Role of Selective Interaction in Wealth Distribution

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Abstract. In our simplified description 'wealth' is money (m). A kinetic theory of a gas like model of money is investigated where two agents interact (trade) selectively and exchange some amount of money between them so that sum of their money is unchanged and thus total money of all the agents remains conserved. The probability distributions of individual money ($P(m)$ vs. m) is seen to be influenced by certain ways of selective interactions. The distributions shift away from Boltzmann-Gibbs like the exponential distribution, and in some cases distributions emerge with power law tails known as Pareto's law ($P(m) \propto m^{-(1+\alpha)}$). The power law is also observed in some other closely related conserved and discrete models. A discussion is provided with numerical support to obtain insight into the emergence of power laws in such models.

Key words: Kinetic theory; selective interaction; disparity; wealth distribution; Pareto's law.

1 Introduction

Econophysics of Wealth distributions is an active area which involves interpreting and analysing real economic data of money, wealth or income distributions of all kinds of people pertaining to different societies and nations [4]. A number of statistical physical models can be found in the literature [1] in connection with the above. Understanding the emergence of Pareto's law ($P(m) \propto m^{-(1+\alpha)}$), now more than a century old, is one of the most important agenda. Some early attempts [5] have been made to understand the wealth distributions, especially the Pareto's law where the index α is generally found to be in the range of 1 to 2.5 more or less universally.

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Some recent works [6,7] assume economic activities to be analogous to elastic collisions to have kinetic theory of gas like models proposed by Chakrabarti and group and later by others (we refer to this class of models as the 'Chakrabarti model'). Analogy is drawn between Money (m) and Energy (E) where temperature (T) is average money ($< m >$) of any individual at 'equilibrium'. There has been a renewed interest in the two-agent exchange model (be it of money, energy or of something else) in the new scenario. For example, a recent work deals with social systems of complex interactions like sex which is based on a granular system of colliding particles (agents) with gaining energy [3].

In this paper we deal with Chakrabarti model kind of systems where it is assumed that any two agents chosen randomly from a total number of agents (N) are allowed to interact (trade) stochastically and thus money is exchanged between them. The interaction is such that one agent wins and the other loses the same amount so that the sum of their money remains constant before and after interaction (trading). Therefore, it is a two-agent zero sum 'game'. This way it ensures the total amount of money of all the agents ($M = \Sigma m_i$) to remain constant. Such a model is thus a conserved model.

2 The models and results

The basic steps of a money exchange (conserved) model are as follows:

$$m_i(t + 1) = m_i(t) + \Delta m, \tag{2.1}$$

$$m_j(t + 1) = m_j(t) - \Delta m, \tag{2.2}$$

where m_i and m_j are money of the i-th and j-th agents respectively. Here we have t as discrete 'time' which is referred to as a single interaction between two agents. The amount Δm (to be won or to be lost by an agent) is given by the nature of interaction. In a pure gambling, $\Delta m = \epsilon(m_i(t) + m_j(t)) - m_i(t)$, where stochasticity is introduced through the parameter ϵ ($0 < \epsilon < 1$).

If the agents are allowed to interact for a long enough time an equilibrium distribution of money of individual agents is achieved. We arrive at a Boltzmann-Gibbs type distribution $[P(m) \propto \exp(-m/ < m >)]$ of individual money which is verified numerically. This is quite the same way in which we arrive at the equilibrium energy distribution of a system of gas particles elastically colliding and exchanging energy with each other. The equilibrium temperature corresponds to average money, $< m >$ per agent.

The study is done through numerical computations and the results reported here are obtained with system sizes (=total number of agents) $N = 1000$. In all cases the system is allowed to equilibrate up to $t = 10^6$ time steps. Averaging is done over 1000 realizations in each case. The final distribution of course should not depend on the initial configuration (initial distribution of money among the agents). The wealth distributions we deal with in this paper are ordinary distributions and not cumulative ones. To obtain a distribution we take the average over many different realizations; that is over a number of ways of the random selection of a pair of agents and also over the stochastic term ϵ.

If we intend to take the average of money of a single agent over a long time, it turns out to be the same for all agents. Therefore, the distribution of individual time averaged money appears to be a delta function as checked by numerical simulation. However, when the average is taken over a short time period, the delta function broadens to appear as a modified exponential distribution. Finally, the distribution of individual money at a certain time turns out to be a purely exponential one as mentioned earlier. This is all with randomly selected pair of agents with stochastic gain or loss (per agent).

However, two randomly selected agents can interact with each other in various numbers of ways other than just pure gambling. Distribution of individual money depends on how the two randomly chosen agents decide to interact when they meet: whether it will be random sharing of their aggregate money or with some rules [6, 7, 9]. We can in general write:

$$\left(\begin{array}{c} m_i(t+1) \\ m_j(t+1) \end{array} \right) = T \left(\begin{array}{c} m_i(t) \\ m_j(t) \end{array} \right),$$

where T is a 2×2 transition matrix given by the kind of interactions. It is interesting to look at some properties of T in connection with the emergence of distributions: Boltzmann-Gibbs type exponential or a modified exponential distribution ($P(m) \propto m^\nu \exp(-m/ < m >)$). This aspect has been investigated in our earlier work [2].

Interactions or trade among agents in a society are often guided by personal choice or some social norms. Individual wealth distributions may be altered due to selective interactions. However, any arbitrary kind of selective interaction (or preferential behaviour) may have no effect. As an example, we examine the role played by the concept of family. A family in a society usually consists of more than one agent. It is quite reasonable to assume that the agents belonging to the same family do not trade or interact among themselves. Such a case of selective interaction does not appear to have any influence on the individual wealth distribution: it remains exponentially distributed as it is checked numerically. In this context we discuss family wealth distribution. In computer simulation we colour the agents belonging to a same family to keep track of. To find wealth distribution of families, we add up the contributions of the family members. In Fig. 1 we plot family wealth distributions for three cases: (i) families consist of 2 members each, (ii) families consist of 4 members each, and (iii) families of mixed sizes between 1 to 4. The distributions are clearly not purely exponential, but modified exponential distributions with different peaks and different widths which is quite expected (The probability of zero income of a family is zero.).

Some special ways of incorporating 'selection' may play a definitive role in the individual wealth distributions as can be seen as follows. Let us define a 'class' of agents by some index. The class may be understood in terms of their efficiency of accumulating money or some other related property. It is assumed that during the interactions, the agents may convert an appropriate amount of money proportional to their efficiency factor in their favour or against. Now the model can be understood in terms of equations (2.1) and (2.2) with the redefined amount of exchange:
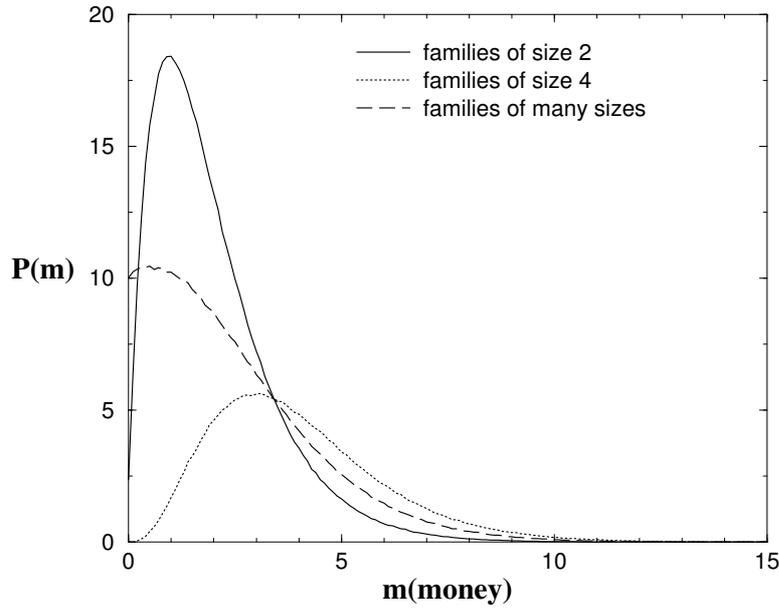
Figure 1: Family wealth (money) distribution: two curves are for families all of equal sizes and one is for families of various sizes between 1 and 4. Distributions are ordinary and not cumulative. Unit is arbitrary.

$$\Delta m = (\epsilon_i - 1)m_i(t) + \epsilon_j m_j(t), \tag{2.3}$$

where the ϵ_i's are random numbers between 0 and 1 and are randomly assigned to the agents at the beginning (frozen in time). Now let us suppose that the agents are given a choice to whom not to interact with. This option, in fact, is not unnatural in the context of a real society where individual or group opinions are important. There has been a lot of work on the process and dynamics of opinion formations in model social systems; a good amount of discussions can be found in [11]. In our model we may imagine that the 'choice' is guided by the relative class index of the two agents. We assume that an interaction takes place when the ratio of two class factors remains within certain upper limit. Our requirement for interaction (trade) to happen is then $1 < \epsilon_i/\epsilon_j < \eta$. Wealth distributions for various values of η are numerically investigated. Power laws in the tails are obtained in all cases. In Fig. 2 we show the distribution for $\eta = 2$. A power law is clearly observed with a power around 3 which means the Pareto index α is close to 2. A straight line in the log-log plot is shown for comparison (a power law with power 3.3).

Many works in Econophysics or Sociophysics are devoted to understand the emergence of power law Pareto tails in the distributions [8]. Pareto's law is observed quite universally across many nations and societies. The power law tails in the distributions are obtained by a number of models like that of Chakrabarti model type (conserved) models [9] and other kinds of (nonconserved) models [10]. However, there is much yet to be understood, particularly how a power law emerges from simple discrete models and algorithms. It is
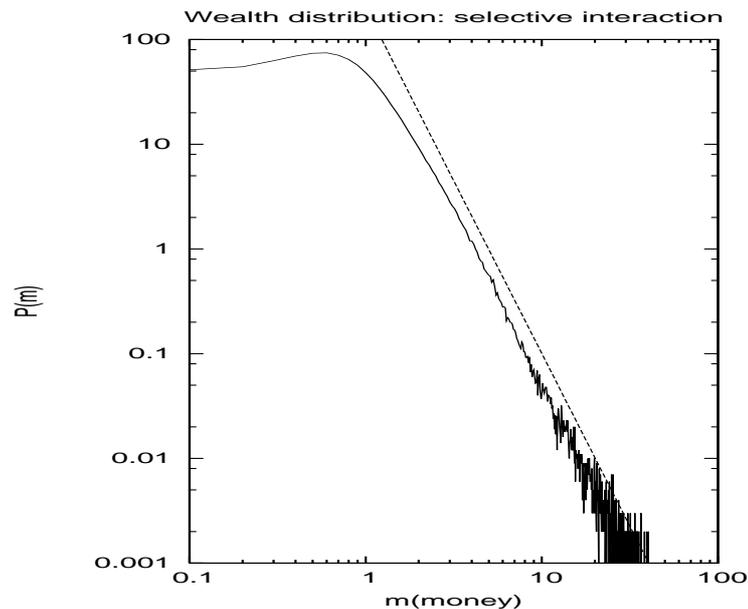
Figure 2: Distribution (not cumulative) of individual money with selective interaction. Power law is evident in the log-log plot where a straight line is drawn with $m^{-3.3}$ for comparison.

known that probability distribution of money of majority is different from that of handful of minority (rich people). 'Disparity' is more or less a reality in an economy. We may then think of a money exchange process within the framework of the Chakrabarti model like discrete and conserved model in a way that the interactions among agents cause increasing variance. It is numerically examined whether the process of forcing the system to have ever increasing variance (measure of disparity) leads to a power law, as the power law is known to be associated with infinite variance. Thus we simulate a process where the random interactions (pure gambling) among agents [as given by equations (2.1)-(2.2)] are allowed only when the variance $\sigma = < m^2 > - < m >^2$ is greater than the previously calculated value where the average value $< m >$ is kept fixed. Thus the variance attains a very high value (not shown here) with iterations under this imposed condition. As a result we arrive at a power law (with a low power) in the individual wealth distribution (shown in Fig. 3).

The above is an artificial situation though. However, we examine two models where we find that the money exchange algorithms are so designed that the resulting variance, in effect, increases monotonically with time. One such case is the model of selective interaction as introduced before. The other prominent example is the model with random saving propensities proposed by Chatterjee and Chakrabarti [9], where $\Delta m = (1 - \epsilon)(1 - \lambda_i)m_i(t) + \epsilon(1 - \lambda_j)m_j(t)$. In both cases it is checked numerically that σ increases with time. In Fig. 4 variance (σ) is plotted with time for our model of selective interaction as discussed earlier. The variance σ is seen to be monotonically increasing with time.
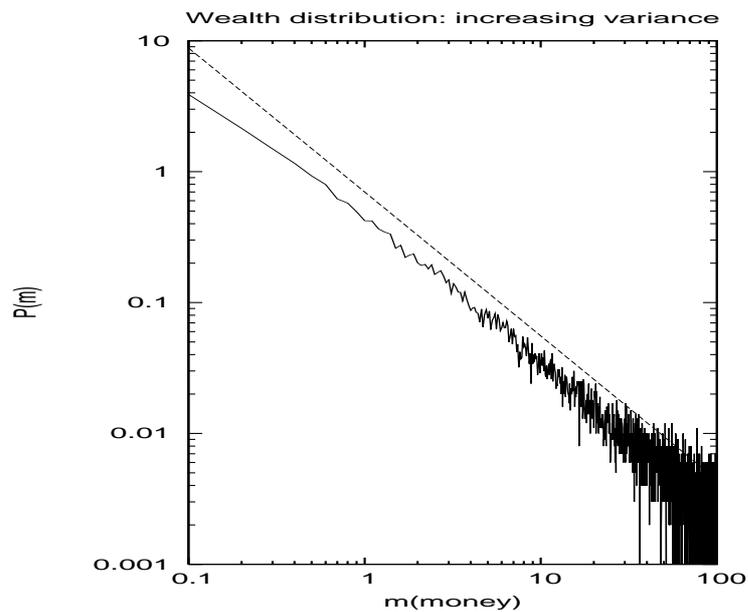
Figure 3: Wealth distribution (not cumulative) with ever increasing variance. Power law is clearly seen and a straight line is drawn with $m^{-1.1}$ to compare.
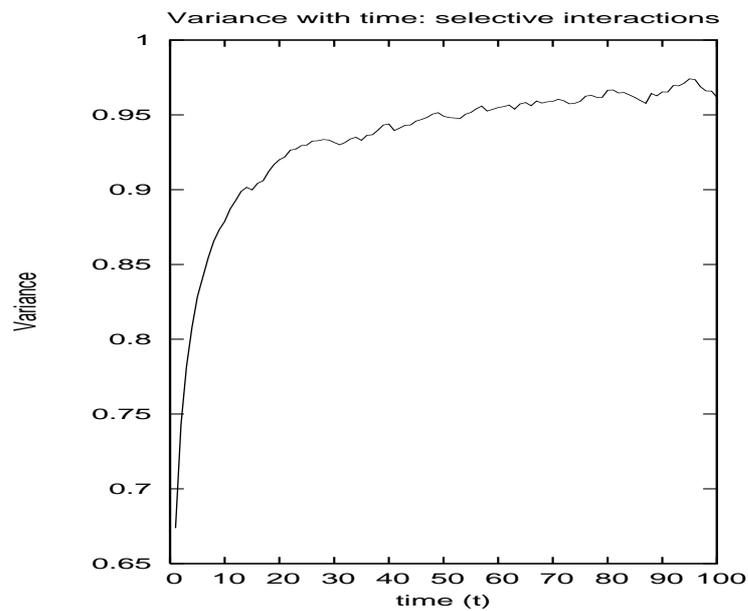

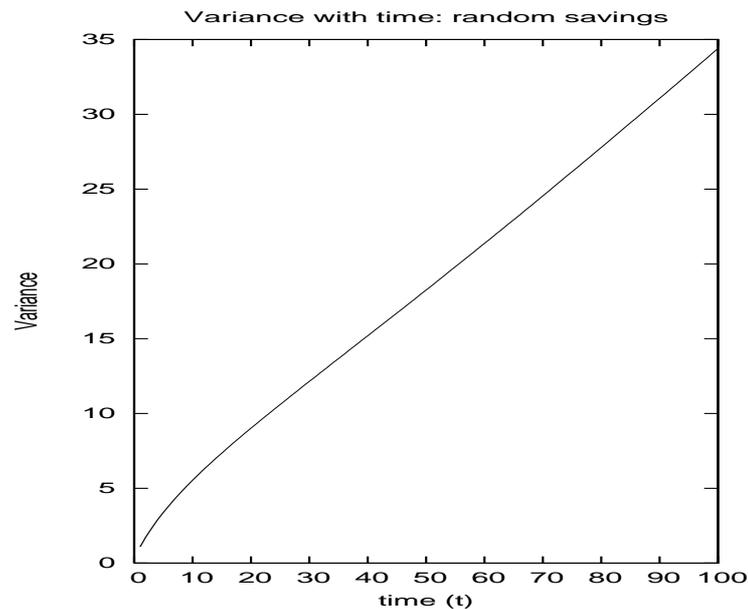Figure 4: Variance against time in the model of selective interaction.

Figure 5: Variance against time in the model of random saving propensities

In Fig. 5 we show variance (σ) against time for the model of random saving propensity. The variance is seen to be increasing faster with time and attaining higher value in this case. The power law coefficients for the respective distributions may be related to the magnitude of variance. In both figures (Figs. 4 and 5), the scale along the x-axis should be multiplied by 10^4 to get the actual time steps (arbitrary unit) in numerical simulation.

3   Discussions and conclusions

In the framework of kinetic theory of gas like (conserved) models, random interactions between randomly selected pairs of agents lead to an exponential distribution of individual wealth. Depending on the type of interaction (that is how the money is shared among agents during an interaction), the distribution is altered from pure exponential to a modified exponential one and in some cases power laws are obtained. We have shown here that the distribution is also influenced by personal choice. Selective interaction (or preferential behaviour) of some kind (within the framework of conserved model) can be connected to a power law in the distribution of individual wealth. The reason for this is not very apparent though. However, this can be given a thought and more appropriate models may be constructed based on this information. In a real society, people usually do not interact arbitrarily but rather do so with purpose and thinking. Some kind of personal preference is always there which may be incorporated in some way or other. On the other hand, a large amount of economic disparity usually exists among people. The detailed mechanism leading to disparity is not always clear but it can be said to be associated with

the emergence of power law tails in wealth distributions. Enhanced variance is observed in our model of selective interaction and a power law tail is obtained in the individual wealth distribution. Monotonically increasing variance (with time) is also seen to be associated with another existing model (model with random saving propensities [9]) which generates power law tails. Therefore, there is a cause of thought on the inherent mechanisms in the kind of discrete and conserved models in relation to large variance and power law tails.

Acknowledgments

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