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ALMOST HOMOMORPHISMS BETWEEN UNITAL C^*-ALGEBRAS: A FIXED POINT APPROACH

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Abstract. Let A,B be two unital C^*-algebras. By using fixed pint methods, we prove that every almost unital almost linear mapping $h : A \longrightarrow B$ which satisfies $h(2^n uy) = h(2^n u)h(y)$ for all $u \in U(A)$, all $y \in A$, and all $n = 0,1,2,\cdots$, is a homomorphism. Also, we establish the generalized Hyers–Ulam–Rassias stability of $*-$homomorphisms on unital C^*-algebras.

Key words: *alternative fixed point, Jordan *-homomorphism*

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1 Introduction

A classical question in the theory of functional equations is that "when is it true that a mapping which approximately satisfies a functional equation $\mathcal{E}$ must be somehow close to an exact solution of $\mathcal{E}$". Such a problem was formulated by S.M. Ulam[27] in 1940 and solved in the next year for the Cauchy functional equation by D.H. Hyers[4]. It gave rise to the *stability* *theory* for functional equations. In 1978, Th. M. Rassias[19] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. This phenomenon of stability that was introduced by Th. M. Rassias [19] is called the Hyers–Ulam–Rassias stability. Subsequently, various approaches to the problem have been introduced by several authors. For the history and various aspects of this theory we refer the reader to monographs [3, 4, 6, 7, 8] and [10]–[26].

Let A be a unital C^*-algebra with unit e, and B a unital C^*-algebra. Let $U(A)$ be the set of unitary elements in A, $A_{sa} := \{x \in A | x = x^*\}$, and $I_1(A_{sa}) = \{v \in A_{sa} | \|v\| = 1, v \in Inv(A)\}$.

A unital C^*-algebra is of real rank zero, if the set of invertible self–adjoint elements is dense in the set of self–adjoint elements (see [1]).

Recently, C. Park, D.-H. Boo and J.-S. An[17] investigated almost homomorphisms between unital C^*-algebras.

In this paper, we will adopt the fixed point alternative of Cădariu and Radu to investigate the $*-$homomorphisms, and the generalized Hyers–Ulam–Rassias stability of $*-$homomorphisms on unital C^*-algebras associated with the Jensen–type functional equation

$$f(\frac{x+y}{2}) + f(\frac{x-y}{2}) = f(x).$$

In section two, we prove that every almost unital almost linear mapping $h : A \longrightarrow B$ is a homomorphism when $h(2^n uy) = h(2^n u)h(y)$ holds for all $u \in U(A)$, all $y \in A$, and all $n = 0, 1, 2, ...,$ and that for a unital C^*-algebra A of real rank zero (see [1]), every almost unital almost linear continuous mapping $h : A \longrightarrow B$ is a homomorphism when $h(2^n uy) = h(2^n u)h(y)$ holds for all $u \in I_1(A_{sa})$, all $y \in A$, and all $n = 0, 1, 2, \cdots$.

In section three, we establish the generalized Hyers-Ulam-Rassias stability of $*$-homomorphisms on unital C^*-algebras.

Throughout this paper assume that A, B are two C^*-algebras. For a given mapping $f : A \to B$, we define

$$\Delta_\mu f(x,y) = \mu f(\frac{x+y}{2}) + \mu f(\frac{x-y}{2}) - f(\mu x)$$

for all $\mu \in \mathbf{T} := \{z \in \mathbf{C}, |z| \leq 1\}$ and all $x, y \in A$. We denote the algebric center of algebra A by $Z(A)$.

2 $*$-Homomorphisms

Before proceeding to the main results, we will state the following theorem (see [19, 27]).

Theorem 2.1. *(The alternative of fixed point [2]) Suppose that we are given a complete generalized metric space (Ω, d) and a strictly contractive mapping $T : \Omega \to \Omega$ with Lipschitz constant L. Then for each given $x \in \Omega$, either*

$d(T^m x, T^{m+1}x) = \infty$ for all $m \geq 0$,

or other exists a natural number m_0 such that

- *$d(T^m x, T^{m+1}x) < \infty$ for all $m \geq m_0$;*
- *the sequence $\{T^m x\}$ is convergent to a fixed point y^* of T;*
- *$y^*$is the unique fixed point of T in the set $\Lambda = \{y \in \Omega : d(T^{m_0}x, y) < \infty\}$;*
- *$d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$ for all $y \in \Lambda$.*

We start our work by providing a proof for the following theorem by using alternative fixed point to investigate almost $*-$homomorphisms between unital C^*-algebras.

Theorem 2.2. *Let $f : A \to B$ be an odd mapping and that*

$$f(2^n uy) = f(2^n u) f(y) \tag{2.1}$$

for all $u \in U(A)$, all $y \in A$, and all $n = 0, 1, 2, ...$. If there exists a function $\phi : A^2 \to [0, \infty)$ such that

$$\|\Delta_\mu f(x,y)\| \leq \phi(x,y), \tag{2.2}$$

$$\|f(u^*) - f(u)^*\| \leq \phi(u,u) \tag{2.3}$$

for all $\mu \in \mathbb{T}$ all $x, y \in A$ and all $u \in (U(A) \cup \{0\})$. Suppose that there exists an $L < 1$ such that $\phi(x,y) \leq 2L\phi(\frac{x}{2}, \frac{y}{2})$ for all $x, y \in A$. If $\lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B)$, then the mapping $f : A \to B$ is a $*-$homomorphism.

Proof. By assumption, it is easy to show that

$$\lim_j 2^{-j} \phi(2^j x, 2^j y) = 0 \tag{2.4}$$

for all $x, y \in A$.

Putting $\mu = 1, y = 3x$ in (2.2), it follows by oddness of f that

$$\|f(2x) - 2f(x)\| \leq \phi(x, 3x)$$

for all $x \in X$. Hence,

$$\|\frac{1}{2} f(2x) - f(x)\| \leq \frac{1}{2}\phi(x,3x) \leq L\phi(x,3x) \tag{2.5}$$

for all $x \in A$.

Consider the set $X' := \{g \mid g : A \to B, g(0) = 0\}$ and introduce the generalized metric on X':

$$d(h,g) := inf\{C \in \mathbf{R}^+ : \|g(x) - h(x)\| \leq C\phi(x,3x), \forall x \in X\}.$$

It is easy to show that (X',d) is complete. Now we define the linear mapping $J : X' \to X'$ by

$$J(h)(x) = \frac{1}{2}h(2x)$$

for all $x \in X$. By Theorem 3.1 of [2],

$$d(J(g),J(h)) \le Ld(g,h)$$

for all $g,h \in X'$.

It follows from (2.5) that

$$d(f,J(f)) \le L.$$

By Theorem 2.1, J has a unique fixed point in the set $X_1 := \{h \in X : d(f,h) < \infty\}$. Let H be the fixed point of J. H is the unique mapping with

$$H(2x) = 2H(x)$$

for all $x \in A$ satisfying there exists $C \in (0,\infty)$ such that

$$\|T(x) - f(x)\| \le C\phi(x,3x)$$

for all $x \in X$. On the other hand we have $\lim_n d(J^n(f),T) = 0$. It follows that

$$\lim_n \frac{1}{2^n} f(2^n x) = H(x)$$

for all $x \in A$.

By the same reasoning as the proof of Theorem 1 of [17], one can show that the mapping $H : A \to B$ is $\mathbb{C}-$linear. On the other hand by using (2.3), we have

$$\begin{aligned} \|H(u^*) - (H(u))^*\| &= \lim_n \|\frac{1}{2^n} f(2^n u^*) - \frac{1}{2^n}(f(2^n u))^*\| \\ &\le \lim_n \frac{1}{2^n}\phi(2^n u, 2^n u) \\ &= 0 \end{aligned} \tag{2.6}$$

for all $u \in U(A)$. Now, let $x \in A$. By Theorem 4.1.7 of [9], x is a finite linear combination of unitary elements, i.e., $x = \sum_{j=1}^{n} c_j u_j \ \ (c_j \in \mathbf{C}, u_j \in U(A))$. Since H is $\mathbb{C}-$linear, it follows from (2.6) that

$$H(x^*) - H(x)^* = H(\sum_{j=1}^{n} c_j u_j^*) - H(\sum_{j=1}^{n} c_j u_j)^* = 0.$$

Hence, H is $*-$preserving. Now, let $u \in U(A), y \in A$. Then by linearity of H and (2.1), we have

$$H(uy) = \lim_n \frac{f(2^n uy)}{2^n} = \lim_n [\frac{f(2^n u)}{2^n} f(y)] = H(u)f(y) \tag{2.7}$$

for all $u \in U(A)$, all $y \in A$. Since H is additive, then by (2.7), we have

$$2^n H(uy) = H(u(2^n y)) = H(u)f(2^n y)$$

for all $u \in U(A)$ and all $y \in A$. Hence,

$$H(uy) = \lim_n [H(u)\frac{f(2^n y)}{2^n}] = H(u)H(y) \tag{2.8}$$

for all $u \in U(A)$ and all $y \in A$.

On the other hand, we have

$$H(e) = \lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B).$$

Hence, it follows from (2.7) and (2.8) that

$$H(e)H(y) = H(e)f(y)$$

for all $y \in A$. Since $H(e)$ is invertible, then $H(y) = f(y)$ for all $y \in A$.

Now, let $x \in A$. Then there are $n \in \mathbf{N}, c_j \in \mathbf{C}, u_j \in U(A), 1 \leq j \leq n$, such that

$$x = \sum_{j=1}^{n} c_j u_j \ ,$$

it follows from (2.8) that

$$\begin{aligned} H(xy) &= H(\sum_{j=1}^{n} c_j u_j y) = \sum_{j=1}^{n} c_j H(u_j y) \\ &= \sum_{j=1}^{n} c_j (H(u_j y)) = \sum_{j=1}^{n} c_j (H(u_j)H(y)) \\ &= H(\sum_{j=1}^{n} c_j u_j) H(y) \\ &= H(x)H(y) \end{aligned}$$

for all $y \in A$. This means that H is a homomorphism. This completes the proof of theorem.

Corollary 2.3. *Let $p \in (0,1), \theta \in [1,\infty)$ be real numbers. Let $f : A \to B$ be an odd mapping such that*

$$f(2^n uy) = f(2^n u)f(y)$$

for all $u \in U(A)$, all $y \in A$, and all $n = 0,1,2,\dots$. Suppose that

$$\|\Delta_\mu f(x,y)\| \le \theta(\|x\|^p + \|y\|^p)$$

for all $\mu \in \mathbb{T}$ and all $x, y \in A$, and that

$$\|f(u^*) - f(u)^*\| \le 2\theta\|u\|^p$$

for all $u \in U(A)$. If $\lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B)$, then the mapping $f : A \to B$ is a $*-$homomorphism.

Proof. It follows from Theorem 2.2, by putting $\phi(x,y) := \theta(\|x\|^p + \|y\|^p)$ all $x, y \in A$ and $L = 2^{p-1}$.

Theorem 2.4. *Let A be a C^*-algebra of real rank zero. Let $f : A \to B$ be an odd mapping such that*

$$f(2^n uy) = f(2^n u) f(y) \tag{2.9}$$

for all $u \in I_1(A_{sa})$, all $y \in A$, and all $n = 0,1,2,\dots$. If there exists a function $\phi : A^2 \to [0,\infty)$ such that

$$\|\Delta_\mu f(x,y)\| \le \phi(x,y), \tag{2.10}$$

for all $\mu \in \mathbb{T}$ and all $x, y \in A$

$$\|f(u^*) - f(u)^*\| \le \phi(u,u) \tag{2.11}$$

for all $u \in I_1(A_{sa})$. Suppose that there exists an $L < 1$ such that $\phi(x,y) \le 2L\phi(\frac{x}{2}, \frac{y}{2})$ for all $x, y \in A$. If $\lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B)$, then the mapping $f : A \to B$ is a $*-$homomorphism.

Proof. By the same reasoning as the proof of Theorem 2.2, the limit

$$H(x) := \lim_n \frac{1}{2^n} f(2^n x)$$

exists for all $x \in A$, also H is $\mathbb{C}-$linear. It follows from (2.9) that

$$H(uy) = \lim_n \frac{f(2^n uy)}{2^n} = \lim_n [\frac{f(2^n u)}{2^n} f(y)] = H(u) f(y) \tag{2.12}$$

for all $u \in I_1(A_{sa})$, and all $y \in A$. By additivity of H and (2.12), we obtain that

$$2^n H(uy) = H(u(2^n y)) = H(u) f(2^n y)$$

for all $u \in I_1(A_{sa})$ and all $y \in A$. Hence,

$$H(uy) = \lim_n [H(u) \frac{f(2^n y)}{2^n}] = H(u) H(y) \tag{2.13}$$

for all $u \in I_1(A_{sa})$ and all $y \in A$. By the assumption, we have

$$H(e) = \lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B).$$

Similar to the proof of Theorem 2.1, it follows from (2.12) and (2.13) that $H = f$ on A. So H is continuous.

It follows from (2.11) that

$$\begin{aligned} \|H(u^*) - (H(u))^*\| &= \lim_n \|\frac{1}{2^n} f(2^n u^*) - \frac{1}{2^n}(f(2^n u))^*\| \\ &\le \lim_n \frac{1}{2^n}\phi(2^n u, 2^n u) \le \lim_n \frac{1}{2^n}\phi(2^n u, 2^n u) \\ &= 0 \end{aligned} \tag{2.14}$$

for all $u \in I_1(A_{sa})$. Since A is real rank zero, it is easy to show that $I_1(A_{sa})$ is dense in $\{x \in A_{sa} : \|x\| = 1\}$. Let $v \in \{x \in A_{sa} : \|x\| = 1\}$. Then there exists a sequence $\{z_n\}$ in $I_1(A_{sa})$ such that $\lim_n z_n = v$. Since H is continuous, it follows from (2.14) that

$$H(v^*) = H(\lim_n (z_n^*)) = \lim_n H(z_n^*) = \lim_n H(z_n)^* = H(\lim_n z_n)^* = H(v)^*. \tag{2.15}$$

Also, it follows from (2.13) that

$$\begin{aligned} H(vy) &= H(\lim_n (z_n y)) = \lim_n H(z_n y) \\ &= \lim_n H(z_n)H(y) \\ &= H(\lim_n z_n)H(y) \\ &= H(v)H(y) \end{aligned} \tag{2.16}$$

for all $y \in A$. Now, let $x \in A$. Then we have $x = x_1 + ix_2$, where $x_1 := \frac{x+x^*}{2}$ and $x_2 := \frac{x-x^*}{2i}$ are self–adjoint.

First, consider the case that $x_1 \neq 0, x_2 \neq 0$. Since H is $\mathbb{C}-$linear, then it follows from (2.15) that

$$\begin{aligned} f(x^*) = H(x^*) = H((x_1 + ix_2)^*) &= H(\|x_1\|\frac{x_1^*}{\|x_1\|}) + H(i\|x_2\|\frac{x_2^*}{\|x_2\|}) \\ &= \|x_1\|H(\frac{x_1^*}{\|x_1\|}) - i\|x_2\|H(\frac{x_2^*}{\|x_2\|}) = \|x_1\|H(\frac{x_1}{\|x_1\|})^* - i\|x_2\|H(\frac{x_2}{\|x_2\|})^* \\ &= H(\|x_1\|\frac{x_1}{\|x_1\|})^* + H(i\|x_2\|\frac{x_2}{\|x_2\|})^* = [H(x_1) + H(ix_2)]^* \\ &= H(x)^* = f(x)^*. \end{aligned}$$

So, it follows from (2.16) that

$$\begin{aligned}
f(xy) &= H(xy) = H(x_1y + ix_2y) \\
&= H(\|x_1\|\frac{x_1}{\|x_1\|}y) + H(i\|x_2\|\frac{x_2}{\|x_2\|}y) \\
&= \|x_1\|H(\frac{x_1}{\|x_1\|}y + i\|x_2\|H(\frac{x_2}{\|x_2\|}y \\
&= \|x_1\|[H(\frac{x_1}{\|x_1\|})H(y)] + i\|x_2\|[H(\frac{x_2}{\|x_2\|})H(y)] \\
&= [H(\|x_1\|\frac{x_1}{\|x_1\|}) + H(i\|x_2\|\frac{x_2}{\|x_2\|})]H(y) + H(i\|x_2\|\frac{x_2}{\|x_2\|})] \\
&= [H(x_1) + H(ix_2)]H(y) \\
&= H(x)H(y) = f(x)f(y)
\end{aligned}$$

for all $y \in A$.

Now, consider the case that $x_1 \neq 0, x_2 = 0$. Then it follows from (2.15) that

$$f(x^*) = H(x^*) = H((x_1)^*) = H(\|x_1\|\frac{x_1^*}{\|x_1\|}) = \|x_1\|H(\frac{x_1^*}{\|x_1\|}) = \|x_1\|H(\frac{x_1}{\|x_1\|})^*$$

$$= H(\|x_1\|\frac{x_1}{\|x_1\|})^* = H(x_1)^* = H(x)^* = f(x)^*.$$

Also, we have

$$\begin{aligned}
f(xy) &= H(xy) = H(x_1y) = H(\|x_1\|\frac{x_1}{\|x_1\|}y) \\
&= \|x_1\|H(\frac{x_1}{\|x_1\|}y) = \|x_1\|[H(\frac{x_1}{\|x_1\|})H(y)] \\
&= H(\|x_1\|\frac{x_1}{\|x_1\|})H(y) = H(x_1)H(y) \\
&= H(x)H(y) = f(x)f(y)
\end{aligned}$$

for all $y \in A$.

Finally, consider the case that $x_1 = 0, x_2 \neq 0$. Then it follows from (2.15) that

$$\begin{aligned}
f(x^*) &= H(x^*) = H((ix_2)^*) = H(i\|x_2\|\frac{x_2^*}{\|x_2\|}) = -i\|x_2\|H(\frac{x_2^*}{\|x_2\|}) = -i\|x_2\|H(\frac{x_2}{\|x_2\|})^* \\
&= H(i\|x_2\|\frac{x_2}{\|x_2\|})^* = H(ix_2)^* = H(x)^* = f(x)^*.
\end{aligned}$$

Similarly we can show that

$$f(xy) = H(xy) = H(x)H(y) = f(x)f(y)$$

for all $y \in A$. Hence, f is a $*-$homomorphism.

Corollary 2.5. *Let $p \in (0,1), \theta \in [1,\infty)$ be real numbers. Let $f : A \to B$ be an odd mapping such that*

$$f(2^n uy) = f(2^n u)f(y)$$

for all $u \in I_1(A_{sa})$, all $y \in A$, and all $n = 0,1,2,\ldots$. Suppose that

$$\|\Delta_\mu f(x,y)\| \le \theta(\|x\|^p + \|y\|^p)$$

for all $\mu \in \mathbb{T}$ and all $x,y \in A$, and that

$$\|f(u^*) - f(u)^*\| \le 2\theta\|u\|^p$$

for all $u \in I_1(A_{sa})$. If

$$\lim_n \frac{f(2^n e)}{2^n} \in U(B) \cap Z(B),$$

then the mapping $f : A \to B$ is a $*-$homomorphism.

Proof. It follows from Theorem 2.4, by putting $\phi(x,y) := \theta(\|x\|^p + \|y\|^p)$ all $x,y \in A$ and $L = 2^{p-1}$.

3 Stability

In this section, we investigate the generalized Hyers–Ulam–Rassias stability of $*-$homomorphisms on unital C^*-algebras.

Theorem 3.1. *Let $f : A \to B$ be a mapping for which there exists a function $\phi : A^4 \to [0,\infty)$ satisfying*

$$\|\mu f(\frac{x+y}{2}) + \mu f(\frac{x-y}{2}) - f(\mu x) + f(uz) - f(u)f(z) + f(u^*) - f(u)^*\| \le \phi(x,y,u,z), \quad (2.17)$$

for all $\mu \in \mathbb{T}$ and all $x,y,z \in A, u \in (U(A) \cup \{0\})$. If there exists an $L < 1$ such that

$$\phi(x,y,u,z) \le 2L\phi(\frac{x}{2},\frac{y}{2},\frac{u}{2},\frac{z}{2})$$

for all $x,y,u,z \in A$, then there exists a unique $^*-$homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\| \le \frac{L}{1-L}\phi(x,0,0,0) \quad (2.18)$$

for all $x \in A$.

Proof. By the same reasoning as the proof of Theorem 2.2, one can show that there exists a unique homomorphism $H : A \to B$ satisfying (2.18). H is given by

$$H(x) = \lim_n \frac{1}{2^n} f(2^n x)$$

for all $x \in A$. We have

$$\begin{aligned}\|H(w^*) - (H(w))^*\| &= \lim_n \|\frac{1}{2^n} f(2^n w^*) - \frac{1}{2^n}(f(2^n w))^*\| \\ &\le \lim_n \frac{1}{2^n}\phi(0,0,2^n w,0) \le \lim_n \frac{1}{2^n}\phi(0,0,2^n w,0) \\ &= 0\end{aligned}$$

for all $w \in A$. Thus $H : A \to B$ is $^*-$preserving. Hence, H is an $^*-$homomorphism satisfying (2.17), as desired.

Theorem 3.2. *Let $f : A \to B$ be a mapping for which there exists a function $\phi : A^4 \to [0,\infty)$ satisfying*

$$\|\mu f(\frac{x+y}{2}) + \mu f(\frac{x-y}{2}) - f(\mu x) + f(uz) - f(u)f(z) + f(u^*) - f(u)^*\| \le \phi(x,y,u,z),$$

for all $\mu \in \mathbb{T}$ and all $x,y,z \in A, u \in (I_1(A_{sa}) \cup \{0\})$. If there exists an $L < 1$ such that

$$\phi(x,y,u,z) \le 2L\phi(\frac{x}{2},\frac{y}{2},\frac{u}{2},\frac{z}{2})$$

for all $x,y,u,z \in A$, then there exists a unique $^*-$homomorphism $H : A \to B$ such that

$$\|f(x) - H(x)\| \le \frac{L}{1-L}\phi(x,0,0,0)$$

for all $x \in A$.

Proof. The proof is similar to that of Theorems 2.4 and 3.1.

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