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Simultaneous suppression of spontaneous emission and thermal effect in two-level trapped ion system

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Abstract. We have proposed a method to suppress simultaneously decoherence induced by spontaneous emission and thermal effect in a single two-level trapped ion system based on bang-bang scheme. The basic idea of this method is that two kinds of decoherences are both suppressed by using a series of large detuned pulses to drive the ion in a precision-designed manner, thus best results can be obtained with minimal resources.

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Key words: decoherence, spontaneous emission, thermal effect, trapped ions

1 Introduction

Recently quantum dynamics of trapped ions system has investigated extensively not only because it is of fundamental interest in quantum optics [1, 2] but also because it plays very important role in quantum computation [3]. Quantum computers have powerful potential of parallel computation. However, an obvious obstacle of physical realization of quantum computer is decoherence. Docoherence of quantum systems generally results from their entanglement with environment. Due to decoherence, quantum linear superposition states will be degenerated into their mixed states. To overcome decoherence, many different strategies have been proposed such as. quantum error-correcting codes (QECC) [4–7], quantum error-avoiding codes (QEAC) [8–11], bang-bang control [12]. The bang-bang controlling scheme, which is also termed as dynamical decoupling scheme, is a very useful and effective method in inhibiting decoherence. So far many dynamical decoupling schemes have been proposed such as periodic dynamical decoupling(PDD) [13], concatenated dynamical decoupling (CDD) [14] and Uhrig dynamical decoupling(UDD) [15]. They can be applied to inhibit a variety of noises such as white noise [12], $1/f^{\alpha}$ noises [16], etc. However, the proposed

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bang-bang schemes have been only used to suppress decoherence induced by single noise resource. According to the authors' knowledge, it is not reported that the bang-bang scheme is used to simultaneously suppress decoherence induced by more noise resources.

In trapped ions system, decoherence induced by the environment seems to be father complex, but basic decoherence factors have spontaneous emission involving electric motion of ion and thermal effect involving vibrational motion of ion. In this paper, we propose a scheme to control simultaneously both decoherence factors by means of external controllable light fields. We find that both decoherence will be effectively suppressed.

Let us consider the interaction of a single two-level ion (the excited state $|e\rangle$ and ground state $|g\rangle$) trapped in a harmonic potential trap with the environment (thermal bath). The environment is modeled as a collection of quantized harmonic oscillators. With approximation of rotating wave, the total Hamiltonian is given by ($\hbar=1$)

$$H=H_0+H_i, \tag{1a}$$

$$H_0=\frac{1}{2}\omega_a\sigma_z+\nu a^+a+\sum_k\omega_k b_k{}^+b_k, \tag{1b}$$

$$H_i=\sum_k\gamma_k(a+b_k{}^++a^+b_k)+\sum_k\lambda_k(\sigma_+b_k+\sigma_-b_k{}^+), \tag{1c}$$

where $b_k{}^+$ and b_k are, respectively, creation and annihilation operator for the k-th field mode, γ_k and λ_k are coupling parameters, σ_z is Pauli operator, $\sigma_+=|e\rangle\langle g|$ and $\sigma_-=|g\rangle\langle e|$ are up and down atomic operators, respectively. It is noted that the first term in Eq. 1c describes thermal effect of vibratioal motion of the ion, while the second term describes spontaneous emission of the ion. The evolution operator of the whole system is represented as

$$U(t,t_0)=e^{-i(t-t_0)(H_0+H_i)}. \tag{2}$$

In order to control decoherence induced by spontaneous emission and thermal effect in two-level trapped ion system, we apply bang-bang control scheme. For this goal we drive

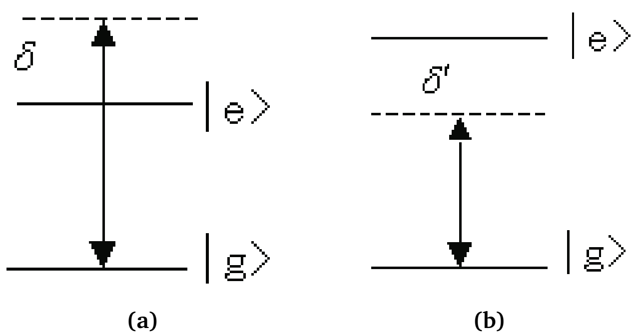

Figure 1: Schematic diagram of the trapped ion driven by optical pulses, (a) $\delta=\omega_L-\omega_a>0$, (b) $\delta'=\omega_a-\omega_L'>0$.

the two-level trapped ion system by means of a classical large-detuned optical pulse with the frequency ω_L, which is shown in Fig. 1(a). The Hamiltonian describing the driving process is

$$H = H_0 + H_I, \tag{3a}$$

$$H_0 = \frac{\omega_a}{2}\sigma_z + \nu a^+ a, \tag{3b}$$

$$H_I = \Omega\sigma_+ e^{i\eta(a+a^+) - i(\omega_L+\nu)t} + H.c, \tag{3c}$$

where Ω is the coupling constant, η is Lamb-Dicke parameter. Applying Lamb-Dicke approximation, we can obtain the interaction Hamiltonian in the ineraction picture as follows

$$H_I(t) = \Omega\sigma_+ e^{i(\delta+\nu)t} + i\eta\Omega\sigma_+ a^+ e^{i(\delta+2\nu)t} + i\eta\Omega\sigma_+ a e^{i\delta t} + H.c, \tag{4}$$

where $\delta = \omega_L + \omega_a$. Setting $\delta > 0$ and considering case of large detuning, and thus neglecting fast varying terms and retaining the slowest term, we can obtain the effective interaction Hamiltonian [17]

$$H_{P1}(t) = -g(2a^+ a + 1)\sigma_z, \tag{5}$$

where $g = \eta^2|\Omega|^2/2\delta$. Similarly, applying a classical large-detuned optical pulsed field with the carrier frequency ω_L' acted on the ion system, which is shown in Fig. 1(b), we can obtain the effective the interaction Hamiltonian [17]

$$H_{P2} = h(2a^+ a + 1)\sigma_z, \tag{6}$$

where $h = \eta'^2|\Omega'|^2/2\delta'$, $\delta' = \omega_a - \omega_L' > 0$. We can adjusts the parameters δ', Ω', δ and Ω to satisfy $h = g$ (supposing two kinds of pulses have the same duration). In order to suppression decoherence, we apply a series of large-detuned optical pulses with two different frequencies ω_L' and ω_L to drive the ion system. We arrange driving process in such a manner that optical pulse with frequency ω_L driving follows by optical pulse with frequency ω_L' driving, and do recurrence like the above. The Hamiltonian describing this driving process is given by

$$\tilde{H}_p(t) = \sum_{n=1} V^{(n)}(t)(2a^+ a + 1)\sigma_z, \tag{7a}$$

where

$$V^{(n)}(t) = (-1)^n g\theta(t - T - (n-1)(T+\tau_0))\theta(n(T+\tau_0) - t). \tag{7b}$$

In (7b), g is the constant parameter, $\theta(t)$ the usual step function, τ_0 the duration of the pulse, T the pulse interval. The evolution $\tilde{U}_j^P(t)$ corresponding to the j-th pulse Hamiltonian can be represented as

$$\tilde{U}_j^P = e^{-i\int V^{(j)}(t)dt} = e^{i(-1)^j V\tau_0(2a^+ a+1)\sigma_z}. \tag{8}$$

In the interaction picture, the interaction Hamiltonian of the whole system(the ion plus the bath) can be represented as

$$H(t) = H_0 + H_i + H_p(t). \tag{9}$$

We now calculate the time evolution of the whole system. For simplicity, we assume that the strength of pulses is so strong that bath effect can be neglected during pulses and the width of pulses is so short that the free evolution of the whole system can be neglected during pulses. We, first of all, consider an elementary cycle that contains two successive pulses. The whole evolution concludes two basic evolutions: (1) evolution $U(t)$ under H between successive pulses; (2) evolution $\tilde{U}_j^P$ $(j=1,2)$ within each pulse. The evolution of the whole system can be obtained with the form

$$U_t\Big(2(T+\tau_0)+t_0,t_0\Big)=\tilde{U}_2^P U(2T+\tau_0+t_0,T+\tau_0+t_0)\tilde{U}_1^P U(T+t_0,t_0). \tag{10}$$

Selecting $2V\tau_0/\hbar=\pi$ (so called π pulses) and taking advantage of (2), (8) and the formulae

$$e^{i\pi(a^+a+1/2)\sigma_z}ae^{-i\pi(a^+a+1/2)\sigma_z}=-a, \tag{11a}$$

$$e^{i\pi(a^+a+1/2)\sigma_z}a^+e^{-i\pi(a^+a+1/2)\sigma_z}=-a^+, \tag{11b}$$

$$e^{i\pi(a^+a+1/2)\sigma_z}\sigma_-e^{-i\pi(a^+a+1/2)}\sigma_-e^{-i\pi(a^+a+1/2)\sigma_z}=-\sigma_-e^{-i2\pi a^+a}, \tag{11c}$$

$$e^{i\pi(a^+a+1/2)\sigma_z}\sigma_-e^{-i\pi(a^+a+1/2)}\sigma_+e^{-i\pi(a^+a+1/2)\sigma_z}=-\sigma_+e^{-i2\pi a^+a}, \tag{11d}$$

we can obtain

$$\tilde{U}_2^P(\tau_0)U(T)\tilde{U}_1^P(\tau_0)=e^{i\pi a^+a\sigma_z}e^{-iT(H_0+H_i)}e^{-i\pi a^+a\sigma_z}=e^{-iT(H_0-H_i+H_\delta)}, \tag{12a}$$

$$H_\delta=\sum_k\lambda_k\Big((1-e^{-i2\pi a^+a})b_k{}^+\sigma_-+(1-e^{i2\pi a^+a})b_k\sigma_+\Big). \tag{12b}$$

The evolution of the whole system

$$\tilde{U}_t(2(T+\tau_0)+t_0,t_0)=e^{-iT(H_0-H_i+H_\delta)}+e^{-iT(H_0+H_i)}. \tag{13}$$

After N cycles, we can obtain the state vector of the whole evolution as

$$\begin{aligned}\Big|\psi(2N(T+\tau_0)+t_0)\Big\rangle&=\tilde{U}_t(N2(T+\tau_0)+t_0,t_0)\ |\psi(t_0)\rangle\\&=\Big(e^{-iT(H_0-H_i+H_\delta)}e^{-iT(H_0+H_i)}\Big)^N\ |\psi(t_0)\rangle.\end{aligned} \tag{14}$$

Nothing that for any state vector $|\psi\rangle$ of the whole system we have $H_\delta|\psi\rangle=0$, we get

$$\Big|\psi(2N(T+\tau_0)+\tau_0)\Big\rangle=\Big(e^{-iT(H_0-H_i)}e^{-iT(H_0+H_i)}\Big)^N\ |\psi(t_0)\rangle, \tag{15}$$

The formula (15) is our starting point for further analysis in the following text. Applying Combell-Baker-Hausdorff formula,

$$\exp\{\mu A\}\exp\{\mu B\}=\exp\left\{\mu A+\mu B+\sum_{m=1}^{\infty}\frac{\mu^{m+1}}{(m+1)!}ad^mA\{B\}\right\}, \tag{16}$$

where $adA\{B\}=[A,B], ad^2A\{B\}=[A,[A,B]],\cdots$, the state vector after N cycles (Eq. (15)) can be represented as

$$\Big|\psi(2N(T+\tau_0)+\tau_0)\Big\rangle=e^{-i2NTH_0+W}|\psi(t_0)\rangle, \tag{17a}$$

where

$$W=N\sum_{m=1}^{\infty}\frac{(-iT)^{m+1}}{(m+1)!}ad^m(H_0-H-i)\{H_0+H_i\}. \tag{17b}$$

From Eq. (17a), it is clear that under condition of parity kicks, the evolution of the whole system is governed by the effective Hamitonlian

$$H_{\text{eff}}=H_0+H_{\text{ieff}}, \tag{18a}$$

where H_{ieff} is the effective interaction Hamiltonian

$$H_{\text{ieff}}=i\frac{W}{2NT}=\frac{1}{2}\sum_{m=1}^{\infty}\frac{(-iT)^m}{(m+1)!}ad^m(H_0-H_i)\{H_0+H_i\}. \tag{18b}$$

When $NT=C$ (constant), we have

$$H_{\text{ieff}}=\frac{1}{2}\sum_{m=1}^{\infty}\frac{(-iC)^m}{(m+1)!}N^{-m}ad^m(H_0-H_i)\{H_0+H_i\}. \tag{19}$$

We can see that if $N\to\infty$, then $H_{\text{ieff}}\to 0$. Therefore, we get

$$\Big|\psi(2N(T+\tau_0)+t_0)\Big\rangle=e^{-i2NTH_0}|\psi(t_0)\rangle. \tag{20}$$

This means that in the limit of infinite parity kicks the interaction of the bare system (the trapped ion) with the bath is completely eliminated and only the free uncoupled evolutions left.

Finally, we discuss this scheme proposed in this text through comparing with the other bang-bang control schemes. The bang-bang controlling scheme is to control decohernce by a series of fast and strong pulses. In presented schemes, suppression of decoherence of different characteristic needs different controlling methods (i.e. controlling Hamiltonian of the different kind). Our scheme may inhibit simultaneously two kinds of decoherences using a kind of controlling Hamiltonian; thus best results can be obtained with minimal resources.

In conclusion, we have found that in a single two-level trapped ion system decoherence due to spontaneous emission and thermal effect can be simultaneously suppressed by using large detuned pulses to drive alternately two-level trapped ion.

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