xy}{k^2}))$. Then, we have

$$\begin{aligned}
D(H(x,y),H(u,v)) &= ||kf(\frac{xy}{k^2})-kf(\frac{uv}{k^2})||+||kg(\frac{xy}{k^2})-kg(\frac{uv}{k^2})||\\
&= ||kf(\frac{x}{k^2})f(y)-kf(\frac{u}{k^2})f(v)||+||kg(\frac{x}{k^2})g(y)-kg(\frac{u}{k^2})g(v)||\\
&= ||kf(\frac{x}{k^2})f(y)-kf(\frac{u}{k^2})f(y)+kf(u)f(\frac{y}{k^2})-kf(u)f(\frac{v}{k^2})||\\
&\quad +||kg(\frac{x}{k^2})g(y)-kg(\frac{u}{k^2})f(y)+kg(u)g(\frac{y}{k^2})-kg(\frac{v}{k^2})g(u)||\\
&\le K(||kf(\frac{x}{k^2})f(y)-kf(\frac{u}{k^2})f(y)||+||kf(\frac{y}{k^2})f(u)-kf(\frac{v}{k^2})f(u)||\\
&\quad +||kg(\frac{x}{k^2})g(y)-kg(\frac{u}{k^2})g(y)||+||kg(u)g(\frac{y}{k^2})-kg(u)g(\frac{v}{k^2})||)\\
&\le KK'(||kf(\frac{x}{k^2})-kf(\frac{u}{k^2})||\cdot||f(y)||+||kf(\frac{y}{k^2})-kf(\frac{v}{k^2})||\cdot||f(u)||\\
&\quad +||kg(\frac{x}{k^2})-kg(\frac{u}{k^2})||\cdot||g(y)||+||kg(\frac{y}{k^2})-kg(\frac{v}{k^2})||\cdot||g(u)||)\\
&\le KK'M_1L1(||k\frac{x}{k^2}-k\frac{u}{k^2}||+||k\frac{y}{k^2}-k\frac{v}{k^2}||)\\
&\quad +KK'M_2L2(||k\frac{x}{k^2}-k\frac{u}{k^2}||+||k\frac{y}{k^2}-k\frac{v}{k^2}||)\\
&\le \frac{2(KK'M_1L_1+KK'M_2L_2)}{k}(||x-u||+||y-v||)\\
&= \frac{2(KK'M_1L_1+KK'M_2L_2)}{k}D((x,y),(u,v)),
\end{aligned}$$

for every $x,y,u,v\in E$. Note that $\alpha:=\dfrac{2(KK'M_1L_1+KK'M_2L_2)}{k}<1$ and $\alpha K<1$. Now, let $z_0\in E\times E$. Set $z_1=Hz_0$ and define sequence $\{z_n\}$ with $z_{n+1}=Hz_n$. It is easy to see that

$$D(z_{n+1},z_n)\le\alpha^nD(z_0,z_1)$$

for all $n\ge1$. Thus, for $m>n$ we have

$$\begin{aligned}
D(z_m,z_n) &\le K^mD(z_m,z_{m-1})+\ldots+K^nD(z_{n+1},z_n)\\
&\le (K^m\alpha^m+\ldots+K^n\alpha^n)D(z_0,z_1)=\beta^n(\frac{1-\beta^{m-n+1}}{1-\beta})D(z_0,z_1),
\end{aligned}$$

where $\beta:=\alpha K<1$. Hence, $\{z_n\}$ is a Cauchy sequence in $E\times E$. Since $(E\times E,D)$ is a complete metric space, there exists $z\in E\times E$ such that $\lim\limits_{n\to\infty}z_n=z$. But, we have

$$D(Hz,z_{n+1})=D(Hz,Hz_n)\le\alpha D(z,z_n),$$

for all $n\ge1$. Thus, $Hz=z$. It implies that there exist $x,y\in E$ such that $kf(\frac{xy}{k^2})=x$ and $kg(\frac{xy}{k^2})=y$. Therefore, $f(\frac{xy}{k^2})g(\frac{xy}{k^2})=\frac{xy}{k^2}$ and the proof is completed.

Example 3.1. Let A be the usual real Banach algebra, $E=[0,1]$ and $f,g:E\to E$ defined by $fx=x^2$ and $gx=x^5$. Then, $f,g:E\to E$ are two bounded Lipschitz homomorphisms satisfying the condition (iii). Hence, f and g have at least one hybrid fixed point. In fact, f and g have the hybrid fixed points $x=0$ and $x=1$.

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