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ESTIMATION OF OPTIMAL ACOUSTIC LINER IMPEDANCE FACTOR FOR REDUCTION OF RADIATED ENGINE NOISE

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Abstract. We study the optimal design problem of acoustic liner to minimize fan noise radiation from commercial aircraft engine nacelles. Specifically we treat the liner impedance factor as a parameter and seek to estimate its optimal value that minimizes far-field radiated noise. The existence of such an optimal parameter is proved under the assumption that the Helmholtz equation governs the noise field. We also present numerical results to demonstrate that the choice of the optimal liner impedance factor does result in significant reduction of noise level in the far-field.

Key Words. liner impedance factor, the Helmholtz equation, optimization problem

1. Introduction

With dynamic growth in aviation forecast well into the 21st century, aircraft noise will remain a challenging environmental problem. Engine noise being a major component of aircraft noise, interest in inlet and acoustic liner design appears to endure([7, 6, 19]). Minimization of fan noise radiation from commercial aircraft engine nacelles may be achieved by (i) acoustic shape optimization of the inlet and (ii) impedance optimization of the liner. The former problem was studied in [3] using a gradient-based method within the context of a nonprogressive wave environment governed by a Helmholtz equation. The existence of optimal shape is proven, which is obtained numerically by spectral element method, and it yielded 25% noise reduction. As a robust and efficient alternative to gradient-based methods, surrogate management framework method is proposed in [16] for shape optimization of a trailing edge flow to control aerodynamic noise. Liner impedance optimization was studied in [5] using a finite duct noise propagation and radiation code based on boundary integral equation method. It was also investigated within the framework of linearized full potential equation (in the frequency domain) and its discrete adjoint in [21].

In this paper, we treat liner impedance optimization as an optimal control and parameter estimation problem. The parameter is the acoustic impedance factor of the acoustic liner. We define a cost function that reflects the amount of noise radiated from the engine inlet. The parameter estimation problem then is to seek the parameter that minimizes the cost function. The focus of the paper is both

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the mathematical analysis and numerical simulation. We show that an optimal parameter exists mathematically. We also show that the spectrum of the state equation is located in the lower half of the complex plane, which guarantees the stability of our numerical algorithm. To find the optimal parameter numerically, we use the finite element method to seek the numerical solution of state equation and an optimization subroutine to find the minimizer of the cost function. Our numerical result indicates that the choice of optimal parameter results in reducing the noise level by about 40%. The technological feasibility of such a liner material is a different issue that we cannot address.

The paper is organized as follows. In §2, we introduce the optimal control problem for noise reduction. In §3, we establish the solution existence of the optimal control problem. The numerical results are presented in the last section, §4.

2. An optimal control problem

We assume the problem to be axisymmetric. The geometry of the domain in which the control problem is posed has the generic shape represented in Figure 1. The modal composition of the noise source is supposed to be known on the source plane Γ_1. The nacelle boundary is made up of two parts, the first part being the interior boundary Γ_2 to which some acoustic liner material is attached, and the second part being Γ_3 that constitutes the rest of boundary of the nacelle geometry. The boundary Γ_4 is assumed to be sufficiently far from the noise source so that the Sommerfeld radiation boundary condition holds. The nacelle symmetry axis is denoted by Γ_5.

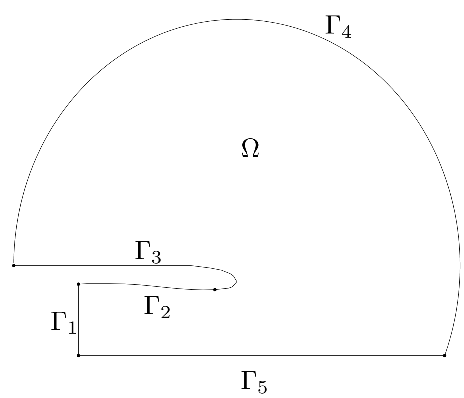


FIGURE 1. The computational domain

If the meanflow is uniform with Mach number M_0, then the governing equation for the acoustic pressure u ([11]) is

$$(1 - M_0^2)\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - 2ikM_0\frac{\partial u}{\partial x} + k^2 u = 0, \tag{2.1}$$

where k is the wavenumber. For simplicity of the presentation, we assume that the mean flow is zero. Then the acoustic pressure u satisfies the Helmholtz equation

$$\Delta u + k^2 u = 0 \quad \text{on } \Omega \tag{2.2}$$

subject to the following boundary conditions on the boundary $\partial\Omega$ of Ω:

$$\begin{array}{rcl} u\,|_{\Gamma_1} &=& g, \\ \left(\frac{\partial u}{\partial n}+\chi u\right)|_{\Gamma_2} &=& 0, \\ \frac{\partial u}{\partial n}\,|_{\Gamma_3} &=& 0, \\ \left(\frac{\partial u}{\partial n}+iku\right)|_{\Gamma_4} &=& 0, \\ \frac{\partial u}{\partial n}\,|_{\Gamma_5} &=& 0. \end{array} \tag{2.3}$$

Here $\chi = ik/\xi$ and ξ the impedance factor, which is complex and whose real part is resistance and the imaginary part is reactance. Both the dependent and the independent variables in the above equations are supposed to be properly non-dimensionalized.

The optimization problem consists in finding the parameter χ so that the least amount of noise propagates to the far field. More specifically, we want to find χ so that the cost functional

$$J(\chi, u) = \alpha \int_\Omega u^2 d\Omega + \beta \int_\Omega |\nabla u|^2 d\Omega + \lambda|\chi - \chi_0|^2 \tag{2.4}$$

is minimized, where $\alpha \geq 0$, $\beta \geq 0$, and $\lambda \geq 0$ are three constants, and χ_0 is a given complex number.

To conclude this section, we introduce notations that will be used throughout the paper. As usual, the space of square integrable complex-valued functions on Ω (or Γ_j) is denoted by $L^2(\Omega)$ (or $L^2(\Gamma_j)$), and its inner product and norm are denoted by $(\cdot,\cdot)$ (or $\langle\cdot,\cdot\rangle_{\Gamma_j}$) and $\|\cdot\|$ (or $\|\cdot\|_{\Gamma_j}$), respectively. Let $H^1_E(\Omega) = \{u \in H^1(\Omega) : u|_{\Gamma_1} = 0\}$, where $H^1(\Omega)$ is the usual Sobelev space and its inner product and norm are denoted by $\|\cdot\|_1$ and $(\cdot,\cdot)_1$, respectively ([1]).

3. Spectrum of the state equation and existence of the optimal parameter

3.1. Spectrum of the state equation. We shall study the existence of the solution of (2.2)-(2.3) and its spectrum. The results will play an important role in defining the set of admissible controls and the construction of numerical algorithms to compute the optimal parameter. To this end, we need to reformulate the PDE problem into a variational problem. Indeed, the variational formulation of problem (2.1)–(2.2) is: Find $u \in H^1(\Omega)$ with $u|_{\Gamma_1} = g$ such that

$$a(\chi, u, v) = 0, \quad \forall v \in H^1_E(\Omega), \tag{3.1}$$

where

$$a(\chi, u, v) = (\nabla u, \nabla v) - k^2(u, v) + \chi\langle u, v\rangle_{\Gamma_2} + ik\langle u, v\rangle_{\Gamma_4}.$$

We have the following theorem about the solution existence of this variational problem.

Theorem 1. *For a given wavenumber k, variational problem (3.1) has a unique solution in $H^1(\Omega)$ for all but a countable set $\mathcal{S}$ of χ having no limit points.*

Proof. It follows from the trace theorem ([1]) that there is a function $u_* \in H^1(\Omega)$ such that $u_*|_{\Gamma_1} = g$. Then variational problem (3.1) is equivalent to finding $w = u - u_* \in H^1_E(\Omega)$ such that

$$a(\chi, w, v) = -a(\chi, u_*, v), \quad \forall v \in H^1_E(\Omega). \tag{3.2}$$

Notice that for $\phi,\ \psi \in H^1_E(\Omega)$,

$$|\langle \phi, \psi \rangle_{\Gamma_j}| \le \|\phi\|_{\Gamma_j} \|\psi\|_{\Gamma_j} \le C_j \|\phi\|_1 \|\psi\|_1, \quad j = 2,\ 4,$$
$$|(\phi, \psi)| \le \|\phi\|_1 \|\psi\|_1,$$

where the trace theorem ([1]) was used and C_j are positive constants. Recall that the embeddings from $H^1(\Omega)$ to $L^2(\Omega)$ and $H^{1/2}(\partial\Omega)$ to $L^2(\partial\Omega)$ are compact ([1]). By the Riez Representation Theorem, we have compact bounded linear operators $B_j (j = 0, 2, 4)$ from $H^1_E(\Omega)$ to itself such that

$$\langle \phi, \psi \rangle_{\Gamma_2} = (B_2 \phi, \psi)_1, \quad \langle \phi, \psi \rangle_{\Gamma_4} = (B_4 \phi, \psi)_1, \quad (\phi, \psi) = (B_0 \phi, \psi)_1$$

for all $\phi,\ \psi \in H^1_E(\Omega)$. Hence,

$$a(\chi, w, v) = ((I - (1 + k^2)B_0 + \chi B_2 + ikB_4)w, v)_1.$$

By the Riez Representation Theorem again, we have for some $f \in H^1_0(\Omega)$

$$-a(\chi, u_*, v) = (f, v)_1, \quad \forall v \in H^1_0(\Omega).$$

Therefore, variational problem (3.2) becomes the following linear operator equation:

$$(I - (1 + k^2)B_0 + \chi B_2 + ikB_4)w = f. \tag{3.3}$$

We first show that the operator $P(\chi) = I - (1 + k^2)B_0 + \chi B_2 + ikB_4$ is invertible for some χ by using the stability argument in the proof of Theorem 2.1 in [2]. It is obvious that $I + B_0 + ikB_4$ is invertible. Since $B_2 - B_0$ is compact, we conclude that $Q(\chi) = I + B_0 + ikB_4 + \chi(B_2 - B_0)$ is invertible except for a countable set of χ ([13]). Since

$$\|P(\chi) - Q(\chi)\|_1 = |\chi - 2 - k^2| \|B_0\|_1 \to 0, \quad \text{as } \chi \to 2 + k^2,$$

it follows from the stability of bounded invertibility (see Chapter 4 of [12]) that $P(\chi_*)$ is invertible for some χ_* sufficiently close to $2 + k^2$. Since $P(\chi) = P(\chi_*) + (\chi - \chi_*)B_2$, the conclusion of the the theorem follows from the fact that B_2 is compact. □

Remark 1. Similarly, we can prove that for a given χ, the variational problem (3.1) has a unique solution for all but possibly a countable set of wavenumbers k.

The complex number $\chi \in \mathcal{S}$ is so-called the Steklov eigenvalue. The Steklov eigenvalue problem is to find χ such that problem (2.1)–(2.2) has some nonzero solution when $g = 0$. We shall call $\mathcal{S}$ the spectrum of the state equation and have the following result concerning its location.

Theorem 2. *If $I - (1 + k^2)B_0$ is invertible, then $\mathcal{S}$ lies in the closed lower half plane of the complex numbers.*

Proof. Let $w \in H^1_E(\Omega)$ be an eigenfunction associated with $\chi \in \mathcal{S}$. Then we have

$$(I - (1 + k^2)B_0 + \chi B_2 + ikB_4)w = 0. \tag{3.4}$$

Thus,

$$((I - (1 + k^2)B_0 + \chi B_2 + ikB_4)w, w) = 0,$$

i.e.,

$$\|\nabla w\|^2 - k^2 \|w\|^2 + \chi \|w\|^2_{\Gamma_2} + ik \|w\|^2_{\Gamma_4} = 0.$$

Hence,

$$\Im(\chi) \|w\|^2_{\Gamma_2} + k \|w\|^2_{\Gamma_4} = 0,$$

where $\Im(\chi)$ is the imaginary part of χ. If $\Im(\chi) > 0$, then $w = 0$ on Γ_2 and Γ_4 and thus equation (3.4) becomes

$$(I - (1 + k^2)B_0)w = 0.$$

Therefore, $w = 0$ and it can not be an eigenfunction. The proof is completed. □

Remark 2. Since B_0 is a self-adjoint compact operator, $I - (1 + k^2)B_0$ is invertible for all but a countable set of k having no limit points. It is easy to see that $I - (1 + k^2)B_0$ is invertible if and only if the following boundary problem has only the trivial solution:

$$\begin{aligned} &\Delta u + k^2 u = 0 \quad \text{in } \Omega, \\ &u\,|_{\Gamma_1} = 0 \quad \text{on } \Gamma_1, \\ &\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega \backslash \Gamma_1. \end{aligned}$$

Our numerical experiment indicates that this boundary value problem does have only the trivial solution for the wavenumber chosen in our numerical examples in Section 4.

Denote by $u(\chi)$ the unique solution of (3.1) for $\chi \in \mathcal{C} \backslash \mathcal{S}$, where $\mathcal{C}$ is the set of complex numbers. We have the following result about the sensitivity derivative of u with respect to χ.

Theorem 3. *Mapping $u : \mathcal{C} \backslash \mathcal{S} \to H_0^1(\Omega)$, $\chi \mapsto u(\chi)$ is differentiable, and for each $\chi \in \mathcal{C} \backslash \mathcal{S}$, $u'(\chi)$ is the unique solution of the following variational equation:*

$$a(\chi, u(\chi), v) = -\langle u(\chi), v\rangle_{\Gamma_2}, \quad \forall v \in H_E^1(\Omega). \tag{3.5}$$

Proof. It follows from (3.3) that $u(\chi)$ is a continuous mapping. For a given $\chi \in \mathcal{C} \backslash \mathcal{S}$, when $h \in \mathcal{C} \backslash \{0\}$ and $|h|$ is sufficiently small, we have $\chi + h \in \mathcal{C} \backslash \mathcal{S}$ and thus by (3.3) again

$$\frac{u(\chi + h) - u(\chi)}{h} = -(I - (1 + k^2)B_0 + \chi B_2 + ikB_4)^{-1} B_2 u(\chi + h),$$

which implies differentiability of $u(\chi)$ and

$$u'(\chi) = -(I - (1 + k^2)B_0 + \chi B_2 + ikB_4)^{-1} B_2 u(\chi),$$

i.e.,

$$(I - (1 + k^2)B_0 + \chi B_2 + ikB_4)u'(\chi) = -B_2 u(\chi).$$

Hence, $u(\chi)$ is the solution of variational equation (3.5). □

A consequence of the above theorem is that the approximations of $u(\chi)$ and $u'(\chi)$ can be computed by using the same finite element procedure. This is very efficient when the cost functional J is numerically minimized by an optimization algorithm requiring derivatives.

3.2. Existence of an optimal parameter. We now restate our minimization problem by using the variation formulation (3.1) as follows:

$$\text{(MIN)} \qquad \begin{cases} \textit{Find } (\chi^*, u^*) \in \mathcal{U}_{ad} \textit{ such that it minimizes the cost function } J(\chi, u) \\ \textit{over the admissible set } \mathcal{U}_{ad}, \end{cases}$$

where

$$\mathcal{U}_{ad} := \{(\chi, u) : \ (\chi, u) \in \mathcal{C} \times H^1(\Omega) \text{ satisfies (3.1)}\},$$

and $\mathcal{C}$ is the set of all complex numbers. It is apparent that $\mathcal{U}_{ad}$ is a nonempty set by Theorem 1.

In practice, the real part of ξ is always positive (see [18]) and thus the imaginary part of χ is positive. It means that we should restrict χ in the upper half complex plane. According to Theorem 2, if $I-(1+k^2)B_0$ is invertible, the variational problem (3.1) has a unique solution for χ with a positive imaginary part. Therefore, the admissible set can be chosen as follows:

$$\mathcal{U}_{ad} := \{(\chi, u) : \Im(\chi) \geq \epsilon_0 \text{ and } u \text{ is the unique solution of (3.1)}\},$$

where ϵ_0 is a small positive number.

It is well-known that the solution to the minimization problem like (MIN) may not be unique. We have the following result about the existence of a minimizer.

Theorem 4. *Problem (MIN) has at least one solution provided that $\alpha > 0$ and $\lambda > 0$.*

Proof. It is easy to see that $\inf_{(\chi,u)\in\mathcal{U}_{ad}} J(\chi, u)$ exists and is finite. Thus there is a sequence $\{(\chi_n, u_n)\}_{n=1}^{\infty} \subset \mathcal{U}_{ad}$ such that

$$\lim_{n\to\infty} J(\chi_n, u_n) = \inf_{(\chi,u)\in\mathcal{U}_{ad}} J(\chi, u).$$

This implies that $\{J(\chi_n, u_n)\}$ is bounded, and thus $\{\chi_n\}$ is bounded and $\{u_n\}$ is bounded in $L^2(\Omega)$. Let C be a constant such that

$$|\chi_n| \leq C, \quad \|u_n\| \leq C, \quad n = 1, 2, \ldots$$

We shall use C to denote all constants independent of u_n and χ_n. Without loss of generality, we assume that χ_n converges to χ^*.

For $\chi = \chi_n$, $u = u_n$ and $v = u_n - u_*$ in (3.1), we have that

$$a(\chi_n, u_n, u_n) = a(\chi_n, u_n, u_*).$$

Since

$$a(\chi_n, u_n, u_n) = \|u_n\|_1^2 - (1+k^2)\|u_n\|^2 + \chi_n\|u_n\|_{\Gamma_2} + ik\|u_n\|_{\Gamma_4},$$

we have

$$\|u_n\|_1^2 \leq (1+k^2)\|u_n\|^2 + (|\chi_n| + k)\|u_n\|_{\partial\Omega}^2 + |a(\chi_n, u_n, u_*)|. \tag{3.6}$$

By Schwarz's inequality and the inequality

$$ab \leq \epsilon a^2 + \frac{1}{4\epsilon}b^2, \quad \epsilon > 0,\ a,\ b \in R,$$

we have

$$|a(\chi_n, u_n, u_*)| \leq \frac{1}{4}\|u_n\|_1^2 + \frac{|\chi_n| + k}{2}\|u_n\|_{\partial\Omega}^2 + C. \tag{3.7}$$

By the trace theorem (see [14]), we get

$$\|u_n\|_{\partial\Omega}^2 \leq C\|u_n\|\|u_n\|_1 \leq \epsilon\|u_n\|_1^2 + C \tag{3.8}$$

for all $\epsilon > 0$. Combining (3.6)–(3.8), we obtain

$$\|u_n\|_1^2 \leq C,$$

which means that $\|u_n\|_1$ is bounded in $H^1(\Omega)$. Thus we can assume that $u_n \to u^*$ weakly in $H^1(\Omega)$ and $u_n \to u^*$ weakly in $L^2(\partial\Omega)$. For each $v \in H_E^1(\Omega)$, by letting $n \to \infty$ in

$$a(\chi_n, u_n, v) = 0,$$

we get

$$a(\chi^*, u^*, v) = 0, \quad \forall v \in H_E^1(\Omega).$$

Thus $(\chi^*, u^*) \in \mathcal{U}_{ad}$. It is well known that J is weekly lower semi-continuous ([15]). Hence, we have

$$J(\chi^*, u^*)) \leq \liminf_{n\to\infty} J(\chi_n, u_n) = \lim_{n\to\infty} J(\chi_n, u_n) = \inf_{(\chi,u)\in\mathcal{U}_{ad}} J(\chi, u),$$

which concludes the proof of the theorem. □

4. Numerical examples

In this section we present examples to show that the choice of optimal parameter results in reducing the noise level significantly. The variational problem (3.1) is solved by a linear finite element method. The computational domain is triangulated in such a way that the coarse meshes are used in the regions far from the noise source. The resulting linear system is solved by BiCG. Initially, we set $\chi = 0$ which means a hard wall condition on Γ_2. Then the direct search algorithm of Hooke and Jeeves is employed to drive the cost function J towards a minimum.

In the following examples, we take $\chi_0 = 0$ and $k = 2\pi$. The percentage of the noise reduction is defined by $E = (J(0) - J(\chi^*))/J(0)$. The mesh size near the noise source is about 0.0195. The profiles of noise sources are displayed in Figure 2. The computational results are presented in Table 1 and Table 2 which show that the noise level is reduced significantly when the optimal impedance factors are used. We also observed that the real part of the optimal impedance factor ξ^* is positive, which agrees with our theoretical analysis (Theorem 2) and the practical requirement (see [18]). Figure 3 presents a graphic illustration of the convergence of the cost function to what looks like a global minimum. The contour maps of the amplitude of the acoustic velocity potential u are displayed in Figure **??** – Figure **??** for $\chi = 0$ and $\chi = \chi^*$. We observe that the distribution of noise is confined near the fan inlet as desired.

TABLE 1. Example I: $g(y) = \exp\left(\pi \cos^2(2\pi y)\right)$

Parameters	$\alpha = 1,\ \beta = \gamma = 0$	$\alpha = \beta = 1,\ \gamma = 0$	$\alpha = \beta = \gamma = 1$
$J(0)$	619	25805	25805
χ^*	$-5.0464 + 1.9717i$	$-4.7223 + 2.2851i$	$-4.7080 + 2.2826i$
ξ^*	$0.42203 - 1.0802i$	$0.5217 - 1.0781i$	$0.52389 - 1.0806i$
$J(\chi^*)$	331	14896	14924
E	46%	42%	42%

TABLE 2. Example II: $g(y) = 20.0 + \exp(3y)\sin(10\pi y)$

Parameters	$\alpha = 1,\ \beta = \gamma = 0$	$\alpha = \beta = 1,\ \gamma = 0$	$\alpha = \beta = \gamma = 1$
$J(0)$	3358	136947	136947
χ^*	$-4.8647 + 1.6859i$	$-4.6694 + 1.9370i$	$-4.6665 + 1.9369i$
ξ^*	$0.39962 - 1.1531i$	$0.47624 - 1.1480i$	$0.47674 - 1.1486i$
$J(\chi^*)$	2070	87293	87318
E	38%	36%	36%

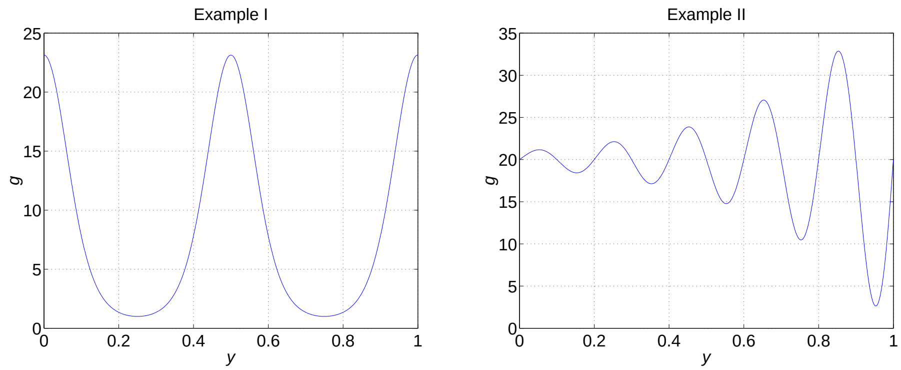
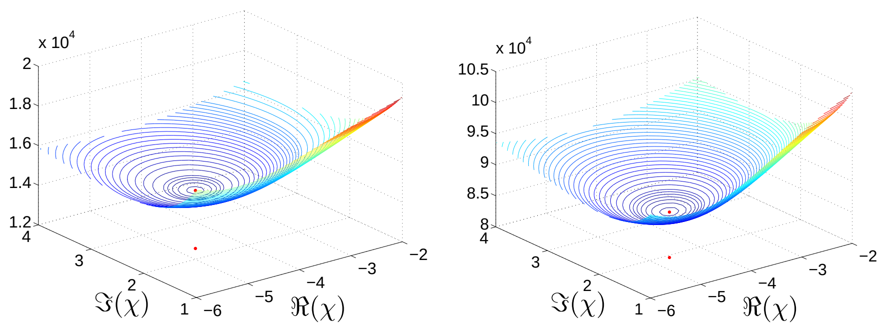


FIGURE 2. Profile of the source function g



Example 1 Example 2

FIGURE 3. 3-D contour plots of cost functions when $\alpha = \beta = \gamma = 1$

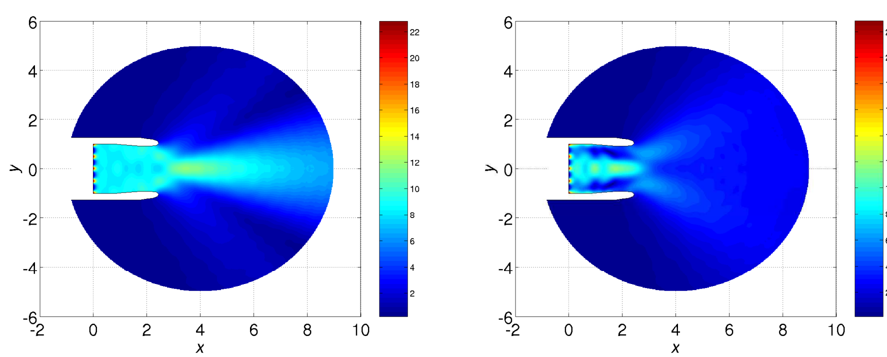


FIGURE 4. Example 1: The contour maps of $|u(0)|$ and $|u(\chi^*)|$ when $\alpha = 1$, $\beta = \gamma = 0$

5. Conclusions

In this paper we posed the optimal design problem of acoustic liner to minimize fan noise radiation from commercial aircraft engine nacelles. The optimization problem is to find the optimal liner impedance factor to minimize far-field radiated

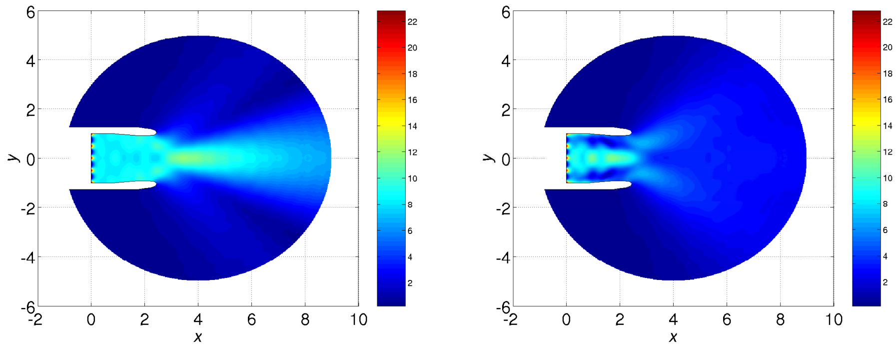


FIGURE 5. Example 1: The contour maps of $|u(0)|$ and $|u(\chi^*)|$ when $\alpha = \beta = 1$, $\gamma = 0$

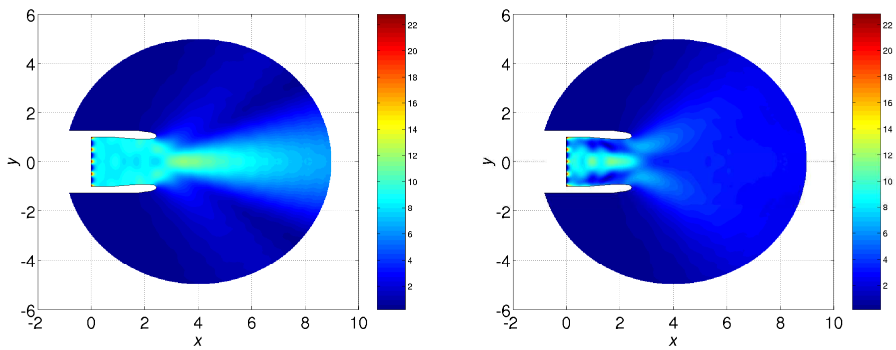


FIGURE 6. Example 1: The contour maps of $|u(0)|$ and $|u(\chi^*)|$ when $\alpha = \beta = \gamma = 1$

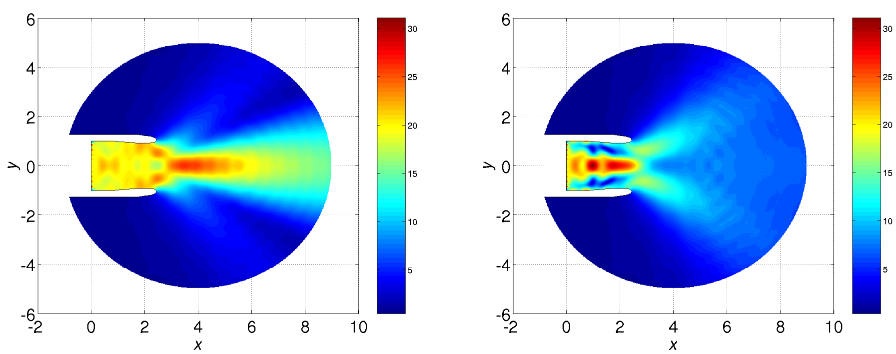


FIGURE 7. Example 2: The contour maps of $|u(0)|$ and $|u(\chi^*)|$ when $\alpha = 1$, $\beta = \gamma = 0$

noise. The cost function is defined to reflect the amount of noise radiated from the engine inlet. We first proved that the governing boundary value problem of the Helmholtz equation is uniquely solvable except a countable set of liner impedance factors having zero as its accumulation point. In particular, under a reasonable

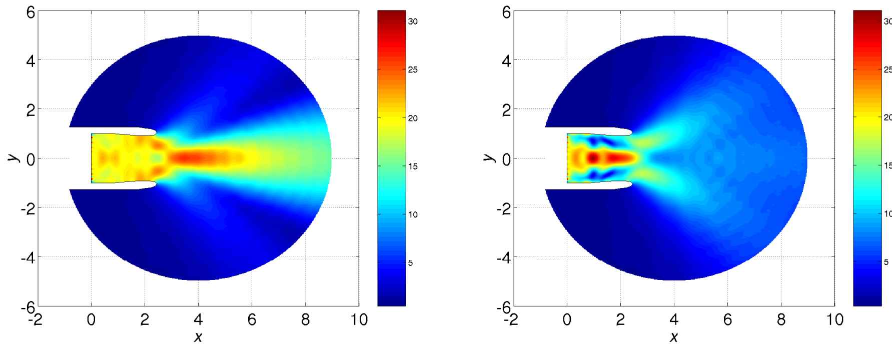


FIGURE 8. Example 2: The contour maps of $|u(0)|$ and $|u(\chi^*)|$ when $\alpha = \beta = 1$, $\gamma = 0$

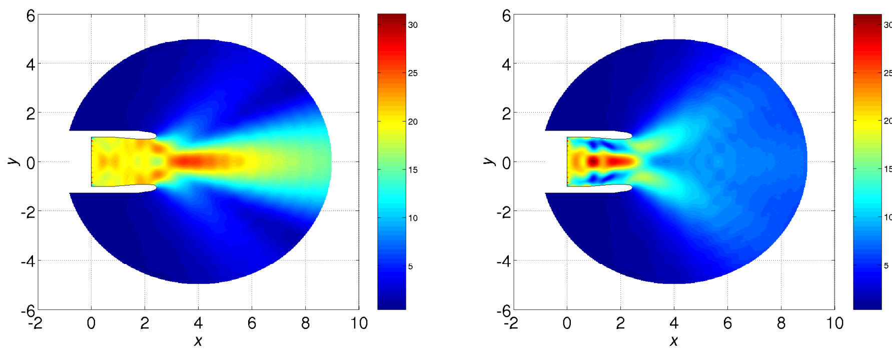


FIGURE 9. Example 2: The contour maps of $|u(0)|$ and $|u(\chi^*)|$ when $\alpha = \beta = \gamma = 1$

assumption, we also showed that this spectral set is located in the left half of the complex plane, which agrees with common practice of engineering communities and guarantees the stability of our numerical algorithm. Then the existence of an optimal liner impedance factor was established. Finally, we presented numerical results to demonstrate that the choice of the optimal liner impedance factors does result in reducing the noise level by about 40%, which is quite significant. In the future we are going to study the corresponding optimization problem when the wave number is considered as a random variable.

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