COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **6**, No. 4, pp. 793-803

Commun. Comput. Phys.
October 2009

A Level Set Method for the Inverse Problem of Wave Equation in the Fluid-Saturated Porous Media

Ying He[1,2] and Bo Han[1,*]

[1] *Department of Mathematics, Harbin Institute of Technology, Harbin, 150001, China.*
[2] *School of Mathematical Sciences, Heilongjiang University, Harbin, 150080, China.*

Received 17 June 2008; Accepted (in revised version) 19 January 2009

Communicated by Pingwen Zhang

Available online 5 March 2009

Abstract. In this paper, a level set method is applied to the inverse problem of 2-D wave equation in the fluid-saturated media. We only consider the situation that the parameter to be recovered takes two different values, which leads to a shape reconstruction problem. A level set function is used to present the discontinuous parameter, and a regularization functional is applied to the level set function for the ill-posed problem. Then the resulting inverse problem with respect to the level set function is solved by using the damped Gauss-Newton method. Numerical experiments show that the method can recover parameter with complicated geometry and the noise in the observation data.

AMS subject classifications: 35R30, 34K28

Key words: Level set method, fluid-saturated porous media, parameter recovery, damped Gauss-Newton method.

1 Introduction

The wave propagation theory in the fluid-saturated porous media was first established by Biot [1, 2], in which equations for acoustic propagation in a porous elastic isotropic solid containing a viscous fluid have been developed. Over the past decades, Biot theory has become a popular model for presenting the property of elastic wave propagation in the fluid-saturated porous media. In this paper, we consider the inverse problem with the coupling governing equations of **u-w** form given by Biot [3]:

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\mathrm{grad}(\mathrm{div}\mathbf{u})+\alpha M\mathrm{grad}(\mathrm{div}\mathbf{w})=\rho\frac{\partial^2\mathbf{u}}{\partial t^2}+\rho_f\frac{\partial^2\mathbf{w}}{\partial t^2}, \tag{1.1a}$$

$$\alpha M\mathrm{grad}(\mathrm{div}\mathbf{u})+M\mathrm{grad}(\mathrm{div}\mathbf{w})=\rho_f\frac{\partial^2\mathbf{u}}{\partial t^2}+m\frac{\partial^2\mathbf{w}}{\partial t^2}+\frac{\eta}{k}\frac{\partial\mathbf{w}}{\partial t}, \tag{1.1b}$$

*Corresponding author. *Email addresses:* `happybirdzhp@sohu.com` (Y. He), `bohan@hit.edu.cn` (B. Han)

where **u** denotes the solid-frame displacement, **w** is the fluid displacement relative to solid-frame, λ is the Lamé coefficient, the quantity κ is the Darcy permeability coefficient, η is the viscosity of the pore fluid, ρ_f is the density of the pore fluid, ρ_s is the density of the solid grain, ρ is the bulk density defined by $\rho=\beta\rho_f+(1-\beta)\rho_s$, and β is the porosity. The relationship of the tortuosity α, added mass density m, and coupling constant M is given by

$$\alpha=1-\frac{K_s}{K_r},\quad M=\frac{K_r^2}{D_r-K_s},\quad D_r=K_r[1+\beta(\frac{K_r}{K_f}-1)],\quad m=\frac{\rho_f}{\beta},$$

where K_r is the bulk modulus of the grain, K_f is the bulk modulus of the pore fluid, and K_s is the bulk modulus of the skeletal frame.

The inverse problem discussed in this paper is to identify the porosity β from the measurements of the solid-frame displacement **u**, which can be viewed as a parametric data-fitting problem. It is possible to formulate such problem as an optimization problem where a functional defined by the discrepancy between the observed and computed data is minimized over a model space. In general, such problem is very difficult to solve, since it is nonlinear and ill-posed. For approximating the ill-posed and nonlinear problem, we utilize a regularized level set method. Level set method, originally introduced by Osher and Sethian [11] is a general framework for computation of evolving interfaces using the implicit representations and has been used successfully in many fields such as image processing [9, 10]. Hintermuller and Ring [7] applied a level set approach for the image segmentation. Recently, the level set method has received growing attention as a flexible algorithm for inverse problem [12] and shape optimization due to the ability to handle topological changes and to compute reconstructions with the minor priori information. Burger [4] studied the level set solution of shape reconstruction problems, in which the rigorous mathematical theory of level set regularization was established. van den Doel and Ascher [6] considered the level set regularization to recover the distributed parameter function with discontinuities for highly ill-posed inverse problem and the regularization functional was applied to the level set function rather than to the discontinuous function to be recovered. Tai and Li [13] applied a piecewise constant level set method to elliptic inverse problems and used the variational penalization method with the variation regularization of the coefficients to solve the inverse problem. Rondi [14] considered the regularization of the inverse conductivity problem with discontinuous conductivities.

In this paper, we will apply a level set method for parameter identification problem with the wave equations in the fluid-saturated porous media. For convenience, we only treat the case of the solution domain $\Omega=\Omega_1\cup\Gamma\cup\Omega_2$, in which the materials in two subregions Ω_1 and Ω_2 are different and separated by closed interface Γ. We assume the unknown parameter β takes the value β_1 in Ω_1 and takes the value β_2 in Ω_2. The purpose of this paper is that instead of recovering the parameter β, we want to implicitly capture the interface Γ by solving the optimal problem for the level set function, as Γ is usually the zero isocontour of the level set function. First, the unknown parameter β can be presented by a level set function. Then we formulate the inverse problem by minimizing an output least-squares functional with respect to the level set function. The resulting optimization problem is solved by using the damped Gauss-Newton method.

The structure of this paper is organized as follows. We start with presenting the elastic wave equations for the inverse problem in the fluid-saturated porous media based on Biot theory in Section 2. In Section 3, we list key terms defined by the level set function and construct a level set algorithm for our problem. In Section 4, in order to illustrate the performance of this algorithm, we carry out some numerical simulations for the wave equations in the fluid-saturated porous media. The results show that the level set method is effective and feasible to the inverse problem.

2 Formulations for the inverse problem in the fluid-saturated porous media

In the present study, we ignore viscous effects and rewrite Eq. (1.1) in 2-D in the Cartesian coordinates:

$$2\frac{\partial}{\partial x}\left(\mu\frac{\partial u_x}{\partial x}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u_x}{\partial z}+\mu\frac{\partial u_z}{\partial x}\right)+\frac{\partial}{\partial x}\left(\lambda\frac{\partial u_x}{\partial x}+\lambda\frac{\partial u_z}{\partial z}\right)+\frac{\partial}{\partial x}\left(\alpha M\frac{\partial \omega_x}{\partial x}+\alpha M\frac{\partial \omega_z}{\partial z}\right)$$
$$=\rho\frac{\partial^2 u_x}{\partial t^2}+\rho_f\frac{\partial^2 \omega_x}{\partial t^2}-f_1, \tag{2.1}$$

$$\frac{\partial}{\partial x}\left(\mu\frac{\partial u_x}{\partial z}+\mu\frac{\partial u_z}{\partial x}\right)+2\frac{\partial}{\partial z}\left(\mu\frac{\partial u_z}{\partial z}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial u_x}{\partial x}+\lambda\frac{\partial u_z}{\partial z}\right)+\frac{\partial}{\partial z}\left(\alpha M\frac{\partial \omega_x}{\partial x}+\alpha M\frac{\partial \omega_z}{\partial z}\right)$$
$$=\rho\frac{\partial^2 u_z}{\partial t^2}+\rho_f\frac{\partial^2 \omega_z}{\partial t^2}-f_2, \tag{2.2}$$

$$\frac{\partial}{\partial x}\left(\alpha M\frac{\partial u_x}{\partial x}+\alpha M\frac{\partial u_z}{\partial z}+M\frac{\partial \omega_x}{\partial x}+M\frac{\partial \omega_z}{\partial z}\right)=\rho_f\frac{\partial^2 u_x}{\partial t^2}+m\frac{\partial^2 \omega_x}{\partial t^2}-f_1, \tag{2.3}$$

$$\frac{\partial}{\partial z}\left(\alpha M\frac{\partial u_x}{\partial x}+\alpha M\frac{\partial u_z}{\partial z}+M\frac{\partial \omega_x}{\partial x}+M\frac{\partial \omega_z}{\partial z}\right)=\rho_f\frac{\partial^2 u_z}{\partial t^2}+m\frac{\partial^2 \omega_z}{\partial t^2}-f_2, \tag{2.4}$$

where $u_x,u_z,\omega_x,\omega_z$ are the components of the displacement vectors $\mathbf{u}$ and $\mathbf{w}$ in the x,z directions, respectively; f_1 and f_2 denote the functions associated with the source function f defined by

$$f_1=\beta f, \quad f_2=(1-\beta)f.$$

The boundary conditions for the problem considered here are

$$\frac{\partial u_x(x,z,t)}{\partial x}\Big|_{x=0}=\frac{\partial u_x(x,z,t)}{\partial x}\Big|_{x=L}=0, \quad \frac{\partial u_x(x,z,t)}{\partial z}\Big|_{z=H}=0, \tag{2.5}$$

$$\frac{\partial u_z(x,z,t)}{\partial x}\Big|_{x=0}=\frac{\partial u_z(x,z,t)}{\partial x}\Big|_{x=L}=0, \quad \frac{\partial u_z(x,z,t)}{\partial z}\Big|_{z=H}=0, \tag{2.6}$$

$$\frac{\partial \omega_x(x,z,t)}{\partial x}\Big|_{x=0}=\frac{\partial \omega_x(x,z,t)}{\partial x}\Big|_{x=L}=0, \quad \frac{\partial \omega_x(x,z,t)}{\partial z}\Big|_{z=H}=0, \tag{2.7}$$

$$\frac{\partial \omega_z(x,z,t)}{\partial x}\Big|_{x=0}=\frac{\partial \omega_z(x,z,t)}{\partial x}\Big|_{x=L}=0, \quad \frac{\partial \omega_z(x,z,t)}{\partial z}\Big|_{z=H}=0, \tag{2.8}$$

where the rectangular domain of solution is $\Omega=\{(x,z):x\in[0,L],z\in(0,H]\}$. Let the initial conditions be given as

$$u_x(x,z,0)=0,\quad \frac{\partial u_x(x,z,0)}{\partial t}=0, \tag{2.9}$$

$$u_z(x,z,0)=0,\quad \frac{\partial u_z(x,z,0)}{\partial t}=0, \tag{2.10}$$

$$\omega_x(x,z,0)=0,\quad \frac{\partial \omega_x(x,z,0)}{\partial t}=0, \tag{2.11}$$

$$\omega_z(x,z,0)=0,\quad \frac{\partial \omega_z(x,z,0)}{\partial t}=0. \tag{2.12}$$

Using Eqs. (2.1)-(2.4) and the boundary and initial conditions (2.5)-(2.12), we determine the wave fields $\mathbf{u}(x,z),\mathbf{w}(x,z)$ with the known porosity $\beta(x,z)$, such problem is identified as a direct problem for the wave equations in the porous media. While the objective of the inverse problem is to identify the porosity $\beta(x,z)$ from measurements u_x^* of the wave function u_x.

In general, the inverse problem in the fluid-saturated porous media can be viewed as a parametric data-fitting problem. It is possible to formulate such problem as an optimization problem where a functional defined by the discrepancy between the observed and computed data is minimized over a model space. Now, for formulating such inverse problem to a nonlinear operator equation, we discretize the Eqs. (2.1)-(2.12) by a second-order finite difference equations as follows:

$$\begin{aligned}
&u_x(i,j,k+1)=\frac{mU_1-\rho_f U_2}{m\rho(i,j)-\rho_f^2}, \qquad u_z(i,j,k+1)=\frac{mU_3-\rho_f U_4}{m\rho(i,j)-\rho_f^2},\\
&\omega_x(i,j,k+1)=\frac{\rho(i,j)U_2-\rho_f U_1}{m\rho(i,j)-\rho_f^2}, \quad \omega_z(i,j,k+1)=\frac{\rho(i,j)U_4-\rho_f U_3}{m\rho(i,j)-\rho_f^2},\\
&u_x(0,j,k)=u_x(1,j,k), \quad u_x(m-1,j,k)=u_x(m,j,k), \quad u_x(i,n-1,k)=u_x(i,n,k),\\
&u_z(0,j,k)=u_z(1,j,k), \quad u_z(m-1,j,k)=u_z(m,j,k), \quad u_z(i,n-1,k)=u_z(i,n,k),\\
&\omega_x(0,j,k)=\omega_x(1,j,k), \quad \omega_x(m-1,j,k)=\omega_x(m,j,k), \quad \omega_x(i,n-1,k)=\omega_x(i,n,k),\\
&\omega_z(0,j,k)=\omega_z(1,j,k), \quad \omega_z(m-1,j,k)=\omega_z(m,j,k), \quad \omega_z(i,n-1,k)=\omega_z(i,n,k),\\
&u_x(i,j,0)=u_x(i,j,1)=0, \quad u_z(i,j,0)=u_z(i,j,1)=0,\\
&\omega_x(i,j,0)=\omega_x(i,j,1)=0, \quad \omega_z(i,j,0)=\omega_z(i,j,1)=0,
\end{aligned}$$

where

$$\begin{aligned}
&\rho(x,z)=\beta(x,z)\rho_f+(1-\beta(x,z))\rho_s,\\
&m(x,z)=\frac{\rho_f}{\beta(x,z)}, \quad M(x,z)=\frac{K_s}{\alpha+\beta(x,z)(\frac{K_s}{K_f}-1)},
\end{aligned}$$

$\beta(x,z)$ is the model to be recovered, u_x is the wave function, $u_x(i,j,k)=u_x(i\times h_x,j\times h_z,k\times\tau)$, $\omega_x(i,j,k)=\omega_x(i\times h_x,j\times h_z,k\times\tau)$, h_x,h_z are the step sizes of the rectangle grid in the x and z directions, respectively, τ is the time step size, $m=L/h_x$, $n=H/h_z$, $l=T/\tau$.

The above difference equations define an operator equation as

$$\mathbf{F}(\beta)=\mathbf{U}, \tag{2.13}$$

where β and $\mathbf{U}$ are vectors with the following form:

$$\begin{aligned}&\beta=(\beta(1,1),\cdots,\beta(1,n),\beta(2,1),\cdots,\beta(2,n),\cdots,\beta(m,n))^T,\\&\mathbf{U}=(u_x(1,1),\cdots,u_x(m,1),u_x(2,1),\cdots,u_x(m,2),\cdots,u_x(m,l))^T.\end{aligned}$$

Here m,n,l denote the number of mesh nodes for the x,z,t components, respectively.

In order to identify the porosity β from the observed data $\mathbf{U}^*$ containing noise, we solve such inverse problem by minimizing an output least-squares functional

$$\min_{\beta}\frac{1}{2}\|\mathbf{F}(\beta)-\mathbf{U}^*\|^2, \tag{2.14}$$

where the notation $\|\cdot\|$ refers to the L_2 norm.

In general, such inverse problem is ill-posed, i.e., the parameter to be reconstructed does not depend continuously on the data. Therefore, regularization methods have to be used in order to compute a stable approximation of the minimizer, i.e., to minimize the associated least-squares problem

$$\frac{1}{2}\|\mathbf{F}(\beta)-\mathbf{U}^*\|^2+\alpha R(\beta)\longrightarrow\min_{\beta}, \tag{2.15}$$

where $R(\beta)$ is the regularization term and $\alpha>0$ is the regularization parameter.

3 Level set method to recover the closed curve Γ

3.1 A regularized level set method

For solving the optimization problem (2.15), in this paper we only consider the solution domain $\Omega=\Omega_1\cup\Gamma\cup\Omega_2$ and assume the unknown parameter β takes on two values β_1, β_2, i.e.,

$$\beta(\mathbf{x})=\begin{cases}\beta_1, & \mathbf{x}\in \text{ interior of }\Gamma,\\ \beta_2, & \mathbf{x}\in \text{ exterior of }\Gamma,\end{cases}$$

where $\mathbf{x}=(x,z)^T$. The interface Γ can be represented implicitly as zero isocontour of a level set function ϕ, i.e., $\Gamma=\{\mathbf{x}:\phi(\mathbf{x})=0\}$. Theoretically, we can use a level set function to represent the curve Γ. However, in practice we take a signed distance function as the level set function for numerical stability and accuracy. This turns out to be a good choice, since steep and flat gradients as well as rapidly changing features are avoided as much as possible. Then the level set function is represented as

$$\phi(\mathbf{x},t)=\begin{cases} \text{distance}(\mathbf{x},\Gamma), & \mathbf{x}\in \text{ interior of } \Gamma, \\ -\text{distance}(\mathbf{x},\Gamma), & \mathbf{x}\in \text{ exterior of } \Gamma. \end{cases}$$

In addition, level set function ϕ satisfies $|\nabla\phi|=1$. However even if we start with a signed distance function, as the interface evolves the level set function will generally drift away from its initialized value as signed distance. We need a procedure called reinitialization to be applied periodically in order to keep the level set function ϕ equal to the signed distance. The function ϕ is the steady state of the following time dependent equation

$$\begin{aligned} &\frac{\partial\phi}{\partial t}+\text{sign}(\phi)(|\nabla\phi|-1)=0, \\ &\phi(x,0)=\tilde{\phi}(x), \end{aligned} \tag{3.1}$$

where $\tilde{\phi}$ is the starting level set function. This reinitialization equation can be solved by using many methods, such as finite-difference method. The details can be found in [10].

If a level set function ϕ satisfying (3.1) is given, the unknown parameter β can be described by the level set function as follows

$$\beta(x,z)=\beta_1 H(\phi)+\beta_2(1-H(\phi)),$$

where $H(\phi)$ is the Heaviside function defined by $H(\phi)=1$ for $\phi>0$ and $H(\phi)=0$ otherwise. Since we want to identify discontinuous parameter β with large jump, in this paper we use the total variation regularization described in [5]. The resulting new minimization problem for the unknown parameter ϕ is given as follows

$$\frac{1}{2}\|\tilde{\mathbf{F}}(\phi)-\mathbf{U}^*\|^2+\alpha R(\phi)\longrightarrow \min_{\phi}, \tag{3.2}$$

where $\tilde{\mathbf{F}}(\phi)=\mathbf{F}(\beta(\phi))$,

$$\alpha R(\phi)=\alpha TV(\phi)=\alpha\int_\Omega |\nabla\phi| dx$$

which was presented and analyzed by Leitao and Scherzer [8].

3.2 The algorithm of the level set function recovery

In the following subsection, we will construct efficient, accurate and flexible algorithm for the parameter minimization problem (3.2) by applying the damped Gauss-Newton method. If ϕ is a minimizer of the problem (3.2), the necessary condition can be expressed as

$$J^T(\phi)(\tilde{F}(\phi)-\mathbf{U}^*)+\alpha R'(\phi)=0, \tag{3.3}$$

where $J(\phi)=\partial\tilde{F}/\partial\phi$ and T denotes the transpose operator.

We use the damped Gauss-Newton method to solve Eq. (3.3). Thus the iteration formulation can be given as

$$\phi_{k+1}=\phi_k-(J^TJ(\phi_k)+\alpha R''(\phi_k))^{-1}J^T(\phi_k)(\tilde{F}(\phi_k)-U^*), \quad k=0,1,\cdots. \tag{3.4}$$

Before using the iteration formulation (3.4), the sensitivity matrix $J(\phi)$ needs to be computed. This can be done with the chain rule

$$J(\phi)=\frac{\partial\tilde{F}}{\partial\phi}=\frac{\partial F}{\partial\beta}\frac{\partial\beta}{\partial\phi}=(\beta_1-\beta_2)\delta(\phi)\frac{\partial F}{\partial\beta}, \tag{3.5}$$

where $\delta(\phi)$ denotes the Dirac function defined by $\delta(\phi)=H'(\phi)$. The Jacobian matrix $\partial F/\partial\beta$ is approximated by using standard central differences.

To find a minimizer of (3.2), we use the following general algorithm.

Algorithm 3.1:

Choose initial level set function $\phi_0(x,z)=\phi(x,z,0)$, and the initial interface $\Gamma_0=\{(x,z)|\ \phi_0(x,z)=0\}$. For $k=1,2,\cdots$, do:

- Choose a regularization parameter α_k.
- Find the minimizer $\phi_k(x,z)$ of the optimization problem (3.2) by using the iteration formulation (3.4).
- Update $\Gamma_k=\{(x,z)|\phi_k(x,z)=0\}$.
- Reinitialize the level set function $\phi_k(x,z)$, i.e., set $\tilde{\phi}=\phi_k$ and choose an appropriate time point t_0. Solve the state equation (3.1) and define the solution $\phi(x,z)$ as $\phi_k(x,z)$.
- Check the convergence, if not converged: set $k=k+1$ and go to step 1.

There are two key details when carrying out Algorithm 3.1. The first is how to determine the regularization parameters $\alpha_k,k=0,1,\cdots$, which plays an important role of the inverse problems. In this paper, the regularization parameters α_k are chosen according to $\alpha_{k+1}=0.9\alpha_k$. The iteration procedure is stopped as soon as the Morozov's discrepancy principle is satisfied. Another important point in Algorithm 3.1 is when to reinitialize the level set function. From the point of view of computational efficiency, it is not necessary for us to reinitialize the level set function at each iteration. After the level set function is solved by using Algorithm 3.1, we check the nodal values of the level set function. If the most sign of nodal values is changed, we just do the reinitialization procedure.

4 Numerical implementation

To test the efficiency of the level set method for the wave equations in the fluid-saturated porous media, we consider some numerical examples.

We take the exact values of distributed parameter $\beta(x,z)$ as

$$\beta(x,z)=\begin{cases} 0.2, & (x,z)\in \text{ interior of } \Gamma, \\ 0.6, & (x,z)\in \text{ exterior of } \Gamma. \end{cases} \tag{4.1}$$

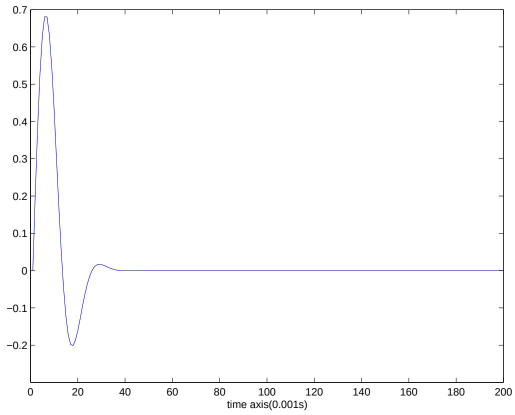

Figure 1: The source function f.

Let the square domain $\Omega=(0,1)\times(0,1)$ to be divided into a square mesh with uniform mesh size h in both x and z directions. We take $h=\frac{1}{32}$, i.e., Ω is divided into $2^5\times 2^5$ grid points. In all examples, the source function is Ricker function with the amplitude 0.8m and the frequency 40Hz, as showed in Fig. 1. The parameters describing the physical properties of the media are given as

$$\lambda=3.3568\times 10^6 Pa,\quad \mu=2.32\times 10^6 Pa,\quad K_f=1.25\times 10^6 Pa,$$
$$K_s=6.296\times 10^6 Pa,\quad \rho_f=1.000kg/m^3,\quad \rho_s=2.400kg/m^3.$$

In our numerical experiments, we do the reinitialization procedures as long as 80% nodal values of the level set function have changed sign. Note that all calculations have been performed on a PC with an Intel Core Dou T2050 processor.

Example 4.1. We first consider a simple example. In this example we assume the initial regularization parameter α_0=2×10^{-4}. To test the effectiveness of the regularized parameter, we also set the regularized parameter α_0=2×10^{-3},α_0=2×10^{-5}, but the results are not sensitive to the values of the regularized parameter. Figs. 2 and 3 show the initial level set curve and the curves after different iterations obtained by applying Algorithm 3.1, respectively. Comparing Fig. 2 with Fig. 3, we find that the interface curve Γ is recovered very well. However, it is observed that the method is not fast.

Example 4.2. In the second example, a complicatedly exact porosity model is considered. The parameters describing the physical properties of the media are the same as the first example. Assume that the initial regularization parameter is $\alpha_0=10^{-4}$. To illustrate the noise sensitivity, we perform this example by adding 5% random noisy to the observed data, and then we recover the parameter from noise data. The results are displayed in Fig. 4, which indicate that the method has the ability of noise suppression.

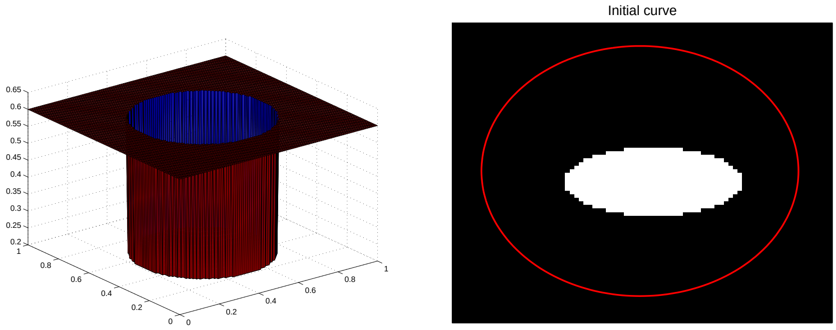

Figure 2: Example 4.1: Initial level set curve Γ and exact porosity $\beta(x,z)$.

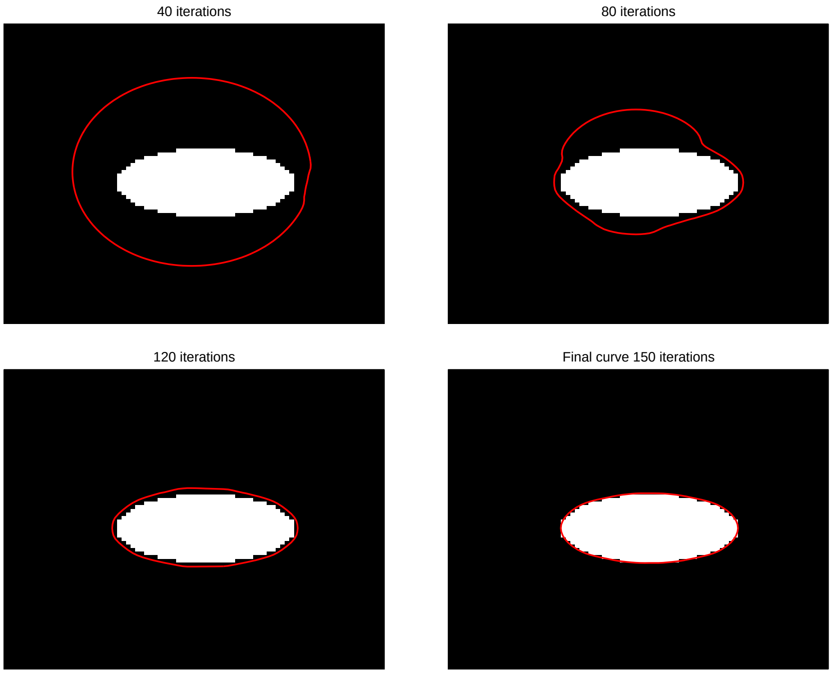

Figure 3: Example 4.1: The computed level set curves at different iterations.

5 Conclusion

In this paper, based on the Biot theory the level set regularized method is used successfully for solving inverse problem in the fluid-saturated porous media. The reinitialization procedure is used to preserve stable curve evolution and ensure desirable results. Through the numerical experiments, the level set method is robust with respect to recovering the parameter with the noise in the observation data. In the future study, we will solve more complicated problems with large solution region and high noise levels.

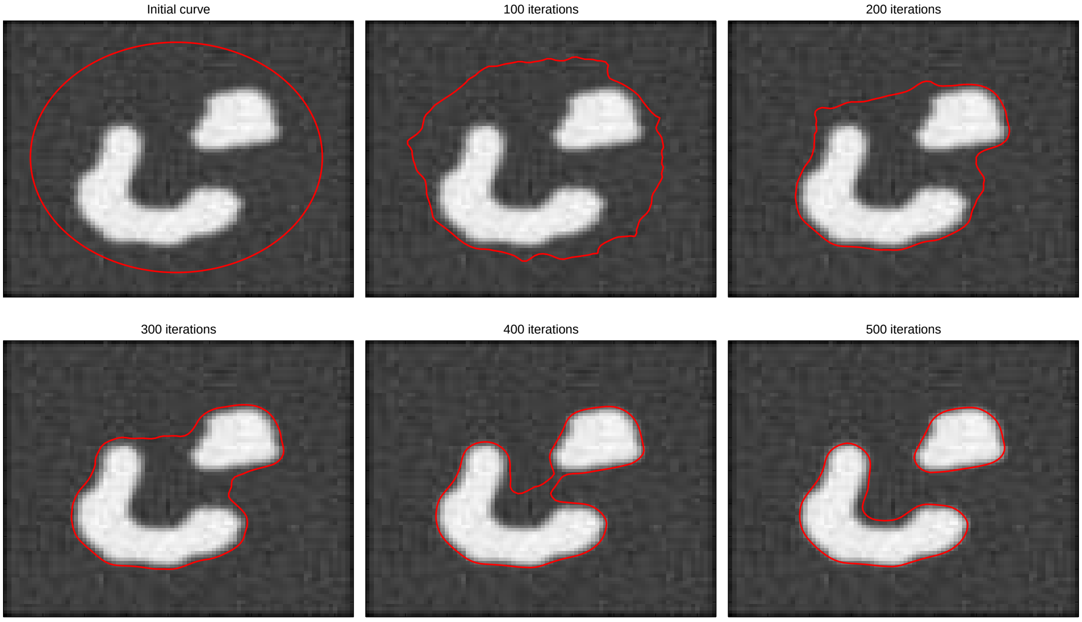

Figure 4: Example 4.2: The computed level set curves at different iterations.

References

[1] M. A. Biot, Theory of propagation of elastic waves in a fluid-saturated porous solid: low-frequency range, J. Acoust. Soc. Amer., 28 (1956), 168-178.

[2] M. A. Biot, Theory of propagation of elastic waves in a fluid-saturated porous solid: higher-frequency range, J. Acoust. Soc. Amer., 28 (1956), 179-191.

[3] M. A. Biot, Mechanics of deformation and acoustic propagation in porous media, J. Appl. Phys., 33 (1962), 1482-1498.

[4] M. Burger, A level set method of inverse problems, Inverse Problems, 17 (2001), 1327-1355.

[5] T. F. Chan and X.-C. Tai, Augmented Lagrangian and total variation methods for recovering discontinuous coefficients from elliptic equations, in: M. Bristeau, G. Etgen, W. Fitzgibbon, J. L. Lions, J. Periaux and M. F. Wheeler (Eds.), Computational Science for the 21st Century, Wiley, New York, 1997, pp. 597-607.

[6] K. van den Doel and U. M. Ascher, On level set regularization for highly ill-posed distributed parameter estimation problems, J. Comp. Phys., 216 (2006), 707-723.

[7] M. Hintermuller and W. Ring, A second order shape optimization approach for image segmentation, SIAM J. Appl. Math., 64 (2003), 442-467.

[8] A. Leitao and O. Scherzer, On the relation between constraint regularization, level sets, and shape optimization, Inverse Problems, 19 (2003), L1-L11.

[9] R. Malladi and J. A. Sethian, Image processing via level set curvature flow, Proceeding of the National Academy of Sciences, 92 (1995), 7046-7050.

[10] S. Osher and R. Fedkiw, Level Set Methods and Dynamic Implicit Surfaces, Springer-Verlag, New York, 2003.

[11] S. Osher and J. A. Sethian, Fronts propagation with curvature-dependent speed: algorithms based on Hamilton-Jacobi formations, J. Comput. Phys., 79 (1988), 19-49.

[12] F. Santosa, A level-set approach for inverse problems involving obstacles, ESAIM: Control, Optimisation and Calculus of Variations, 1 (1996), 17-33.
[13] X. C. Tai and H. W. Li, A piecewise constant level set method for elliptic inverse problems, Appl. Num. Math., 57 (2007), 686-696.
[14] L. Rondi, On the regularization of the inverse conductivity problem with discontinuous conductivities, Inverse Probl. Imag., 2 (2008), 397-409.