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Points of Coincidence and Common Fixed Points for II-Expansive Mappings on Complex Valued Metric Spaces

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Abstract. We use the two mappings satisfying II-expansive conditions on complex valued metric spaces to construct the convergent sequences and prove that the unique limit of the sequences is the point of coincidence or common fixed point of the two mappings. Also, we discuss the uniqueness of points of coincidence or common fixed points and give the existence theorems of unique fixed points. The obtained results generalize and improve the corresponding conclusions in references.

Key Words: Complex valued metric space, II-expansive mapping, Cauchy sequence, point of coincidence, common fixed point.

AMS Subject Classifications: 47H05, 47H10, 54E40, 54H25

1 Introduction and preliminaries

Real metric spaces have been widely generalized and improved by cone metric spaces [1] and topological vector space-valued cone metric spaces [2, 3] and so on. A number of authors discussed and obtained some fixed point and common fixed point theorems on these spaces, greatly generalized and improved some corresponding results.

Recently, the author in [4] defined complex valued metric spaces on a nonempty set X and obtained coincidence point theorems and common fixed point theorems for two mappings satisfying some contractive conditions on this space. The authors in [5–7] generalized and extended the results in [4]. The authors [8, 9] discussed the existence problems of common fixed points for two mappings satisfying expansive conditions. These results further enrich and improve the existence theory of coincidence points and common fixed points on complex valued metric spaces.

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In this paper, by weakening the condition and using a new method of proof, we generalize a known result [9, Theorem 3.1] and give another unique common fixed point theorem for two mappings satisfying a II-expansive condition.

In what follows, we recall some notations and definitions that will be utilized in our subsequent discussion.

Let $\mathbb{C}$ be the set of complex numbers and $z_1,z_2\in\mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:

$$z_1\precsim z_2 \Leftrightarrow [\mathrm{Re}(z_1)\le \mathrm{Re}(z_2)]\wedge[\mathrm{Im}(z_1)\le \mathrm{Im}(z_2)].$$

Consequently, $z_1\precsim z_2$ if and only if one of the following conditions is satisfied:

(C1) $\mathrm{Re}(z_1)=\mathrm{Re}z_2$, $\mathrm{Im}z_1=\mathrm{Im}z_2$;

(C2) $\mathrm{Re}(z_1)<\mathrm{Re}z_2$, $\mathrm{Im}z_1=\mathrm{Im}z_2$;

(C3) $\mathrm{Re}(z_1)=\mathrm{Re}z_2$, $\mathrm{Im}z_1<\mathrm{Im}z_2$;

(C4) $\mathrm{Re}(z_1)<\mathrm{Re}z_2$, $\mathrm{Im}z_1<\mathrm{Im}z_2$.

In particular, we write $z_1\precnsim z_2$ if $z_1\neq z_2$ and one of (C2), (C3), (C4) is satisfied, and we write $z_1<z_2$ if only (C4) is satisfied.

Obviously, the following statements hold:

(i) If $b\ge a\ge 0$, then $az\precsim bz$ for any $z\in\mathbb{C}$ with $0\precsim z$;

(ii) if $0\precsim z_1\precnsim z_2$, then $|z_1|<|z_2|$;

(iii) if $z_1\precsim z_2$ and $z_2<z_3$, then $z_1<z_3$;

(iv) if $z_1\precsim z_2$ and $z\in\mathbb{C}$, then $z+z_1\precsim z+z_2$.

Definition 1.1 (see [4–7])**.** Let X be a nonempty set. If a mapping $d:X\times X\to\mathbb{C}$ satisfies the following conditions:

(i) $0\precsim d(x,y)$ for all $x,y\in X$, and $d(x,y)=0$ if and only if $x=y$;

(ii) $d(x,y)=d(y,x)$ for all $x,y\in X$;

(iii) $d(x,z)\precsim d(x,y)+d(y,z)$ for all $x,y,z,\in X$.

Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

Example 1.1 (see [4])**.** Let $X=\mathbb{C}$. Define a mapping $d:X\times X\to\mathbb{C}$ as follows

$$d(z_1,z_2)=e^{ik}|z_1-z_2|,\quad \forall z_1,z_2\in X,$$

where $k\in\mathbb{R}$. Then (X,d) is a complex valued metric space.

Example 1.2. Let $X=\{a,b,c\}$. Define a mapping $d:X\times X\to\mathbb{C}$ by

$$\begin{aligned} &d(a,a)=d(b,b)=d(c,c)=0, && d(a,b)=d(b,a)=3+4i,\\ &d(a,c)=d(c,a)=2+3i, && d(b,c)=d(c,b)=4+5i. \end{aligned}$$

Obviously, (X,d) is a complex valued metric space.

Definition 1.2 (see [4–7])**.** Let $\{x_n\}_{n\geq 1}$ be a sequence in a complex valued metric space (X,d) and $x\in X$.

(i) If for any $c\in\mathbb{C}$ with $0<c$, there exists $n_0\in\mathbb{N}$ such that for all $n>n_0$, $d(x_n,x)<c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to $x\in X$ and x is the limit point of $\{x_n\}$. We denote this by $\lim_{n\to\infty}x_n=x$ or $x_n\to x$ as $n\to\infty$.

(ii) If for any $c\in\mathbb{C}$ with $0<c$, there exists $n_0\in\mathbb{N}$ such that for all $n>n_0$ and any $m\in\mathbb{N}$, $d(x_n,x_{n+m})<c$, then $\{x_n\}$ is said to be a Cauchy sequence.

(iii) If every Cauchy sequence in X is convergent, then X is said to be complete.

Definition 1.3. Let (X,d) a complex valued metric space, $f,g:X\to X$ two mappings. If for each $x,y\in X$,

$$d(fx,fy)\gtrsim \alpha d(gx,fx)+\beta d(gy,fy)+\gamma d(gx,gy),$$

where $\alpha,\beta,\gamma\geq 0$ and $\alpha+\beta+\gamma>1$. Then f is called II-expansive with respect to g.

If X is a real metric space and $g=1_X$, then f is called a II-expansive mapping [10].

Definition 1.4 (see [11])**.** Let X be a nonempty set, $f,g:X\to X$ two mappings. f and g are called weakly compatible if $x\in X$ and $fx=gx$, then $fgx=gfx$.

Definition 1.5 (see [12])**.** Let X be a nonempty set, $f,g:X\to X$ two mappings. If there exist $x,w\in X$ such that $w=fx=gx$, then x is called a coincident point of f and g, w is called a point of coincidence of f and g.

Lemma 1.1 (see [8])**.** *If two sequences $\{x_n\}$ and $\{y_n\}$ in a complex valued metric space (X,d) converge to x and y in X respectively. Then*

$$\lim_{n\to\infty}d(x_n,y_n)=d(x,y);\quad \lim_{n\to\infty}|d(x_n,y_n)|=|d(x,y)|.$$

Lemma 1.2 (Cauchy Principle, see [9])**.** *Let $\{x_n\}$ be a sequence in a complex valued metric space (X,d). If there exists $0\leq h<1$ such that for all $n\in\mathbb{N}$,*

$$d(x_{n+1},x_n)\lesssim hd(x_n,x_{n-1}).$$

Then $\{x_n\}$ is a Cauchy sequence.

Lemma 1.3 (see [12])**.** *If $f,g:X\to X$ are weakly compatible and have an unique point of coincidence w, then w is the unique common fixed point of f and g.*

2 Point of coincidence and common fixed point

Theorem 2.1. *Let (X,d) be a complex valued metric space, $f,g:X\to X$ two mappings. Suppose that $fX\supset gX$ and for each $x,y\in X$ with $x\neq y$,*

$$d(fx,fy)\gtrsim \alpha d(gx,fx)+\beta d(gy,fy)+\gamma d(gx,gy), \tag{2.1}$$

where $\alpha,\beta\in\mathbb{R}$, $\gamma\geq -1$. If (i) fX or gX is complete; (ii) $\alpha+\beta+\gamma>1$. Then f and g have coincidence point.

Proof. At first, (ii) implies $\alpha+\gamma>0$ or $\beta+\gamma>0$. Otherwise, if $\alpha+\gamma\leq 0$ and $\beta+\gamma\leq 0$, hence $\alpha+\beta+2\gamma\leq 0$, therefore $\alpha+\beta+\gamma\leq -\gamma\leq 1$. This is a contradiction.

Take $x_0\in X$. By $fX\supset gX$, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ such that

$$y_n=gx_n=fx_{n+1},\quad n=0,1,2,\cdots.$$

If there exists n such that $x_n=x_{n+1}$, then x_n is the coincidence point of f and g. Hence we may assume that $x_n\neq x_{n+1}$, $\forall n=0,1,\cdots$.

Suppose that $\beta+\gamma>0$. Taking $x=x_{n+1}$ and $y=x_{n+2}$, using (2.1) and calculating, we obtain

$$d(y_n,y_{n+1})\gtrsim \alpha d(y_n,y_{n+1})+(\beta+\gamma)d(y_{n+1},y_{n+2}),$$

i.e.,

$$(1-\alpha)d(y_n,y_{n+1})\gtrsim(\beta+\gamma)d(y_{n+1},y_{n+2}),\quad \forall n=0,1,2.$$

If $1-\alpha<0$, then $d(y_n,y_{n+1})=d(y_{n+1},y_{n+2})=0$ for all $n\in\mathbb{N}$ by the above, this means that $\{y_n\}$ is a constant sequence, hence it is a Cauchy sequence. So we may assume that $1-\alpha\geq 0$. In this case, we obtain $0\leq\frac{1-\alpha}{\beta+\gamma}<1$ and

$$d(y_{n+1},y_{n+2})\lesssim\frac{1-\alpha}{\beta+\gamma}d(y_n,y_{n+1}),\quad \forall n=0,1,2,\cdots. \tag{2.2}$$

Suppose that $\alpha+\gamma>0$. Taking $x=x_{n+2}$ and $y=x_{n+1}$, using (2.1) and calculating, we have

$$d(y_n,y_{n+1},a)\gtrsim(\alpha+\gamma)d(y_{n+1},y_{n+2})+\beta d(y_n,y_{n+1}),$$

i.e.,

$$(1-\beta)d(y_n,y_{n+1})\gtrsim(\alpha+\gamma)d(y_{n+1},y_{n+2}),\quad \forall n=0,1,2,\cdots.$$

As the case of $\beta+\gamma>0$, we may assume that $1-\beta\geq 0$. In this case, we have $0\leq\frac{1-\beta}{\alpha+\gamma}<1$ and

$$d(y_{n+1},y_{n+2})\lesssim\frac{1-\beta}{\alpha+\gamma}d(y_n,y_{n+1}),\quad \forall n=0,1,2,\cdots. \tag{2.3}$$

Let

$$h=\max\Big\{\frac{1-\beta}{\alpha+\gamma},\frac{1-\alpha}{\beta+\gamma}\Big\},$$

then $0\leq h<1$. Combining (2.2) and (2.3), we have

$$d(y_{n+1},y_{n+2})\lesssim hd(y_n,y_{n+1}),\quad \forall n=0,1,2,\cdots.$$

Hence $\{y_n\}$ is a Cauchy sequence by Lemma 1.2.

Suppose that fX is complete. Since $y_n = gx_n = fx_{n+1} \in fX$, there exist $u,p \in X$ such that $y_n \to u = fp$.

If $\beta+\gamma>0$, then taking $x=x_{n+1}$ and $y=p$, using (2.1) and calculating, we have

$$d(y_n,fp) \gtrsim \alpha d(y_n,y_{n+1}) + \beta d(gp,fp) + \gamma d(y_{n+1},gp).$$

Let $n\to\infty$ in the above, then using Lemma 1.1 and Cauchy sequence $\{y_n\}$, we obtain

$$0 \gtrsim (\beta+\gamma) d(fp,gp),$$

hence

$$fp = gp = u.$$

If $\alpha+\gamma>0$, then taking $x=p$ and $y=x_{n+1}$, using (2.1) and calculating, we have,

$$d(fp,y_n) \gtrsim \alpha d(gp,fp) + \beta d(y_{n+1},y_n) + \gamma d(gp,y_{n+1}).$$

Let $n\to\infty$ in the above, then similarly,we have

$$fp = gp = u.$$

Hence u is a point of coincidence of f and g, p is a coincidence point of f and g for any case.

If gX is complete, then there exist $u,p,q \in X$ such that $y_n = gx_n \to u = gq = fp$. The rest proof is similar to the case of fX being complete. So we omit the proof. □

Remark 2.1. The conditions in Theorem 2.1 are vary weaker than those in [9, Theorem 3.1]. In [9], $\alpha,\beta,\gamma \geq 0$ satisfy $\alpha+\beta+\gamma>1$ and $\alpha<1$ or $\beta<1$. On the other hand, The method of proof in Theorem 2.1 is better than that in [9, Theorem 3.1] and the process of proof is very simple and easy to understand.

Theorem 2.2. *Let (X,d) be a complex valued metric space, $f,g: X\to X$ two mappings such that $fX \supset gX$ and* (2.1) *holds, where $\alpha,\beta,\gamma \in \mathbb{R}$. If (i) fX or gX is complete; (ii)* $\min\{\alpha+\beta+\gamma,\gamma\}>1$, *(iii) f and g are weakly compatible. Then f and g have an unique common fixed point.*

Proof. By Theorem 2.1, there exist $u,p \in X$ such that $u = fp = gp$. Suppose that there exist $v,z \in X$ such that $v = fz = gz$, then taking $x=p$ and $y=z$, using (2.1), we obtain

$$d(u,v) = d(fp,fz) \gtrsim \gamma d(gp,gz) = \gamma d(u,v),$$

hence $d(u,v)=0$ by $\gamma>1$, so $u=v$, that is, f and g have an unique point u of coincidence. Hence u is an unique common fixed point of f and g by Lemma 1.3. □

Example 2.1. Consider the complete complex valued metric space (X,d) in Example 1.2. Define two mappings $f,g: X\to X$ as follows

$$fa=a,\quad fb=c,\quad fc=b,\quad ga=a,\quad gb=a,\quad gc=c.$$

Obviously, $gX \subset fX = X$, f and g are weakly compatible. Let $\alpha=\frac{1}{16}$, $\beta=\frac{-2}{16}$, $\gamma=\frac{18}{16}$, then $\gamma>\alpha+\beta+\gamma>1$.

It is easy to check

$$
\begin{aligned}
d(fa,fb)&=2+3i\gtrsim\frac{1}{16}0+\frac{-2}{16}(2+3i)+\frac{18}{16}0=\alpha d(ga,fa)+\beta d(gb,fb)+\gamma d(ga,gb),\\
d(fa,fc)&=3+4i\gtrsim\frac{1}{16}0+\frac{-2}{16}(4+5i)+\frac{18}{16}(2+3i)=\alpha d(ga,fa)+\beta d(gc,fc)+\gamma d(ga,gc),\\
d(fb,fa)&=2+3i\gtrsim\frac{1}{16}(2+3i)+\frac{-2}{16}0+\frac{18}{16}0=\alpha d(gb,fb)+\beta d(ga,fa)+\gamma d(gb,ga),\\
d(fb,fc)&=4+5i\gtrsim\frac{1}{16}(2+3i)+\frac{-2}{16}(4+5i)+\frac{18}{16}(2+3i)=\alpha d(gb,fb)+\beta d(gc,fc)+\gamma d(gb,gc),\\
d(fc,fa)&=3+4i\gtrsim\frac{1}{16}(4+5i)+\frac{-2}{16}0+\frac{18}{16}(2+3i)=\alpha d(gc,fc)+\beta d(ga,fa)+\gamma d(gc,ga),\\
d(fc,fb)&=4+5i\gtrsim\frac{1}{16}(4+5i)+\frac{-2}{16}(2+3i)+\frac{18}{16}(2+3i)=\alpha d(gc,fc)+\beta d(gb,fb)+\gamma d(gc,gb).
\end{aligned}
$$

Hence f, g, α, β, γ satisfy all conditions of Theorem 2.2, so f,g have an unique common fixed point a. If take $\alpha=\frac{-1}{16}$, $\beta=\frac{2}{16}$, $\gamma=\frac{17}{16}$, then $\alpha+\beta+\gamma>\gamma>1$. In this case, we are also easy to check that f, g, α, β, γ satisfy all of the conditions of Theorem 2.2, hence f, g have an unique common fixed point a.

Now, we give two unique fixed point theorems:

Theorem 2.3. *Let (X,d) be a complex valued metric space, $f:X\to X$ a mapping such that for each $x,y\in X$, $x\neq y$,*

$$d(fx,fy)\gtrsim\alpha d(f^2x,fx)+\beta d(f^2y,fy)+\gamma d(f^2x,f^2y),$$

where $\alpha,\beta,\gamma\in\mathbb{R}$. If (i) fX is complete; (ii) $\min\{\alpha+\beta+\gamma,\gamma\}>1$. Then f has an unique fixed point.

Proof. Let $g=f^2$, then the conclusion follows easily from Theorem 2.2. ☐

Theorem 2.4. *Let (X,d) be complex valued metric space, $f:X\to X$ a mapping such that $f^2X=fX$ and for each $x,y\in X$, $x\neq y$,*

$$d(f^2x,f^2y)\gtrsim\alpha d(fx,f^2x)+\beta d(fy,f^2y)+\gamma d(fx,fy),$$

where $\alpha,\beta,\gamma\in\mathbb{R}$. If (i) fX is complete; (ii) $\min\{\alpha+\beta+\gamma,\gamma\}>1$. Then f has an unique fixed point.

Proof. Let $F=f^2$, $G=f$, then F and G are weakly compatible and satisfy all conditions of Theorem 2.2, hence $G=f$ and $F=f^2$ have a point of coincidence $u=Gp=Fp$.

If $v=Ft=Gt$ is also a point of coincidence of F and G, then we obtain

$$d(Fp,Ft)\gtrsim\alpha d(Gp,Fp)+\beta d(Gt,Ft)+\gamma d(Gp,Gt),$$

hence

$$d(u,v)\gtrsim\gamma d(u,v),$$

so $u=v$. Therefore u is the unique common fixed point of $f=G$ and $f^2=F$ by Lemma 1.3. Obviously, u is the unique fixed point of f.

Finally, we give a new common fixed point theorem for II-expansive mappings with another type. $\square$

Theorem 2.5. *Let (X,d) be a complete complex valued metric space, $f,g:X\to X$ two onto mappings such that for each $x,y\in X$,*

$$d(fx,gy)\gtrsim Ad(x,y)+Bd(x,fx)+Cd(y,gy), \tag{2.4}$$

where A, B, C are real numbers satisfying

$$A+B>0,\quad A+C>0,\quad A+B+C>1.$$

Then f and g have common fixed point. Furthermore, if $A>1$, then f and g have an unique common fixed point.

Proof. Take $x_0\in X$ and construct a sequence $\{x_k\}_{k=0}^{\infty}$ satisfying

$$x_{2k}=fx_{2k+1},\quad x_{2k+1}=gx_{2k+2},\quad k=0,1,2,\cdots.$$

For any $k=0,1,2,\cdots$, using (2.4), we obtain

$$\begin{aligned}d(x_{2k+1},x_{2k+2})=&d(fx_{2k+3},gx_{2k+2})\\ \gtrsim&Ad(x_{2k+3},x_{2k+2})+Bd(x_{2k+3},fx_{2k+3})+Cd(x_{2k+2},gx_{2k+2})\\ =&(A+B)d(x_{2k+3},x_{2k+2})+Cd(x_{2k+2},x_{2k+1}),\end{aligned}$$

hence

$$(1-C)d(x_{2k+1},x_{2k+2})\gtrsim(A+B)d(x_{2k+2},x_{2k+3}).$$

If $1-C<0$, then using $A+B>0$, we obtain $d(x_{2k+1},x_{2k+2})=d(x_{2k+2},x_{2k+3})=0$, hence $\{y_k\}$ is a constant sequence, so it is Cauchy. Therefore, we may assume that $1-C\geq 0$, in this case, we have $0\leq\frac{1-C}{A+B}<1$ and

$$d(x_{2k+2},x_{2k+3})\lesssim\frac{1-C}{A+B}d(x_{2k+1},x_{2k+2}),\quad\forall k\in\mathbb{N},\quad a\in X. \tag{2.5}$$

Similarly, for $k=-1,0,1,2,\cdots$, using (2.4), we obtain

$$\begin{aligned}d(x_{2k+2},x_{2k+3})=&d(fx_{2k+3},gx_{2k+4})\\ \gtrsim&Ad(x_{2k+3},x_{2k+4})+Bd(x_{2k+3},fx_{2k+3})+Cd(x_{2k+4},gx_{2k+4})\\ =&(A+C)d(x_{2k+3},x_{2k+4})+Bd(x_{2k+2},x_{2k+3}),\end{aligned}$$

hence

$$(1-B)d(x_{2k+2},x_{2k+3})\gtrsim(A+C)d(x_{2k+3},x_{2k+4}).$$

And we may assume that $1-B\geq 0$, hence we have $0\leq \frac{1-B}{A+C}<1$ and

$$d(x_{2k+3},x_{2k+4})\precsim \frac{1-B}{A+C}d(x_{2k+2},x_{2k+3}),\quad \forall k\in\mathbb{N}. \tag{2.6}$$

Let $h=\max\{\frac{1-B}{A+C},\frac{1-C}{A+B}\}$, then $0\leq h<1$. Combing (2.5) and (2.6), we obtain

$$d(x_{k+1},x_{k+2})\precsim hd(x_k,x_{k+1}),\quad \forall k=0,1,2,\cdots.$$

Hence $\{x_k\}$ is Cauchy by Lemma 1.2.

Since X is complete, there exists $u\in X$ such that $\lim_{k\to\infty}x_k=u$, and there exist $v,w\in X$ such that $fv=gw=u$ since f and g are both onto.

Using (2.4), we obtain

$$d(x_{2k},u)=d(fx_{2k+1},gw)\succsim Ad(x_{2k+1},w)+Bd(x_{2k+1},x_{2k})+Cd(w,u).$$

Let $k\to\infty$ in the above, then we obtain

$$0=d(u,u)\succsim (A+C)d(u,w),$$

hence $A+C>0$ implies that

$$w=u=gw. \tag{2.7}$$

Similarly, from

$$d(u,x_{2k+1})=d(fv,gx_{2k+2})\succsim Ad(v,x_{2k+2})+Bd(v,u)+Cd(x_{2k+2},x_{2k+1}),$$

we obtain that

$$v=u=fv. \tag{2.8}$$

Combing (2.7) and (2.8), we have

$$u=fu=gu.$$

That is, u is a common fixed point of f and g.

Suppose that $A>1$ and u' is also a common fixed point of f and g, then by (2.4),

$$d(u,u')=d(fu,gu')\succsim Ad(u,u')+Bd(u,fu)+Cd(u',gu')=Ad(u,u'),$$

hence $u=u'$. So u is the unique common fixed point of f and g. □

Remark 2.2. Using Theorem 2.5, we can give many fixed point results with different forms, but we omit this part. Particularly, If $f=g$, $B=C=0$, $A>1$, then Theorem 2.5 is the version of unique fixed point theorem for a I-expansive mapping in [10] on complex valued metric spaces.

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