Analysis in Theory and Applications
Anal. Theory Appl., Vol. **31**, No. 4 (2015), pp. 373-380
DOI: 10.4208/ata.2015.v31.n4.3

Boundedness of Multilinear Oscillatory Singular Integral on Weighted Weak Hardy Spaces

Yali Pan and Changwen Li*

School of Mathematical Science, Huaibei Normal University, Huaibei 235000, China

Received 8 April 2015; Accepted (in revised version) 8 June 2015

Abstract. In this paper, by using the atomic decomposition of the weighted weak Hardy space $WH^1_\omega(\mathbb{R}^n)$, the authors discuss a class of multilinear oscillatory singular integrals and obtain their boundedness from the weighted weak Hardy space $WH^1_\omega(\mathbb{R}^n)$ to the weighted weak Lebesgue space $WL^1_\omega(\mathbb{R}^n)$ for $\omega\in A_1(\mathbb{R}^n)$.

Key Words: Multilinear oscillatory singular integral, $A_1(\mathbb{R}^n)$, weighted weak Hardy space.
AMS Subject Classifications: 42B20, 42B25, 42B35

1 Introduction and main results

Recently, some people studied the boundedness of oscillatory singular integrals with polynomial phase functions. Let $P(x,y)$ be a real-valued polynomial defined on $\mathbb{R}^n\times\mathbb{R}^n$ and K be a standard Calderón-Zygmund kernel, that is, K is C^1 on $\mathbb{R}^n$ away from the origin and has mean value zero on the unit sphere centered at the origin. Define the oscillatory singular integral operator T by

$$Tf(x)=p.v.\int_{\mathbb{R}^n} e^{iP(x,y)}K(x-y)f(y)dy. \tag{1.1}$$

Ricci-Stein in [1] stated that T is bounded on $L^p(\mathbb{R}^n)$ for $1<p<\infty$. Chanillo and Christ [2] proved that T is also bounded from $L^1(\mathbb{R}^n)$ to weak $L^1(\mathbb{R}^n)$. Pan [3] introduced a variant of the Hardy space $H^1_E(\mathbb{R}^n)$, and proved that T is bounded from $H^1_E(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$. For the special form $P(x,y)=P(x-y)$, Hu and Pan [4] considered the behavior of T on $H^1(\mathbb{R}^n)$.

The purpose of this paper is to study a class of multilinear operators which are closely related to the operator T defined by (1.1). Let $m\in\mathbb{N}$ and $m\geq 2$. Let Ω be homogeneous of degree zero, belong to the space $Lip_1(S^{n-1})$ and satisfy the following conditions

$$\int_{S^{n-1}}\Omega(\theta)\theta^\alpha d\theta=0 \quad \text{for} \quad \alpha\in(N\cup\{0\})^n \quad \text{and} \quad |\alpha|=m. \tag{1.2}$$

*Corresponding author. *Email addresses:* `panlijiyang@163.com` (Y. L. Pan), `cwli2008@163.com` (C. W. Li)

Let A have derivatives of order m in $BMO(\mathbb{R}^n)$ and let $R_m(A;x,y)$ denote the *m-th* order Taylor series remainder of A at x about y, that is,

$$R_m(A;x,y)=A(x)-\sum_{|\alpha|\le m-1}\frac{1}{\alpha}D^\alpha A(y)(x-y)^\alpha.$$

The operator we will consider here is defined by

$$T^A f(x)=\int_{\mathbb{R}^n}e^{iP(x,y)}\frac{\Omega(x-y)}{|x-y|^{n+m}}Q_{m+1}(A;x,y)f(y)dy, \tag{1.3}$$

where

$$Q_{m+1}(A;x,y)=R_m(A;x,y)-\sum_{|\alpha|=m}\frac{1}{\alpha!}D^\alpha A(y)(x-y)^\alpha.$$

Hu and Yang [5] considered the boundedness of the operator T^A on $H^1(\mathbb{R}^n)$ when $P(x,y)=P(x-y)$. They proved that T^A is bounded from $H^1(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$. In 2000, Hu , Wu and Yang [6] proved that T^A is also bounded from weighted Hardy space $H^1_\omega(\mathbb{R}^n)$ to the weighted Lebesgue space $L^1_\omega(\mathbb{R}^n)$ for $\omega\in A_1(\mathbb{R}^n)$ and from the weighted Herz-type Hardy space to the weighted Herz space.

The classical A_p weighted theory was first introduced by Muckenhoupt in [7]. And we denote the weighted measure of E by $\omega(E)$, i.e., $\omega(E)=\int_E\omega(x)dx$.

We say that $\omega\in A_1(\mathbb{R}^n)$ if

$$\frac{1}{|B|}\int_B\omega(x)dx\le C\operatorname{ess}\inf_{x\in B}\omega(x)\quad\text{for every ball}\quad B\subseteq\mathbb{R}^n.$$

And we say $\omega\in A_p(\mathbb{R}^n)$ with $1<p<\infty$, if there exists a constant $C>0$, such that

$$\Big(\frac{1}{|B|}\int_B\omega(x)dx\Big)\Big(\frac{1}{|B|}\int_B\omega(x)^{-\frac{1}{(p-1)}}dx\Big)^{p-1}\le C$$

for every ball $B\subseteq\mathbb{R}^n$.

We define $A_\infty(\mathbb{R}^n)=\bigcup\limits_{p\ge1}A_p(\mathbb{R}^n)$. So we can get

$$\varphi\in A_1(\mathbb{R}^n)\Longrightarrow\varphi\in A_\infty(\mathbb{R}^n).$$

If $\omega\in A_\infty(\mathbb{R}^n)$, we write

$$q_\omega=\inf(q>1:\omega\in A_q(\mathbb{R}^n))$$

to denote the critical index of ω.

Definition 1.1. Let ω be a weight function on $\mathbb{R}^n$, for $1\le p<\infty$, the weighted Lebesgue space is defined by

$$L^p_\omega(\mathbb{R}^n)=\Big\{f:\|f\|_{L^p_\omega}=\Big(\int_{\mathbb{R}^n}|f(x)|^p\omega(x)dx\Big)^{1/p}<\infty\Big\}.$$

And the weighted weak Lebesgue space is defined by

$$WL^p_\omega(\mathbb{R}^n)=\Big\{f:\|f\|_{WL^p_\omega}=\sup_{\lambda>0}\lambda\cdot\omega(x\in\mathbb{R}^n:|f(x)|>\lambda)^{1/p}<\infty\Big\}.$$

Let $\omega\in A_\infty(\mathbb{R}^n)$, $0<p\le 1$ and $N=[n(q_\omega/p-1)]$, we define

$$\mathcal{A}_{p,\omega}=\Big\{\phi\in S(\mathbb{R}^n):\sup_{x\in\mathbb{R}^n}\sup_{|\alpha|\le N+1}(1+|x|)^{N+n+1}|D^\alpha\phi(x)|\le 1\Big\},$$

where $S(\mathbb{R}^n)$ denotes Schwartz function class. Let $S'(\mathbb{R}^n)$ be the dual space of $S(\mathbf{R}^n)$.

For $f\in S'(\mathbb{R}^n)$, the grand maximal function of f is defined by

$$G_{p,\omega}f(x)=\sup_{\phi\in\mathcal{A}_{p,\omega},\ \|\phi\|_{\mathcal{A}_{p,\omega}}\le 1}\ \sup_{|x-y|<t}|(\phi_t*f)(y)|.$$

Definition 1.2 (see [8]). Let $\omega\in A_\infty(\mathbb{R}^n)$, $0<p\le 1$. Then the weighted weak Hardy spaces $WH^p_\omega(\mathbb{R}^n)$ is defined by

$$WH^p_\omega(\mathbb{R}^n)=\{f\in S':G_{p,\omega}f\in WL^p_\omega(\mathbb{R}^n)\}.$$

Moreover, we set $\|f\|_{WH^p_\omega}=\|G_\omega f\|_{WL^p_\omega}$.

In 2001, Quek and Yang [8] got the atomic decompositions theorem of weighted weak Hardy spaces. In this paper, we will discuss the boundedness of the operator T^A on the weighted weak Hardy spaces. Our main result of this paper is the following theorem.

Theorem 1.1. *Let $\omega\in A_1(\mathbb{R}^n)$, T^A be defined as in* (1.3) *and $P(x,y)$ be a real-valued polynomial on $\mathbb{R}^n\times\mathbb{R}^n$ with $\nabla_y P(0,y)=0$. Then T^A is bounded from $WH^1_\omega(\mathbb{R}^n)$ to $WL^1_\omega(\mathbb{R}^n)$, that is, there exists a constant $C>0$ such that*

$$\|T^Af\|_{WL^1_\omega}\le C\sum_{|\alpha|=m}\|D^\alpha A\|_{BMO}\|f\|_{WH^1_\omega},$$

where C depends only on n, m, $degP$ and $A_1(\omega)$, the $A_1(\mathbb{R}^n)$-constant of ω.

2 Notations and preliminary lemmas

Let us begin by giving the atomic decompositions of weighted weak Hardy spaces.

Lemma 2.1 (see [8]). *Suppose $\omega\in A_\infty(\mathbb{R}^n)$, $0<p\le 1$, For $f\in WH^p_\omega(\mathbb{R}^n)$, there exists a sequence of bounded and measurable functions $\{f_k\}_{k=-\infty}^{\infty}$ such that*

(1) $f(x)=\sum_{k=-\infty}^{\infty}f_k(x)$, in $S'(\mathbb{R}^n)$,

(2) each f_k can be further decomposed into $f_k(x)=\sum_i h^k_i(x)$, where $h^k_i(x)$ satisfies the following conditions:

(a) $supp(h_i^k)\subset B_i^k=B(x_i^k,r_i^k)$. *Moreover,*

$$\sum_i \omega(B_i^k)\le C_1 2^{-kp},\sum_i \chi_{B_i^k}(x)\le C_1,$$

where χ_E *denotes the characteristic function of the set E and* $C_1\le \|f\|_{WH_\omega^p}^p$*;*

(b) $\|h_i^k\|_{L^\infty}\le C2^k$*;*

(c) $\int h_i^k x^\alpha dx=0$, *for every multi-index* α *with* $|\alpha|\le[n(q_\omega/p-1)]$.

Lemma 2.2 (see [6]). *Let* T^A *be defined as in* (1.3). *Then for* $1<p<\infty$ *and* $\omega\in A_p(\mathbb{R}^n)$, T^A *is bounded on* $L^p_\omega(\mathbb{R}^n)$, *that is, for all* $f\in L^p_\omega(\mathbb{R}^n)$,

$$\|T^A(f)\|_{L^p_\omega}\le C\sum_{|\alpha|=m}\|D^\alpha A\|_{BMO}\|f\|_{L^p_\omega}.$$

Lemma 2.3 (see [9]). *Let* $\omega\in A_p(\mathbb{R}^n)$, $p\ge 1$, *and* $r>0$, *Then for any ball B and* $\lambda>1$,

$$\omega(2B)\le C\omega(B),\quad \omega(\lambda B)\le C\lambda^{np}\omega(B),$$

where C does not depend on B nor on λ.

Lemma 2.4 (see [10]). *Let* $b(x)$ *be a function on* $\mathbb{R}^n$ *with mth order derivatives in* $L^q_{loc}(\mathbb{R}^n)$ *for some* $q>n$. *Then*

$$|R_m(b;x,y)|\le C_{m,n}|x-y|^m\sum_{|\alpha|=m}\Big(\frac{1}{|\tilde{Q}(x,y)|}\int_{\tilde{Q}(x,y)}|D^\alpha b(z)|^q dz\Big)^{1/q},$$

where $\tilde{Q}(x,y)$ *is the cube centered at x and having diameter* $5\sqrt{n}|x-y|$.

Lemma 2.5 (see [6]). *Let* $\varphi\in C_0^\infty(\mathbb{R}^n)$ *satisfy that* $\mathrm{supp}\varphi\subset\{x\in\mathbb{R}^n:|x|\le 2\}$ *and* $\varphi(x)=1$ *for* $|x|\le 1$ *and* $\psi\in C_0^\infty(\mathbb{R}^n)$ *satisfy that* $\mathrm{supp}\psi\subset\{x\in\mathbb{R}^n:1/4\le|x|\le 4\}$ *and* $\psi(x)=1$ *for* $1/2\le|x|\le 2$. *Define*

$$T_k f(x)=\psi(2^{-k}x)\int_{\mathbb{R}^n}e^{iP(x,y)}\varphi(y)f(y)dy.$$

If the polynomial $P(x,y)$ *has the form*

$$P(x,y)=\sum_{|\mu|\ge 1,|\nu|=l}a_{\mu\nu}x^\mu y^\nu+Q(x,y),$$

where $Q(x,y)$ *is a polynomial with degree in y smaller than l, then for each sufficiently large positive integer N,*

$$\|T_k f\|_{L^2}\le C_N 2^{nk/2}|a_{\mu_0\nu_0}|^{-1/(2Nl)}2^{-k|\mu_0|/(2Nl)}\|f\|_{L^2},$$

where $|a_{\mu_0\nu_0}|^{1/|\mu_0|}=\max_{|\mu|\ge 1,|\nu|=l}|a_{\mu\nu}|^{1/|\mu}$.

Throughout the rest of the paper the letter C will stand for a positive constant not necessarily the same one at each occurrence.

3 Proof of the theorem

Proof of Theorem 1.1. For every $\lambda > 0$, there exists a $k_0 \in \mathbb{Z}$ such that $2^{k_0} \le \lambda < 2^{k_0+1}$. For every $f \in WH^1_\omega(\mathbb{R}^n)$, by Lemma 2.1, we can write

$$f = \sum_{k=-\infty}^{\infty} f_k = \sum_{k=-\infty}^{k_0} \sum_i h_i^k(x) + \sum_{k=k_0+1}^{\infty} \sum_i h_i^k(x) := F_1 + F_2,$$

where $\{h_i^k\}$ satisfies (a)-(c) in Lemma 2.1. Then we have

$$\begin{aligned}&\lambda\omega(\{x\in\mathbb{R}^n : |T^A f(x)| > \lambda\})\\ \le&\lambda\omega(\{x\in\mathbb{R}^n : |T^A F_1(x)| > \lambda/2\}) + \lambda\omega(\{x\in\mathbb{R}^n : |T^A F_2(x)| > \lambda/2\})\\ =&I_1 + I_2.\end{aligned}$$

We write $A_k = suppf_k$, then $A_k = \bigcup_i B_i^k$ and

$$\omega(A_k) \le \sum_i \omega(B_i^k) \le C_1 2^{-kp} \|f\|^p_{WH^1_\omega}.$$

First we claim that the following inequality holds:

$$\|F_1\|_{L^2_\omega} \le C\lambda^{1/2} \|f\|^{1/2}_{WH^1_\omega}. \tag{3.1}$$

In fact, by Lemma 2.1, we have $\text{supp}(h_i^k) \subseteq B_i^k = B(x_i^k, r_i^k)$, $\|h_i^k\|_{L^\infty} \le C2^k$, then it follows from Minkowski's integral inequality that

$$\|F_1\|_{L^2_\omega} \le \sum_{k=-\infty}^{k_0} \sum_i \|h_i^k\|_{L^2_\omega} \le \sum_{k=-\infty}^{k_0} \sum_i \|h_i^k\|_{L^\infty} \omega(B_i^k)^{1/2}.$$

For each $k\in\mathbb{Z}$, by using the bounded overlapping property of the cubes $\{B_i^k\}$ and the fact that $f \in WH^1_\omega(\mathbb{R}^n)$, we thus obtain

$$\begin{aligned}\|F_1\|_{L^2_\omega} \le& C\sum_{k=-\infty}^{k_0} 2^k \Big(\sum_i \omega(B_i^k)\Big)^{1/2} \le C \sum_{k=-\infty}^{k_0} 2^{k/2} \|f\|^{1/2}_{WH^1_\omega}\\ \le& C\sum_{k=-\infty}^{k_0} 2^{(k-k_0)/2} \cdot \lambda^{1/2} \|f\|^{1/2}_{WH^1_\omega} \le C\lambda^{1/2}\|f\|^{1/2}_{WH^1_\omega}.\end{aligned}$$

By the hypothesis $\omega \in A_1(\mathbb{R}^n)$, we have $\omega \in A_2(\mathbb{R}^n)$. Applying Chebyshev's inequality, Lemma 2.2 and the inequality (3.1), we have

$$I_1 \le \frac{4}{\lambda} \|T^A F_1\|^2_{L^2_\omega} \le C\lambda^{-1} \|F_1\|^2_{L^2_\omega} \le C\|f\|^1_{WH^1_\omega}. \tag{3.2}$$

Now we turn our attention to the estimate of I_2. We write

$$\overline{B}_i^k=B\Big(x_i^k,2\Big(\frac{3}{2}\Big)^{(k-k_0)/n}r_i^k\Big),\quad B_{k_0}=\bigcup_{k\geq k_0+1}\bigcup_i\overline{B}_i^k,$$

then we have

$$\begin{aligned}\omega(B_{k_0})&\leq C\sum_{k=k_0+1}^{\infty}\sum_i\omega(\overline{B}_i^k)\leq C\sum_{k=k_0+1}^{\infty}\sum_i\omega(B_i^k)2^n(\frac{3}{2})^{k-k_0}\\&\leq C\sum_{k=k_0+1}^{\infty}\Big(\frac{3}{2}\Big)^{k-k_0}2^{-k}\|f\|^1_{WH^1_\omega}\leq C\lambda^{-1}\|f\|^1_{WH^1_\omega}.\end{aligned}\tag{3.3}$$

Now, we divide I_2 into two parts.

$$\begin{aligned}I_2&\leq\lambda\omega(\{x\in B_{k_0}:|T^AF_2(x)|>\lambda/2\})+\lambda\omega(\{x\in(B_{k_0})^c:|T^AF_2(x)|>\lambda/2\})\\&=I_{21}+I_{22}.\end{aligned}$$

By (3.3), we have

$$I_{21}\leq\lambda\omega(B_{k_0})\leq C\|f\|^1_{WH^1_\omega}.\tag{3.4}$$

On the other hand, it follows immediately from Chebyshev's inequality that

$$I_{22}\leq 2\int_{(B_{k_0})^c}|T^AF_2(x)|\omega(x)dx\leq 2\sum_{k=k_0+1}^{\infty}\sum_i\int_{\overline{B}_i^k}|T^Ah_i^k(x)|\omega(x)dx.$$

So we only need to consider $T^Ah_i^k(x)$. Set

$$A_k(x)=A(x)-\sum_{|\alpha|=m}\frac{1}{\alpha!}m_{B_{k+n_0}}(D^\alpha A)x^\alpha,$$

where $m_{B_{k+n_0}}(D^\alpha A)$ denotes the mean value of $D^\alpha A$ on $B_{k+n_0}=B(0,2^{k+n_0})$, and n_0 is any fixed integer satisfying $2^{n_0}\geq 20\sqrt{n}$. Then $Q_{m+1}(A;x,y)=Q_{m+1}(A_k;x,y)$ and for every $q\geq 1$,

$$\Big(2^{-kn}\int_{B_{k+n_0}}|D^\alpha A_k(x)|^q dx\Big)^{1/q}\leq C\|D^\alpha A\|_{BMO}.\tag{3.5}$$

In what follows, we suppose $k\geq 2$. For $2^{k-1}<|x|\leq 2^k$, we write

$$\begin{aligned}|T^Ah_i^k(x)|&=\Big|\int_{\mathbb{R}^n}e^{iP(x,y)}\frac{\Omega(x-y)}{|x-y|^{n+m}}Q_{m+1}(A_k;x,y)h_i^k(y)dy\Big|\\&\leq\int_{\mathbb{R}^n}\Big|\frac{\Omega(x-y)}{|x-y|^{n+m}}R_m(A_k;x,y)-\frac{\Omega(x)}{|x|^{n+m}}R_m(A_k;x,0)\Big||h_i^k(y)|dy\\&\quad+\sum_{|\alpha|=m}\frac{1}{\alpha!}|D^\alpha A_k(x)|\int_{\mathbb{R}^n}\Big|\frac{\Omega(x-y)(x-y)^\alpha}{|x-y|^{n+m}}-\frac{\Omega(x)x^\alpha}{|x|^{n+m}}\Big||h_i^k(y)|dy\end{aligned}$$

$$
\begin{aligned}
&+\Big(\Big|\frac{\Omega(x)}{|x|^{n+m}}R_m(A_k;x,0)\Big|+\sum_{|\alpha|=m}\frac{1}{\alpha!}\frac{|D^\alpha A_k(x)|}{|x|^n}\Big)\Big|\int_{\mathbb{R}^n}e^{iP(x,y)}h_i^k(y)dy\Big|\\
\equiv&T^{A,1}h_i^k(x)+T^{A,2}h_i^k(x)+T^{A,3}h_i^k(x).
\end{aligned}
$$

Now by an argument similar to the proof of Theorem 1 in [6], we can obtain from Lemma 2.4 and (3.5) that

$$
\begin{aligned}
&\Big|\frac{\Omega(x-y)}{|x-y|^{n+m}}R_m(A_k;x,y)-\frac{\Omega(x)}{|x|^{n+m}}R_m(A_k;x,0)\Big|\\
\le&\Big|\frac{\Omega(x-y)}{|x-y|^{n+m}}-\frac{\Omega(x)}{|x|^{n+m}}\Big||R_m(A_k;x,y)|+\frac{|\Omega(x)|}{|x|^{n+m}}|R_m(A_k;x,y)-R_m(A_k;x,0)|\\
\le&C(|x-y|^{-n-1}+\sum_{l=0}^{m-1}|x|^{-n-m}|x-y|^l)\le C|x|^{-n-1}.
\end{aligned}
$$

Then

$$
T^{A,1}h_i^k(x)\le C|x|^{-n-1}\int_{\mathbb{R}^n}|h_i^k(y)|dy\le C2^k|x|^{-n-1}. \tag{3.6}
$$

On the other hand, using $\Omega\in Lip_1(S^{n-1})$, $2^{k-1}<|x|\le 2^k$ and $|y|\le 1$, it is easy to see that for $|\alpha|=m$,

$$
\Big|\frac{\Omega(x-y)(x-y)^\alpha}{|x-y|^{n+m}}-\frac{\Omega(x)x^\alpha}{|x|^{n+m}}\Big|\le C|x|^{-n-1},
$$

where C is independent of x and y. We can easily deduce that for $2^{k-1}<|x|\le 2^k$,

$$
T^{A,2}h_i^k(x)\le C2^k|x|^{-n-1}\sum_{|\alpha|=m}|D^\alpha A_k(x)|. \tag{3.7}
$$

For $T^{A,3}$, let φ, ψ and T_k be the same as in Lemma 2.5; then another application of Lemma 2.4 and (3.3) leads to that

$$
T^{A,3}h_i^k(x)\le C2^k|x|^{-n-1}\Big(1+\sum_{|\alpha|=m}|D^\alpha A_k(x)|\Big)|T_kh_i^k(x)|. \tag{3.8}
$$

Therefor,

$$
\begin{aligned}
I_{22}\le&C\sum_{k=k_0+1}^{\infty}\sum_i 2^k\int_{\overline{B}_i^k}|x|^{-n-1}\Big(1+\sum_{|\alpha|=m}|D^\alpha A_k(x)|\Big)\omega(x)dx\\
&+C\sum_{k=k_0+1}^{\infty}\sum_i 2^k\int_{\overline{B}_i^k}|x|^{-n-1}\Big(1+\sum_{|\alpha|=m}|D^\alpha A_k(x)|\Big)|T_kh_i^k(x)|\omega(x)dx.
\end{aligned}
$$

By Lemma 2.5 and Lemma 2.3, we have

$$
I_{22}\le C\|f\|^1_{WH^1_\omega}. \tag{3.9}
$$

Combining the above inequality (3.2) with (3.4) and (3.9), and then taking the supremum over all $\lambda>0$, we complete the proof of Theorem 1.1. □

Remark 3.1. Theorem 1.1 is new even when $\omega(x)\equiv 1$ and $P(x,y)=P(x-y)$, which also generalizes Theorem 1 in [6].

Acknowledgments

The authors would like to thank the referees for their careful review to the paper and their great help to improve the quality of this paper. The author's research was supported by the National Natural Science Foundation of China (Grant No. 11501233), China Postdoctoral Science Foundation (No. 2015M572327), Humanities and Social Sciences Program of the Ministry of Education (No. 15YJC630053), Natural Science Foundation of Anhui Province (No. 1408085MA08 and No. 1508085SMA204) and Natural Science Foundation of the Education Department of Anhui Province (No. KJ2015A335 and No. KJ2015A270).

References

[1] F. Ricci and E. Stein, Hormonic analysis on nilpotent groups and singular integrals (I), oscillatory integral, J. Funct. Anal., 73 (1987), 179–194.
[2] S. Chanillo and M. Christ, Weak $(1,1)$ bounds for oscillatory singular integrals, Duke Math. J., 55(1) (1987), 141–155.
[3] Y. B. Pan, Hardy space and oscillatory singular integrals, Rev. Math. Iberoamericana., 7 (1991), 55–64.
[4] Y. Hu and Y. B. Pan, Boundedness of oscillatory singular integrals on Hardy spaces, Ark. Math., 30 (1992), 311–320.
[5] G. E. Hu and D. C. Yang, Multilinear oscillatory singular integral oprators on Hardy spaces, Chinese J. Contemporary Math., 18 (1997), 403–413.
[6] G. E. Hu, Q. Wu and D. C. Yang, Boundedness of multilinear oscillatory singular integrals on Hardy type spaces, Hiroshima Math. J., 30 (2000), 301–315.
[7] B. Muckenhoupt, Weighted norm inequalities for the Hardy maximal function, Trans. Amer. Math. Soc., 165 (1972), 207–226.
[8] T. S. Quek and D. C. Yang, Calerón-Zygmund type operators on weight weak Hardy spaces over $\mathbb{R}^n$, Acta. Math. Sin., 44(3) (2001), 517–526.
[9] J. Garcia-Cuerva and J. Rubio de Francia, Weighted Norm Inequalities and Related Topics, North-Holland, Amsterdam, 1985.
[10] J. Cohen and J. Gosselin, A BMO estimate for multilinear singular integrals, Illinois J. Math., 30 (1986), 445–464.