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Stability Results for Jungck-Kirk-Mann and Jungck-Kirk Hybrid Iterative Algorithms

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Abstract. In this paper, we introduce two hybrid iterative algorithms of Jungck-Kirk-Mann (J-K-M) and Jungck-Kirk (J-K) types to obtain some stability results for non-selfmappings by employing a certain general contractive condition. Our results generalize and extend most of the existing ones in the literature.

Key Words: Stability result, Jungck-Kirk-Mann, Jungck-Kirk.

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1 Introduction

Let (E,d) be a complete metric space, and for $T:E\to E$ a selfmap of E, let

$$F_T=\{p\in E|Tp=p\}$$

be the set of fixed points of T. Also, for $S,T:Y\to E$, let $C(S,T)=\{z\in Y|Sz=Tz=p\}$ be the set of all coincidence points of S and T.

Definition 1.1. (see [20]) Let $S,T:Y\to E,T(Y)\subseteq S(Y)$ and z a coincidence point of S and T, that is,

$$Sz=Tz=p.$$

For any $x_0\in Y$, let the sequence $\{Sx_n\}_{n=0}^{\infty}$ generated by the iterative procedure

$$Sx_{n+1}=f(T,x_n),\quad n=0,1,\cdots, \tag{1.1}$$

converge to p. Let $\{Sy_n\}_{n=0}^{\infty}\subset E$ be an arbitrary sequence, and set

$$\epsilon_n=d\big(Sy_{n+1},f(T,y_n)\big),\quad n=0,1,\cdots.$$

Then, the iterative procedure (1.1) will be called (S,T)-stable if and only if $\lim_{n\to\infty}\epsilon_n=0$ implies that $\lim_{n\to\infty}Sy_n=p$.

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This definition reduces to that of the stability of iterative process in the sense of Harder and Hicks [6] when $Y=E$ and $S=I$ (identity operator).

Remark 1.1. (i) If in (1.1), for $x_0\in E$,

$$Sx_{n+1}=f(T,x_n)=(1-\alpha_n)Sx_n+\alpha_n Tx_n,\quad n=0,1,\cdots,\quad \alpha_n\in[0,1], \tag{1.2}$$

then we get the iterative process of Singh et al. [25].

(ii) If in (1.1), for $x_0\in E$, $Y=E$, $S=I$ (identity operator), then we obtain

$$x_{n+1}=f(T,x_n)=(1-\alpha_n)x_n+\alpha_n Tx_n,\quad n=0,1,\cdots,\quad \alpha_n\in[0,1], \tag{1.3}$$

which is known as the Mann iterative process (see Mann [11]).

(iii) Also, if in (1.1), for $x_0\in E$, $Y=E$, $S=I$ (identity operator), we have

$$x_{n+1}=f(T,x_n)=\sum_{i=0}^{k}\alpha_i T^i x_n,\quad \sum_{i=0}^{k}\alpha_i=1,\quad n=0,1,\cdots, \tag{1.4}$$

where k is a fixed integer and $\alpha_i\geq 0$, $\alpha_0\neq 0$, $\alpha_i\in[0,1]$, and (1.4) is the Kirk's iterative process [9].

For several stability results that have been obtained by various authors and different contractive definitions, we refer to Berinde [3], Harder and Hicks [6], Osilike [14], Rhoades [17, 18] and others in the reference of this paper.

We introduce the following hybrid iterative algorithms to establish our results:

Let $(E,\|.\|)$ be a normed linear space, $S,T:Y\to E$ and $T(Y)\subseteq S(Y)$. Then, for $x_0\in Y$, consider the sequence $\{Sx_n\}_{n=0}^{\infty}\subset E$ defined by

$$Sx_{n+1}=\alpha_{n,0}Sx_n+\sum_{i=1}^{k}\alpha_{n,i}T^i x_n,\quad n=0,1,\cdots,\quad \sum_{i=0}^{k}\alpha_{n,i}=1, \tag{1.5}$$

$\alpha_{n,i}\geq 0$, $\alpha_{n,0}\neq 0$, $\alpha_{n,i}\in[0,1]$, where k is a fixed integer.

If in (1.5), $\alpha_{n,i}=\alpha_i$, then we obtain the following interesting iterative scheme: For $x_0\in Y$, define the sequence $\{Sx_n\}_{n=0}^{\infty}\subset E$ by

$$Sx_{n+1}=\alpha_0 Sx_n+\sum_{i=1}^{k}\alpha_i T^i x_n,\quad n=0,1,\cdots,\quad \sum_{i=0}^{k}\alpha_i=1, \tag{1.6}$$

$\alpha_i\geq 0$, $\alpha_0\neq 0$, $\alpha_i\in[0,1]$, where k is a fixed integer.

Remark 1.2. The scheme defined in (1.5) shall be called the Jungck-Kirk-Mann iterative algorithm while that of (1.6) shall be called the Jungck-Kirk iterative algorithm.

Remark 1.3. (i) If in (1.5), $k=1$, $Y=E$, $S=I$ (I being the identity operator), then we obtain the Mann iterative process in (1.3).

(ii) Also, if in (1.5), $k=1$, then we obtain the Jungck-Mann iterative process in (1.2).

(iii) The iterative processes of Picard [1], Jungck [8], Krasnoselskij [10] and Schaefer [19] are also particular cases of those defined in (1.5) and (1.6). Indeed, (1.5) also generalizes one of the iterative processes defined by the author in [16] but independent of those introduced in [14, 15, 17, 18].

Lemma 1.1. (see [3, 4]) *If δ is a real number such that $0\le\delta<1$, and $\{\epsilon'_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty}\epsilon'_n=0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying $u_{n+1}\le\delta u_n+\epsilon'_n$, $n=0,1,\cdots$, we have $\lim_{n\to\infty}u_n=0$.*

We shall employ the following contractive condition: For commuting operators S,T: $Y\to E$ on an arbitrary set Y with values in $E, T(Y)\subseteq S(Y)$, there exist $a\in[0,1)$ and φ: $\mathbf{R}^+\to\mathbf{R}^+$, a sublinear, monotone increasing function with $\varphi(0)=0$ such that

$$\|Tx-Ty\|\le\varphi(\|Sx-Tx\|)+a\|Sx-Sy\|,\quad \forall x,y\in Y. \tag{1.7}$$

Remark 1.4. The contractive condition (1.7) is more general than those of Singh et al. [25] and several others in the literature. For instance, if in (1.7), $\varphi(u)=Lu$, $\forall u\in\mathbf{R}^+$, $L\ge 0$, then we obtain one of the contractive conditions of [25].

Also, if in (1.7), $\varphi(u)=0$, $\forall u\in\mathbf{R}^+$, then we obtain the second contractive condition in [25].

The contractive condition (1.7) also reduces to those of [7, 13, 16, 19, 20] and so on, in the selfmapping setting.

Lemma 1.2. *Let $(E,\|\cdot\|)$ be a normed linear space and $S,T:Y\to E$ be commuting operators on an arbitrary set Y with values in E satisfying* (1.7) *such that*

$$T(Y)\subseteq S(Y),\quad \|S^2x-T(Sx)\|\le\|Sx-Tx\|,\quad \forall x\in Y,$$

and

$$\|S^2x-S^2y)\|\le\|Sx-Sy\|,\quad \forall x,y\in Y.$$

Let $\varphi:\mathbf{R}^+\to\mathbf{R}^+$ be a sublinear, monotone increasing function such that $\varphi(0)=0$. Let z be a coincidence point of S,T,S^i and T^i, i.e.,

$$Sz=Tz=p\quad\textit{and}\quad S^iz=T^iz=p.$$

Then, $\forall i\in\mathbf{N}$ and $\forall x,y\in Y$, we have

$$\|T^ix-T^iy\|\le\sum_{j=1}^{i}\binom{i}{j}a^{i-j}\varphi^j(\|Sx-Tx\|)+a^i\|Sx-Sy\|. \tag{1.8}$$

Proof. We now prove that φ is sublinear: In order to show that φ^i (i.e., iterate of φ) is sublinear, we have to show that φ is both subadditive and positively homogeneous.

We first establish that φ subadditive implies that each iterate φ^j of φ is also subadditive: Since φ is subadditive, we have

$$\varphi(x+y) \leq \varphi(x)+\varphi(y), \quad \forall x,y \in \mathbf{R}_+.$$

Therefore,using subadditivity of φ in φ yields

$$\varphi^2(x+y)=\varphi(\varphi(x+y)) \leq \varphi(\varphi(x)+\varphi(y)) \leq \varphi(\varphi(x))+\varphi(\varphi(y))=\varphi^2(x)+\varphi^2(y),$$

which implies that φ^2 is subadditive.

Similarly, applying subadditivity of φ^2 in φ^3, we get

$$\begin{aligned}\varphi^3(x+y)=&\varphi(\varphi^2(x+y)) \leq \varphi(\varphi^2(x)+\varphi^2(y)) \leq \varphi(\varphi^2(x))+\varphi(\varphi^2(y))\\ =&\varphi^3(x)+\varphi^3(y),\end{aligned}$$

which implies that φ^3 is also subadditive. Hence, in general, each φ^n, $n=1,2,\cdots$, is subadditive.

We now prove that φ positively homogeneous implies that each iterate φ^i of φ is also positively homogeneous: Therefore, we have

$$\varphi(\alpha x)=\alpha\varphi(x), \quad \forall x \in \mathbf{R}^+ \quad \text{and} \quad \alpha>0.$$

Using positive homogeneity of φ in φ^2, we have

$$\begin{aligned}\varphi^2(\alpha x)=&\varphi(\varphi(\alpha x))=\varphi(\alpha\varphi(x))\\ =&\alpha\varphi(\varphi(x))=\alpha\varphi^2(x), \quad \forall x \in \mathbf{R}_+ \quad \text{and} \quad \alpha>0,\end{aligned}$$

which implies that φ^2 is positively homogeneous.

Hence, in general, each φ^n, $n=1,2,\cdots$, is positively homogeneous.

The second part of the proof of this Lemma is by mathematical induction (i.e., induction on i). If $i=1$, then (1.8) becomes

$$\|Tx-Ty\| \leq \sum_{j=1}^{1}\binom{1}{j}a^{1-j}\varphi^j(\|Sx-Tx\|)+a\|Sx-Sy\|=\varphi(\|Sx-Tx\|)+a\|Sx-Sy\|.$$

i.e., (1.8) reduces to (1.7) when $i=1$ and hence the result holds. ☐

Assume as an inductive hypothesis that (1.8) holds for $i=m$, $m \in \mathbf{N}$, i.e.,

$$\|T^m x-T^m y\| \leq \sum_{j=1}^{m}\binom{m}{j}a^{m-j}\varphi^j(\|Sx-Tx)\|)+a^m\|Sx-Sy\|.$$

We then show that the statement is true for $i=m+1$;

$$
\begin{aligned}
&\|T^{m+1}x-T^{m+1}y\|=\|T^m(Tx)-T^m(Ty)\|\\
\leq&\sum_{j=1}^{m}\binom{m}{j}a^{m-j}\varphi^j(\|S(Tx)-T^2x\|)+a^m\|S(Tx)-S(Ty)\|\\
=&\sum_{j=1}^{m}\binom{m}{j}a^{m-j}\varphi^j(\|T(Sx)-T(Tx)\|)+a^m\|T(Sx)-T(Sy)\|\\
\leq&\sum_{j=1}^{m}\binom{m}{j}a^{m-j}\varphi^j(\varphi(\|S^2x-T(Sx)\|)+a\|S^2x-S(Tx)\|)\\
&+a^m(\varphi(\|S^2x-T(Sx)\|)+a\|S^2x-S^2y\|))\\
\leq&\sum_{j=1}^{m}\binom{m}{j}a^{m-j}\varphi^j(\varphi(\|Sx-Tx\|)+a\|Sx-Tx\|)\\
&+a^m(\varphi(\|Sx-Tx\|)+a\|Sx-Sy\|))\\
\leq&\sum_{j=1}^{m}\binom{m}{j}a^{m-j}\varphi^{j+1}(\|Sx-Tx\|)+\sum_{j=1}^{m}\binom{m}{j}a^{m+1-j}\varphi^j(\|Sx-Tx\|)\\
&+a^m\varphi(\|Sx-Tx\|)+a^{m+1}\|Sx-Sy\|\\
=&\binom{m}{m}\varphi^{m+1}(\|Sx-Tx\|)+\left[\binom{m}{m-1}+\binom{m}{m}\right]a\varphi^m(\|Sx-Tx\|)\\
&+\left[\binom{m}{m-2}+\binom{m}{m-1}\right]a^2\varphi^{m-1}(\|Sx-Tx\|)+\cdots\\
&+\left[\binom{m}{2}+\binom{m}{3}\right]a^{m-2}\varphi^3(\|Sx-Tx\|)+\left[\binom{m}{1}+\binom{m}{2}\right]a^{m-1}\varphi^2(\|Sx-Tx\|)\\
&+\left[\binom{m}{1}+\binom{m}{0}\right]a^m\varphi(\|Sx-Tx\|)+a^{m+1}\|Sx-Sy\|\\
=&\binom{m+1}{m+1}\varphi^{m+1}(\|Sx-Tx\|)+\binom{m+1}{m}a\varphi^m(\|Sx-Tx\|)\\
&+\binom{m+1}{m-1}a^2\varphi^{m-1}(\|Sx-Tx\|)+\cdots+\binom{m+1}{2}a^{m-1}\varphi^2(\|Sx-Tx\|)\\
&+\binom{m+1}{1}a^m\varphi(\|Sx-Tx\|)+a^{m+1}\|Sx-Sy\|\\
=&\sum_{j=1}^{m+1}\binom{m+1}{j}a^{m+1-j}\varphi^j(\|Sx-Tx\|)+a^{m+1}\|Sx-Sy\|.
\end{aligned}
$$

2 Main results

Theorem 2.1. *Let $(E,\|.\|)$ be a normed linear space. Let S, $T:Y\to E$ be commuting operators on an arbitrary set Y with values in E such that $T(Y)\subseteq S(Y)$ and $S(Y)$ or $T(Y)$ is a complete* *subspace of E. Let z be a coincidence point of S, T, S^i and T^i, i.e.,*

$$S(z)=T(z)=p, \quad S^iz=T^iz=p.$$

Let $x_0\in Y$ and let $\{Sx_n\}_{n=0}^{\infty}\subset E$ defined by (1.5) *be the Jungck-Kirk-Mann iterative process converging to p, with $\alpha_{n,i}\in[0,1]$, $i=0,1,\cdots,k$ and $\sum_{i=0}^{k}\alpha_{n,i}=1$. Suppose that S and T satisfy the contractive condition* (1.7)*, where $\varphi:\mathbf{R}^+\to\mathbf{R}^+$ is a sublinear monotone increasing function such that $\varphi(0)=0$. Then, the Jungck-Kirk-Mann iterative process is (S,T)-stable.*

Proof. Let

$$\{Sy_n\}_{n=0}^{\infty}\subset E$$

and

$$\epsilon_n=\Big\|Sy_{n+1}-\alpha_{n,0}Sy_n-\sum_{i=1}^{k}\alpha_{n,i}T^iy_n\Big\|.$$

Let $\lim_{n\to\infty}\epsilon_n=0$. Then, we shall prove that $\lim_{n\to\infty}Sy_n=p$, by using both Lemma 1.1 and Lemma 1.2 as well as the triangle inequality. Therefore,

$$\begin{aligned}
\|Sy_{n+1}-p\|\le&\Big\|Sy_{n+1}-\alpha_{n,0}Sy_n-\sum_{i=1}^{k}\alpha_{n,i}T^iy_n\Big\|+\Big\|\alpha_{n,0}Sy_n+\sum_{i=1}^{k}\alpha_{n,i}T^iy_n-p\Big\|\\
=&\epsilon_n+\Big\|\alpha_{n,0}Sy_n+\sum_{i=1}^{k}\alpha_{n,i}T^iy_n-\sum_{i=0}^{k}\alpha_{n,i}p\Big\|\\
=&\epsilon_n+\Big\|\alpha_{n,0}(Sy_n-p)+\sum_{i=1}^{k}\alpha_{n,i}(T^iy_n-p)\Big\|\\
\le&\|\alpha_{n,0}\|\|Sy_n-p\|+\sum_{i=1}^{k}\alpha_{n,i}\|p-T^iy_n\|+\epsilon_n\\
=&\alpha_{n,0}\|Sy_n-p\|+\sum_{i=1}^{k}\alpha_{n,i}\|T^iz-T^iy_n\|+\epsilon_n\\
\le&\alpha_{n,0}\|Sy_n-p\|+\sum_{i=1}^{k}\alpha_{n,i}\bigg\{\sum_{j=1}^{i}\binom{i}{j}a^{i-j}\varphi^j(\|Sz-Tz\|)+a^i\|Sz-Sy_n\|\bigg\}+\epsilon_n\\
=&\alpha_{n,0}\|Sy_n-p\|+\sum_{i=1}^{k}\alpha_i\bigg\{\sum_{j=1}^{i}\binom{i}{j}a^{i-j}\varphi^j(0)+a^i\|p-Sy_n\|\bigg\}+\epsilon_n\\
=&\Big(\sum_{i=0}^{k}\alpha_{n,i}a^i\Big)\|Sy_n-p\|+\epsilon_n, \qquad (2.1)
\end{aligned}$$

where $\varphi^j(0)=0$. Since

$$0\le\sum_{i=0}^{k}\alpha_{n,i}a^i\le\Big|\sum_{i=0}^{k}\alpha_{n,i}a^i\Big|\le\sum_{i=0}^{k}|\alpha_{n,i}||a^i|<1,$$

using Lemma 1.1 in (2.1) yields

$$\lim_{n\to\infty}\|Sy_n-p\|=0,$$

that is,

$$\lim_{n\to\infty}Sy_n=p.$$

Conversely, let $\lim_{n\to\infty}Sy_n=p$. Then, by applying both Lemma 1.2 and the triangle inequality, we have

$$\begin{aligned}\epsilon_n&=\Big\|Sy_{n+1}-\alpha_{n,0}Sy_n-\sum_{i=1}^{k}\alpha_{n,i}T^iy_n\Big\|\\&\le\|Sy_{n+1}-p\|+\Big\|p-\alpha_{n,0}Sy_n-\sum_{i=1}^{k}\alpha_{n,i}T^iy_n\Big\|\\&\le\|Sy_{n+1}-p\|+\alpha_{n,0}\|Sy_n-p\|+\sum_{i=1}^{k}\alpha_{n,i}\|p-T^iy_n\|\\&=\|Sy_{n+1}-p\|+\alpha_{n,0}\|Sy_n-p\|+\sum_{i=1}^{k}\alpha_{n,i}\|T^iz-T^iy_n\|\\&\le\|Sy_{n+1}-p\|+\Big(\sum_{i=0}^{k}\alpha_ia^i\Big)\|Sy_n-p\|\to0\quad\text{as n}\to\infty.\end{aligned}$$

The proof is complete. □

Theorem 2.2. *Let $(E,\|.\|)$ be a normed linear space. Let $S,T:Y\to E$ be commuting operators on an arbitrary set Y with values in E such that $T(Y)\subseteq S(Y)$ and $S(Y)$ or $T(Y)$ is a complete subspace of E. Let z be a coincidence point of S, T, S^i and T^i, i.e.,*

$$S(z)=T(z)=p,\quad S^iz=T^iz=p.$$

Let $x_0\in Y$ and let $\{Sx_n\}_{n=0}^{\infty}\subset E$ defined by (1.6) *be the Jungck-Kirk iterative process converging to p, with $\alpha_i\in[0,1]$, $i=0,1,\cdots,k$ and $\sum_{i=0}^{k}\alpha_i=1$. Suppose that S and T satisfy the contractive condition* (1.7)*, where $\varphi:\mathbf{R}^+\to\mathbf{R}^+$ is a sublinear monotone increasing function such that $\varphi(0)=0$. Then, the Jungck-Kirk iterative process is (S,T)-stable.*

Proof. The proof is similar to that of Theorem 2.1. □

Remark 2.1. Both Theorem 2.1 and Theorem 2.2 are generalizations and extensions of several results in the literature. In particular, we refer to some similar results in Berinde [3], Osilike [14], Osilike and Udomene [15], Rhoades [17, 18] Singh et al. [20] and some of the results of the author [7, 9, 13, 16]. However, our results are independent of those of [14, 15, 17, 18].

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