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Oscillatory Strongly Singular Integral Associated to the Convex Surfaces of Revolution

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Abstract. Here we consider the following strongly singular integral

$$T_{\Omega,\gamma,\alpha,\beta}f(x,t)=\int_{R^n} e^{i|y|^{-\beta}}\frac{\Omega(\frac{y}{|y|})}{|y|^{n+\alpha}}f(x-y,t-\gamma(|y|))dy,$$

where $\Omega\in L^p(S^{n-1})$, $p>1$, $n>1$, $\alpha>0$ and γ is convex on $(0,\infty)$.

We prove that there exists $A(p,n)>0$ such that if $\beta>A(p,n)(1+\alpha)$, then $T_{\Omega,\gamma,\alpha,\beta}$ is bounded from $L^2(R^{n+1})$ to itself and the constant is independent of γ. Furthermore, when $\Omega\in C^\infty(S^{n-1})$, we will show that $T_{\Omega,\gamma,\alpha,\beta}$ is bounded from $L^2(R^{n+1})$ to itself only if $\beta>2\alpha$ and the constant is independent of γ.

Key Words: Oscillatory strongly rough singular integral, rough kernel, surfaces of revolution.

AMS Subject Classifications: 42B20, 42B35

1 Introduction

The standard Hilbert transform along a curve is defined as

$$H_\Gamma f(x)=P.V.\int_{-1}^{1} f(x-\Gamma(t))\frac{dt}{t},$$

where $\Gamma:(-1,1)\to R^n$ is a continuous curve in R^n. The study of these operators was initiated by Fabes and Rivière [7]. In [18], Stein and Wainger proved that H_Γ is bounded on $L^p(1<p<\infty)$ if Γ is well-curved in R^n. Here we say that Γ is well-curved, if Γ is smooth with $\Gamma(0)=0$ and a segment of the curve containing the origin lies in a subspace of R^n spanned by

$$\frac{d^{(k)}\Gamma(t)}{dt}\Big|_{t=0},\quad k=1,2,\cdots.$$

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When $n=2$, $\Gamma(t)$ can be written as $(t,\gamma(t))$. If $\gamma(t)$ is flat at the origin, i.e.,

$$\frac{d^{(k)}\gamma(t)}{dt}\Big|_{t=0}=0$$

for $k=1,2,\cdots$, then it is easy to see that $\Gamma(t)$ is not well-curved in R^2. The main contributions on the Hilbert transforms along the flat curves were made by Wainger and his colleagues. Readers can see [1,12–15,19,21,22] among numerous references, in particular, the good survey papers [18] and [23].

Another interesting operator in harmonic analysis is the hyper Hilbert transform

$$H_\alpha f(x)=P.V.\int_{-1}^{1} f(x-t)\frac{dt}{t|t|^\alpha},\quad 0<\alpha<1.$$

We know that the operator H_α is bounded from the Sobolev space L^p_α to the Lebesgue space L^p for $1<p<\infty$, because of the mean zero of the kernel of H_α. One naturally expected that, without the assumption of the mean zero on the kernel, the worsened singularity of H_α near the origin can be counterbalanced by an oscillatory factor $e^{i|t|^{-\beta}}(\beta>0)$ as t approaches zero. This idea motivated the study of the oscillatory hyper Hilbert transforms (see [10]) and the strongly singular integral operators in high dimensional spaces. More details of the strongly singular integral operators can be found in [8, 9, 16, 20].

Consider the following oscillatory hyper Hilbert transforms,

$$H_{\Gamma,\alpha,\beta}f(x)=\int_{-1}^{1} f(x-\Gamma(t))e^{i|t|^{-\beta}}\frac{dt}{t|t|^\alpha},\quad \alpha,\beta\geq 0,$$

where $\Gamma(0)=0$, $\beta>\alpha$.

Zielinski [24] studied the L^2-boundedness of $H_{\Gamma,\alpha,\beta}$ along the parabola (t,t^2). In [2], for $\Gamma(t)=(t,|t|^q), q\geq 2$, Chandarana proved that $H_{\Gamma,\alpha,\beta}$ is bounded on L^2 if and only if $\beta\geq 3\alpha$. When $n=3$, a similar result was proved in [3].

In [4] and [5], we generalized these results in R^n and removed all assumptions on the indexes. At the same time, Laghi and Lyall in [11] proved that if $\Gamma(t)$ is well-curved, then $H_{\Gamma,\alpha,\beta}$ is bounded on $L^2(R^n)$ if and only if $\beta\geq(n+1)\alpha$.

In [6] we study the general case $\Gamma(t)=(t,\gamma(t))$ in R^2 where γ is flat on $(0,1)$. We obtain some interesting results. In the same paper, we construct some examples to illustrate the complexity of this problem.

Here we consider the following oscillatory strongly singular integral associated to the surfaces of revolution,

$$T_{\Omega,\gamma,\alpha,\beta}f(x,t)=\int_{R^n} e^{i|y|^{-\beta}}\frac{\Omega(\frac{y}{|y|})}{|y|^{n+\alpha}}f(x-y,t-\gamma(|y|))dy. \tag{1.1}$$

At first, when $\alpha=0$ and Ω is mean zero on S^{n-1}, by virtue of the decay estimates of the Fourier transform of Ω on S^{n-1}, T is bounded from L^2 to itself without any assumptions on γ. It is easy to see that this arguments does not work when $\alpha>0$.

If $H_{\gamma,\alpha,\beta}$ is bounded from L^2 to itself when $n=1$, then $T_{\Omega,\gamma,\alpha,\beta}$ is bounded from L^2 to itself for any $n>1$ only if $\Omega\in L^1(S^{n-1})$. But in [6] we have seen that, for general $\alpha,\beta>0$ and a convex curve $\Gamma(t)=(t,\gamma(t))$, $H_{\gamma,\alpha,\beta}$ is not bounded from L^2 to itself in general.

Here we will see that for the general convex curve, the L^2-boundedness of $T_{\Omega,\gamma,\alpha,\beta}$ is more simple, even if the kernel is rough. Our main results are stated as the following two theorems.

Theorem 1.1. *Suppose that $\Omega\in L^p(S^{n-1})$, $p>1$, $n>1$, $\alpha>0$ and γ is convex on $(0,\infty)$. There exists $A(p,n)>0$ which depends only on p, n such that if $\beta>A(p,n)(1+\alpha)$, then we have*

$$\|T_{\Omega,\gamma,\alpha,\beta}f\|_2\le C\|f\|_2,$$

where C is independent of γ and f.

Furthermore, if $\Omega\in C^\infty(S^{n-1})$, we have

Theorem 1.2. *Assume that $\Omega\in C^\infty(S^{n-1})$, $n>1$, $\alpha>0$ and γ is convex on $(0,\infty)$. If $\beta>2\alpha$, then we have*

$$\|T_{\Omega,\gamma,\alpha,\beta}f\|_2\le C\|f\|_2.$$

Throughout this note the letters C and c always denote two positive constants which depend only on α, β, p and n. They maybe vary in different cases. In general, we take C big enough and c small enough.

2 Proof of the main theorems

Firstly, we need the following Van der Corput lemma.

Lemma 2.1 (Van der Corput"s Lemma). *If ψ, ϕ are two smooth functions on the interval (a,b) and $|\psi^{(k)}(t)|\ge\lambda>1$ on $t\in(a,b)$ for some $k\in N$, then we have*

$$\Big|\int_a^b e^{i\psi(t)}\phi(t)dt\Big|\le C_k\lambda^{-1/k}(|\phi(b)|+\int_a^b|\phi'(t)|dt),\quad (k\ge2),$$

or

$$\Big|\int_a^b e^{i\psi(t)}\phi(t)dt\Big|\le C_1\lambda^{-1}\Big(|\phi(b)|+\int_a^b|\phi'(t)|dt+\int_a^b\frac{|\phi(t)\psi''(t)|}{|\psi'(t)|^2}dt\Big),\quad (k=1).$$

The lemma and its proof can be found in [17]. When $k=1$, the result can be checked directly by the integral by parts.

We also need the following decay estimates of the Fourier transform of Ω on S^{n-1}.

Lemma 2.2 (see [17]). *If $\Omega\in C^\infty(S^{n-1})$ and $d\mu=\Omega d\theta$, then we have the following estimate*

$$|\widehat{d\mu}(\xi)|=\Big|\int_{S^{n-1}}\Omega(\theta)e^{i\theta\cdot\xi}d\theta\Big|\le C(1+|\xi|)^{-\frac{n-1}{2}}.$$

Lemma 2.3 (see [17]). *If $\Omega\in L^p(S^{n-1})$ for some $p>1$ and $d\mu=\Omega d\theta$, then there exists two positive constants C and ϵ (<1) such that*

$$\int_0^1|\widehat{d\mu}(t\xi)|^2dt\le C(1+|\xi|)^{-\epsilon}.$$

Proof of Theorem 1.1. Now we begin to prove Theorem 1.1. Using the inverse of Fourier transform we have

$$\begin{aligned}T_{\Omega,\gamma,\alpha,\beta}f(x)&=\int_{R^n}e^{i|y|^{-\beta}}\frac{\Omega(\frac{y}{|y|})}{|y|^{n+\alpha}}f(x-y,t-\gamma(|y|))dy\\&=\int_{R^n}\int_{R^{n+1}}e^{i[(x-y)\xi+(t-\gamma(|y|))\eta]}\hat{f}(\xi,\eta)d\xi d\eta e^{i|y|^{-\beta}}\frac{\Omega(\frac{y}{|y|})}{|y|^{n+\alpha}}dy\\&=\int_{R^{n+1}}e^{i(x\xi+t\eta)}\int_{R^n}e^{i(|y|^{-\beta}-y\cdot\xi-\gamma(|y|)\eta)}\frac{\Omega(\frac{y}{|y|})}{|y|^{n+\alpha}}dy\hat{f}(\xi,\eta)d\xi d\eta\\&=\int_{R^{n+1}}e^{i(x\xi+t\eta)}\int_0^\infty\int_{S^{n-1}}e^{i(r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\hat{f}(\xi,\eta)d\xi d\eta,\end{aligned}$$

which implies that

$$\widehat{T_{\Omega,\gamma,\alpha,\beta}f}(\xi,\eta)=m(\xi,\eta)\hat{f}(\xi,\eta),\tag{2.1}$$

where

$$\begin{aligned}m(\xi,\eta)&=\int_0^\infty\int_{S^{n-1}}e^{i(r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\\&=\int_0^1\int_{S^{n-1}}e^{i(r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr+\int_1^\infty\int_{S^{n-1}}e^{i(r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\\&=\bar{m}(\xi,\eta)+m_0(\xi,\eta).\end{aligned}\tag{2.2}$$

To prove the L^2 boundedness of $T_{\Omega,\gamma,\alpha,\beta}$, by Plancherel equality, we only need to show that $m(\xi,\eta)$ is a bounded function.

It is easy to check that when $\alpha>0$,

$$\begin{aligned}|m_0(\xi,\eta)|&=\Big|\int_1^\infty\int_{S^{n-1}}e^{i(r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\Big|\\&\le\int_1^\infty\int_{S^{n-1}}\frac{|\Omega(\theta)|}{r^{1+\alpha}}d\theta dr\\&\le C\|\Omega\|_{L^1(S^{n-1})}.\end{aligned}\tag{2.3}$$

Set $\psi(r)=r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta$. For any $(\xi,\eta)\in R^n\times R$, we estimate the bounds of $\bar{m}(\xi,\eta)$ in two case.

Case 1: $\eta\leq 0$. As $\gamma''(r)\geq 0$ on $(0,\infty)$, there holds

$$\psi''(r)=\beta(\beta+1)r^{-\beta-2}-\gamma''(r)\eta\geq\beta(\beta+1)r^{-\beta-2}.$$

If $\beta>2\alpha$, by Van der Corput's Lemma, we can get that

$$\begin{aligned}\Big|\int_0^1 e^{i\psi(r)}\frac{dr}{r^{1+\alpha}}\Big|&\leq\sum_{j\geq 1}\Big|\int_{2^{-j}}^{2^{1-j}} e^{i\psi(r)}\frac{dr}{r^{1+\alpha}}\Big|\\&\leq C\sum_{j\geq 1}2^{-\frac{j(\beta+2)}{2}}\Big(2^{j(1+\alpha)}+\int_{2^{-j}}^{2^{1-j}}\frac{dr}{r^{2+\alpha}}\Big)\\&\leq C\sum_{j\geq 1}2^{j(\alpha-\frac{\beta}{2})}\\&\leq C,\end{aligned}$$

which implies that

$$\begin{aligned}|\bar{m}(\xi,\eta)|&=\Big|\int_0^1\int_{S^{n-1}}e^{i\psi(r)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\Big|\\&=\Big|\int_{S^{n-1}}\Omega(\theta)\int_0^1 e^{i\psi(r)}\frac{dr}{r^{1+\alpha}}d\theta\Big|\\&\leq C\int_{S^{n-1}}|\Omega(\theta)|d\theta\\&=C\|\Omega\|_{L^1(S^{n-1})},\end{aligned}\tag{2.4}$$

where C does not depend on ξ, η and γ.

Case 2: $\eta>0$. In this case we divide $\bar{m}(\xi,\eta)$ into two parts. Set

$$t(\xi)=\Big(\frac{2}{\beta}(1+|\xi|)\Big)^{-\frac{1}{\beta+1}}.$$

Then one have

$$\begin{aligned}\bar{m}(\xi,\eta)&=\int_0^1\int_{S^{n-1}}e^{i\psi(r)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\\&=\int_{t(\xi)}^1\int_{S^{n-1}}e^{i\psi(r)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr+\int_0^{t(\xi)}\int_{S^{n-1}}e^{i\psi(r)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\\&=m_1(\xi,\eta)+m_2(\xi,\eta).\end{aligned}\tag{2.5}$$

Set $d\mu=\Omega(\theta)d\theta$. For the term $m_1(\xi,\eta)$, by Lemma 2.3 we can obtain that

$$\begin{aligned}|m_1(\xi,\eta)|=&\Big|\int_{t(\xi)}^1\int_{S^{n-1}}e^{i(r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\Big|\\ =&\Big|\int_{t(\xi)}^1\int_{S^{n-1}}e^{ir\theta\cdot\xi}\Omega(\theta)d\theta e^{i(r^{-\beta}-\gamma(r)\eta)}r^{-1-\alpha}dr\Big|\\ \leq&\int_{t(\xi)}^1|\widehat{d\mu}(r\xi)|r^{-1-\alpha}dr\\ \leq&\Big(\int_0^1|\widehat{d\mu}(r\xi)|^2dr\Big)^{\frac{1}{2}}\Big(\int_{t(\xi)}^1r^{-2-2\alpha}dr\Big)^{\frac{1}{2}}\\ \leq&C(1+|\xi|)^{-\frac{\epsilon}{2}}t(\xi)^{-\frac{1}{2}-\alpha}\\ \leq&C(1+|\xi|)^{\frac{\frac{1}{2}+\alpha}{\beta+1}-\frac{\epsilon}{2}}.\end{aligned}\tag{2.6}$$

If $\beta\geq\frac{2}{\epsilon}(\alpha+1)$, then

$$\frac{\frac{1}{2}+\alpha}{\beta+1}-\frac{\epsilon}{2}<\frac{\epsilon}{2}\Big(\frac{\frac{1}{2}+\alpha}{\alpha+2}-1\Big)<0.$$

So (2.6) yields that

$$|m_1(\xi,\eta)|\leq C(1+|\xi|)^{\frac{\frac{1}{2}+\alpha}{\beta+1}-\frac{\epsilon}{2}}\leq C.\tag{2.7}$$

On the other hand, for the term $m_2(\xi,\eta)$, as

$$t(\xi)=\Big(\frac{2}{\beta}(1+|\xi|)\Big)^{-\frac{1}{\beta+1}}>r,$$

we get that

$$|\xi|<\frac{\beta}{2}r^{-\beta-1}.$$

So for the derivative of $\psi(r)$ when $r<t(\xi)$ we have

$$\begin{aligned}|\psi'(r)|=&|-\beta r^{-\beta-1}-\theta\cdot\xi-\gamma'(r)\eta|\geq\beta r^{-\beta-1}+\gamma'(r)\eta-|\xi|\\ \geq&\frac{\beta}{2}r^{-\beta-1}+\gamma'(r)\eta\geq\frac{\beta}{2}r^{-\beta-1}.\end{aligned}\tag{2.8}$$

Now, by Lemma 2.1, (2.8) and the convexity of γ one can obtain that

$$\begin{aligned}\Big|\int_0^{t(\xi)}e^{i\psi(r)}\frac{dr}{r^{1+\alpha}}\Big|\leq&\sum_{j>0}\Big|\int_{2^{-j}t(\xi)}^{2^{1-j}t(\xi)}e^{i\psi(r)}\frac{dr}{r^{1+\alpha}}\Big|\\ \leq&C\sum_{j>0}(2^{-j}t(\xi))^{\beta+1}\Big((2^{-j}t(\xi))^{-\alpha-1}+\int_{2^{-j}t(\xi)}^{2^{1-j}t(\xi)}\frac{|\psi''(r)|}{r^{1+\alpha}\psi'(r)^2}dr\Big)\end{aligned}$$

$$
\begin{aligned}
&\le C\sum_{j>0}(2^{-j}t(\xi))^{\beta-\alpha}\Big(1+\int_{2^{-j}t(\xi)}^{2^{1-j}t(\xi)}\frac{|\beta(\beta+1)r^{-\beta-2}-\gamma''(r)\eta|}{(\frac{\beta}{2}r^{-\beta-1}+\gamma'(r)\eta)^2}dr\Big)\\
&\le C\sum_{j>0}(2^{-j}t(\xi))^{\beta-\alpha}\Big(1+\int_{2^{-j}t(\xi)}^{2^{1-j}t(\xi)}\Big(\frac{\beta(\beta+1)r^{-\beta-2}}{(\frac{\beta}{2}r^{-\beta-1}+\gamma'(r)\eta)^2}+\frac{\gamma''(r)\eta}{(\frac{\beta}{2}r^{-\beta-1}+\gamma'(r)\eta)^2}\Big)dr\Big)\\
&\le C\sum_{j>0}(2^{-j}t(\xi))^{\beta-\alpha}\Big(1+\int_{2^{-j}t(\xi)}^{2^{1-j}t(\xi)}\Big(r^{\beta}+\frac{\gamma''(r)\eta}{(1+\gamma'(r)\eta)^2}\Big)dr\Big)\\
&\le C\sum_{j>0}(2^{-j}t(\xi))^{\beta-\alpha}\Big(1+\frac{1}{1+\gamma'(2^{-j}t(\xi))\eta}-\frac{1}{1+\gamma'(2^{1-j}t(\xi))\eta}\Big)\\
&\le C\sum_{j>0}(2^{-j}t(\xi))^{\beta-\alpha}=Ct(\xi)^{\beta-\alpha}\le C. \qquad (2.9)
\end{aligned}
$$

So we have

$$
\begin{aligned}
|m_2(\xi,\eta)|&=\Big|\int_0^{t(\xi)}\int_{S^{n-1}}e^{i\psi(r)}\frac{dr}{r^{1+\alpha}}\Omega(\theta)d\theta dr\Big|\\
&\le\Big|\int_{S^{n-1}}\Omega(\theta)\int_0^{t(\xi)}e^{i\psi(r)}\frac{dr}{r^{1+\alpha}}drd\theta\Big|\\
&\le C\|\Omega\|_{L^1(S^{n-1})}. \qquad (2.10)
\end{aligned}
$$

(2.5), (2.7) and (2.10) yield that when $\eta>0$, there holds

$$
|\bar{m}(\xi,\eta)|\le|m_1(\xi,\eta)|+|m_2(\xi,\eta)|\le C. \qquad (2.11)
$$

At last let everything together. When $\beta>(\frac{2}{\epsilon})(\alpha+1)$ and γ is convex, from (2.3), (2.4) and (2.11) we can obtain that

$$
|m(\xi,\eta)|\le|m_0(\xi,\eta)|+|\bar{m}(\xi,\eta)|\le C,
$$

which means that the operator $T_{\Omega,\gamma,\alpha,\beta}$ is bounded from L^2 to itself. So we complete the proof of Theorem 1.1. □

Proof of Theorem 1.2. Now we turn to prove Theorem 1.2. In the proof of Theorem 1.1, it is easy to see that the estimates (2.3), (2.4) and (2.10) remain true only if $\beta>2\alpha$. When $\Omega\in C^{\infty}(S^{n-1})$, $n>1$, we check that the estimate (2.7) holds only if $\beta>2\alpha$. By Lemma 2.2 and the similar computations as in (2.6), we get that

$$
\begin{aligned}
|m_1(\xi,\eta)|&=\Big|\int_{t(\xi)}^1\int_{S^{n-1}}e^{i(r^{-\beta}-r\theta\cdot\xi-\gamma(r)\eta)}\frac{\Omega(\theta)}{r^{1+\alpha}}d\theta dr\Big|\\
&=\Big|\int_{t(\xi)}^1\int_{S^{n-1}}e^{ir\theta\cdot\xi}\Omega(\theta)d\theta e^{i(r^{-\beta}-\gamma(r)\eta)}r^{-1-\alpha}dr\Big|\\
&\le\int_{t(\xi)}^1|\widehat{d\mu}(r\xi)|r^{-1-\alpha}dr
\end{aligned}
$$

$$\begin{aligned}
&\leq C\int_{t(\xi)}^{1}(1+r|\xi|)^{-\frac{n-1}{2}}r^{-1-\alpha}dr\\
&\leq C(1+|\xi|)^{-\frac{n-1}{2}}\int_{t(\xi)}^{1}r^{-\frac{n-1}{2}}r^{-1-\alpha}dr\\
&\leq C(1+|\xi|)^{-\frac{n-1}{2}}t(\xi)^{-\frac{n-1}{2}-\alpha}\\
&=C(1+|\xi|)^{\frac{2\alpha-(n-1)\beta}{2(\beta+1)}}\\
&\leq C.
\end{aligned}\tag{2.12}$$

At last, by the same arguments as in the proof of Theorem 1.1 we can show that the operator $T_{\Omega,\gamma,\alpha,\beta}$ is bounded from L^2 to itself. So we prove Theorem 1.2. □

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