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Dissipative dynamics of quantum and classical correlations for two-qubit under two-side and one-side decoherence

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Abstract. The dissipative dynamics of classical correlation(CC), quantum discord(QD) and entanglement (QE) of two qubits in two-side and one-side decoherence models are investigated under Markovian environments. We find the sudden change QD as well as CC and sudden death of entanglement (ESD). The results show that QD and QE decay faster with the increasing of squeezing parameter r; the dipole-dipole interaction Ω under two-side decoherence leads to the oscillation of quantum discord and concurrence for initial non-eigenstates; while in all cases when entanglement suddenly disappears, quantum discord keeps nonzero under same conditions.

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Key words: quantum discord, entanglement, classical correlation, dipole-dipole interaction, squeezing parameter

1 Introduction

Quantum entanglement is a vital resource and has the computational advantage of quantum over classical algorithms. Hence it has been playing a central role in quantum computation and information processing [1]. However, there are other non-classical correlations apart from the entanglement [2–5] that can be very important to these fields. In order to characterize all non-classical correlations, Ollivier and Zurek introduced a concept of quantum discord [2], it is a different type of quantum correlation than the entanglement, and it can be considered as a more universal resource because separable mixed states (without entanglement) can have nonzero quantum discord. This measure of quantum correlations holds a

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fundamental feature of classical bipartite states. When the discord is zero, the information is locally accessible and can be obtained by distant independent observers without perturbing the bipartite state. In addition, it was shown theoretically [6, 7] and later experimentally [8] that, some separable states may also speed up certain tasks over their classical counterparts. Therefore, much attention has been paid to many relative topics of quantum discord [9–13].

It is well known that all quantum systems interact inevitably with their surrounding environments, this leads to decoherence which degrades the entanglement of the quantum system [14]. Thus it is important to know the influence of the environment on quantum correlation. Recently, the quantum correlation dynamics in open quantum systems have been studied [15–18]. It was shown that the quantum correlation measured by quantum discord is more resistant against the environment than quantum entanglement. For a certain class of states under Markovian dynamics, the quantum entanglement can disappear within a finite time, namely, entanglement sudden death (ESD) [19], but quantum discord only vanishes asymptotically at infinite time. In addition, for some special initial states, quantum correlation in a bipartite quantum system will not be affected by the decoherence environment during an initial time interval.

On the other hand, many typical environments have been investigated, such as, vacuum, squeezed vacuum, multi-mode vacuum cavity, single mode cavity and so on. The effects on the entanglement, such as dipole-dipole interaction between the particles and the couplings of particles with the same cavity field have been studied extensively. Zhang *et al.* [20] studied the entanglement character between two identical two-level atoms in a two-mode cavity field, where the authors discussed the influence of dipole-dipole interaction on entanglement between atoms. Chen *et al.* [21] study the influence of the dipole-dipole interaction on the evolution of entanglement between two atoms, they obtained different results. However, the dissipative dynamics of quantum discord in coupled qubit system under Markovian environments are rarely discussed. In this paper we investigate the dissipative dynamics of quantum discord and entanglement for two coupled qubit system subjected to Markovian environments, each of which interacts with a multi-mode squeezed vacuum field reservoir. We also compare the dynamics of quantum discord with that of the entanglement by using the standard numerical method, and examine the influence of dipole-dipole interaction and squeezing parameter on quantum discord and entanglement.

2 The model and its solution

The system consists of two identical qubits. By one-side decoherence, we mean that only one qubit A (or B) is subject to the multi-mode squeezed vacuum field reservoir. For the two-side decoherence, the two atoms characterized by an excited state $|1\rangle$ and a ground state $|0\rangle$, are independently subject to their respective reservoirs (assumed to be the same for both) described by annihilation b_k and creation operators b_k^+. In the interaction picture and the rotating-wave approximation, the interaction between the qubits and their reservoirs is generally represented by the simple Hamiltonian ($\hbar=1$)

$$V(t)=\sum_{j=1}^{2}\sum_{k}g_k\left(b_k^+\sigma_-^j e^{-i(\omega-r_k)t}+b_k\sigma_+^j e^{i(\omega-r_k)t}\right), \tag{1}$$

where j labels A_j, $\sigma_-=|0\rangle\langle 1|$ ($\sigma_+=|1\rangle\langle 0|$), and ω are lowering (raising) operator and transition frequency between the ground and excited states of a qubit, and $r_k=ck$ are the frequencies associated with the reservoir. Here, we consider each qubit is coupled to a multi-mode squeezed vacuum field reservoir represented by the reduced density operator

$$\rho_R=\prod_k S_k(\xi)|0_k\rangle\langle 0_k|S_k^+(\xi), \tag{2}$$

$$S_k(\xi)=\exp\left(\xi^* b_{k_0+k}b_{k_0-k}-\xi b_{k_0+k}^+ b_{k_0-k}^+\right). \tag{3}$$

Here, $S_k(\xi)$ is the squeeze operator, $\xi=r\exp(i\phi)$ with r being the squeezing parameter and ϕ being the phase for squeezed field. Under the Markovian approximation, the reduced density matrix ρ describing the two non-coupled qubit system is then expressed by a master equation [22]

$$\frac{d\rho}{dt}=\sum_{i=1}^{2}\left(\frac{\Gamma}{2}c_r^2\left(\sigma_+^i\sigma_-^i\rho-2\sigma_-^i\rho\sigma_+^i+\rho\sigma_+^i\sigma_-^i\right)+\frac{\Gamma}{2}s_r^2\left(\sigma_-^i\sigma_+^i\rho-2\sigma_+^i\rho\sigma_-^i+\rho\sigma_+^i\sigma_-^i\right)\right.$$
$$\left.-\Gamma e^{-i\phi}s_r c_r\sigma_-^i\rho\sigma_-^i-\Gamma e^{i\phi}s_r c_r\sigma_+^i\rho\sigma_+^i\right), \tag{4}$$

where $s_r=\sinh(r)$, $c_r=\cosh(r)$, Γ is the spontaneous decay rate of the qubit to the reservoir. When the two qubits are coupled to each other , the dipole-dipole interaction [23] of two qubits can be expressed as $H=\Omega(\sigma_+^1\sigma_-^2+\sigma_-^1\sigma_+^2)$, where Ω is the interaction strength between two qubits, and taken real for simplicity. The dynamics of the density matrix ρ describing the system of two-coupled qubits is then expressed by

$$\frac{d\rho}{dt}=-i[H,\rho]+\sum_{i=1}^{2}\left(\frac{\Gamma}{2}c_r^2\left(\sigma_+^i\sigma_-^i\rho-2\sigma_-^i\rho\sigma_+^i+\rho\sigma_+^i\sigma_-^i\right)\right.$$
$$\left.-\frac{\Gamma}{2}s_r^2\left(\sigma_-^i\sigma_+^i\rho-2\sigma_+^i\rho\sigma_-^i+\rho\sigma_+^i\sigma_-^i\right)-\Gamma e^{i\phi}s_r c_r\sigma_-^i\rho\sigma_-^i-\Gamma e^{i\phi}s_r c_r\sigma_+^i\rho\sigma_+^i\right). \tag{5}$$

Assuming that the two-qubit system is initially an “X-state” form, in the basis of the separable product states $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$, the initial state is

$$\rho_0=\begin{pmatrix} a_0 & 0 & 0 & \mu_0 \\ 0 & b_0 & \nu_0 & 0 \\ 0 & \nu^* & c_0 & 0 \\ \mu_0^* & 0 & 0 & d_0 \end{pmatrix}. \tag{6}$$

One of the important features is that the initial density matrix of Eq. (6) preserves its form during time evolution governed by Eq. (5),

$$\rho_{AB}=\begin{pmatrix} a & 0 & 0 & \mu \\ 0 & b & \nu & 0 \\ 0 & \nu^* & c & 0 \\ \mu^* & 0 & 0 & d \end{pmatrix}. \tag{7}$$

Using Eq. (5), we can obtain the resolve of coupled differential equations. For simplicity, we set $\phi=0$.Then given an initial state, the behavior of quantum discord and entanglement between the two qubits finally depends on squeeze parameter γ and dipole-dipole interaction Ω.

3 Dynamics of quantum and classical correlations

A bipartite quantum state contains both classical and quantum correlations. These correlations are quantified jointly by their "quantum mutual information", an information-theoretic measure of the total correlation in a bipartite quantum state [24]. Such the quantum mutual information can be defined as

$$I(\rho_{AB})=S(\rho_A)+S(\rho_B)-S(\rho_{AB}). \tag{8}$$

ρ_{AB} denotes the density operator of a composite bipartite system, ρ_A (ρ_B) is the reduced density operator of part A (B) respectively, and $S(\rho)=-Tr(\rho\log_2\rho)$ is the joint entropy of the system. Quantum mutual information may be written as a sum of classical correlation $C(\rho_{AB})$ and quantum correlation $D(\rho_{AB})$, namely, $I(\rho_{AB})=D(\rho_{AB})+C(\rho_{AB})$ [2–4]. This quantum part $D(\rho_{AB})$ has been called quantum discord [2]. The classical correlation $C(\rho_{AB})$ is defined as

$$C(\rho_{AB})=\max_{\{B_k\}}\Big(S(\rho_A)-S(\rho_{AB}|\{B_k\})\Big)=S(\rho_A)-\min_{\{B_k\}}S(\rho_{AB}|\{B_k\}), \tag{9}$$

where the maximum (or the minimum) is taken over all the complete sets of projective measurements $\{B_k\}$ on the subsystem B and $S(\rho_{AB}|\{B_k\})=\sum_k p_k S(\rho_k)$ is the quantum conditional entropy with respect to this measurement, the conditional density operator ρ_k associated with the measurement result k is

$$\rho_k=\frac{1}{p_k}\Big(Tr_B(I\otimes B_k)\rho_{AB}(I\otimes B_k)\Big),$$

where the probability

$$p_k=Tr_{AB}(I\otimes B_k)\rho_{AB}(I\otimes B_k).$$

To evaluate the entanglement and the discord dynamics presented in this paper we determine an analytical expression for a subclass of the X structured density operator same as Eq. (7).

For the discord, we choose the set of projectors $\{|\varphi_1\rangle\langle\varphi_1|, |\varphi_2\rangle\langle\varphi_2|\}$ where $|\varphi_1\rangle = \cos\theta|1\rangle + e^{i\delta}\sin\theta|0\rangle$, $|\varphi_2\rangle = \sin\theta|1\rangle - e^{i\delta}\sin\theta|0\rangle$ to measure one of the subsystem. The minimum of discord $D(\rho_{AB}) = I(\rho_{AB}) - C(\rho_{AB})$ can be obtained by the maximum of Eq. (9), which can be achieved by choosing appropriate values of θ and δ [13].

The usual way to identify entanglement between two qubits in a mixed state is to examine the concurrence. Using Wootters' formula [25], the concurrence for the state of Eq. (7) is written as $C = 2\max\{0, C_1 = |\mu| - \sqrt{bc}, C_2 = |\nu| - \sqrt{ad}\}$.Then given an initial state, the behavior of quantum discord and entanglement between the two qubits finally depends on squeeze parameter r.

3.1 Dynamics of QD, CC and QE under two-side decoherence

Firstly, we take the initial state

$$\rho(0) = (1-p)|\psi_\alpha\rangle\langle\psi_\alpha| + p|\phi_\beta\rangle\langle\phi_\beta|,$$

where $|\psi_\alpha\rangle = \cos\alpha|01\rangle + \sin\alpha|10\rangle$ and $|\psi_\beta\rangle = \cos\beta|00\rangle + \sin\beta|11\rangle$ are two Bell-like states. The initial state of the two-qubit is given by the parameters $(p, \alpha, \beta) = (0.6, \pi/10, \pi/3)$, we draw the dynamics of quantum and classical correlation and concurrence in Fig. 1, we can see CC, QD and QE decrease with time, but the sudden change of CC and QD is present for this initial state, and it occurs earlier when r is large. Additionally, the initial order is CC>QE>QD, after a short evolution time, the order becomes CC>QD>QE, this is because entanglement undergoes sudden death, while correlations are long lived. The results imply that QE disappear completely after a finite time independently with the initial states, while QD as well as CC decreases and tends to be a stable value according to the initial-state parameter for a very-long-time interval. In this sense, the quantum discord is more robust than entanglement for the quantum system exposed to the environment. The inset in Fig. 1(a) shows r versus ESD time, and indicates that the disentanglement time decreases with the increase of squeeze parameter r due to the increasing of average photon number of every mode in the squeezed vacuum reservoirs.

Fig. 2 gives the dynamics of QD and QE for dipole-dipole interaction strength $\Omega = 8$, we can see that QD decreases oscillatory with time and the amplitude is reduced with r increasing. When r is large, quantum discord shows no evident oscillation after long time evolution, and remains to be non-zero value. Instead, concurrence falls abruptly to zero, i.e., the ESD occurs .The oscillations of QD and QE disappear due to the fact that large r dominates over Ω, consequently the effects of Ω could be neglected at present. But QD behaves obviously differently from the QE, implying that quantum algorithms based only on quantum discord correlations are more robust than those based on entanglement.

The influences of dipole-dipole interaction strength Ω on the dynamics can be seen from Fig. 3, obviously, both quantum discord and concurrence drop with time. For $\Omega = 0$, they display no oscillation but concurrence present "ESD" feature without dark-bright periods. When $\Omega > 0$, QD and QE decreases oscillatory with Ω. For strong Ω, the concurrence evolution exhibits dark-bright periods, and the period of oscillation becomes shorter. However, quantum

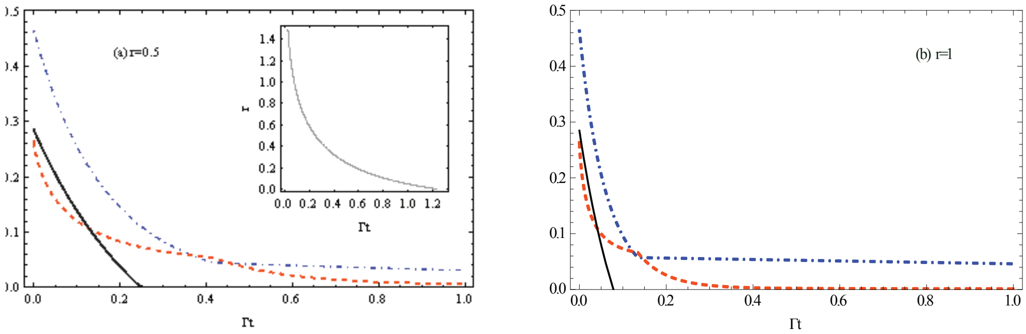

Figure 1: Dissipative dynamics of QD (dashed line), CC(dot-dashed line) and QE (solid line) under two-side decoherence for (a) $r=0.5$ (b) $r=1$, $(p,\alpha,\beta)=(0.6,\pi/10,\pi/3)$, $\Omega=0$.

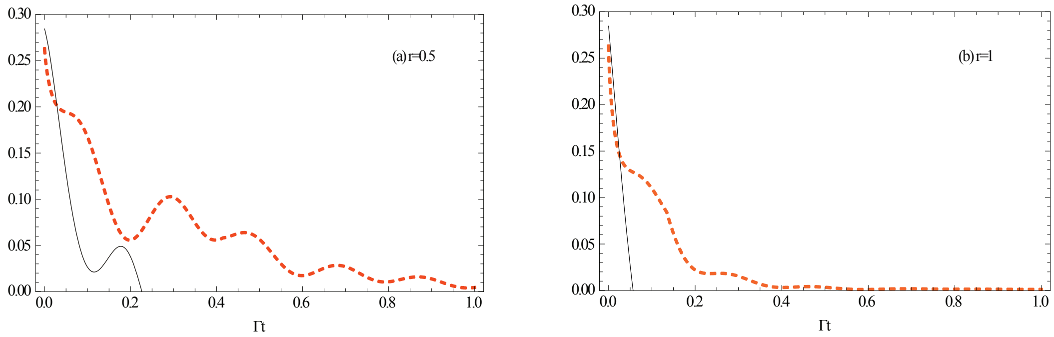

Figure 2: Dissipative dynamics of QD (dashed line) and QE (solid line) for (a) $r=0.5$, (b) $r=1$, $(p,\alpha,\beta)=(0.6,\pi/10,\pi/3)$, $\Omega=8$.

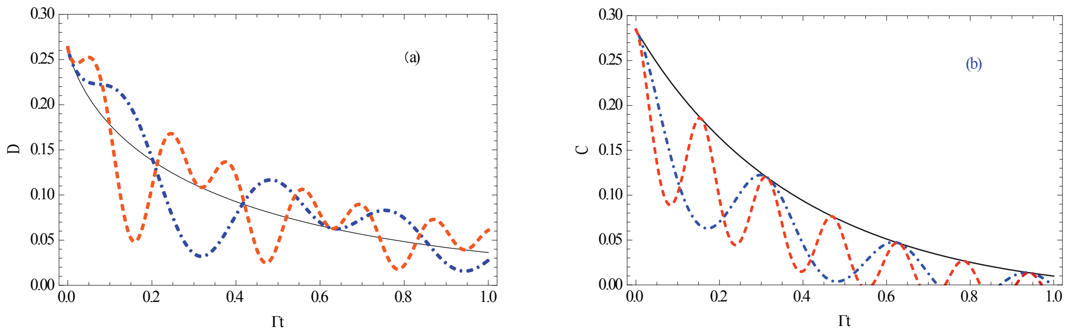

Figure 3: Dissipative dynamics of quantum discord (a) and concurrence (b) for $r=0$, $(p,\alpha,\beta)=(0.6,\pi/10,\pi/3)$, $\Omega=0$ (solid line), $\Omega=5$ (dot-dashed line), $\Omega=10$ (dashed line).

discord goes down oscillatory and more slowly and without dark-bright periods. The concurrence dynamics is confined with the region determined by the below solid line ($\Omega = 0$), but it is not same for the QD. This mean Ω can make the quantum correlation stronger or weaker periodically. Moreover, the investigation shows that, the dipole-dipole interaction leads to the oscillation when the initial states are not the eigenstates of the coupling Hamiltonian $H = \Omega(\sigma_+^1\sigma_-^2 + \sigma_-^1\sigma_+^2)$.

3.2 Dynamics of QD, CC and QE under one-side decoherence

By one-side decoherence, we mean that only one qubit A (or B) is subject to the heat reservoir. The Solution of the quantum-Louiville equation is outlined in Appendix, The dynamics behaviors of CC, QD and QE, are very similar with that present in two-side decoherence model, for example, there exist sudden-change behavior in the dynamics of CC and QD, long-lived CC and QD, and ESD, etc. However, in comparison with the dynamics under the two-side decoherence, in one-side decoherence QD, CC and QE decay slowly under same initial conditions.

It is worth pointing out that when the system is initially the eigenstates of the coupling Hamiltonian, for example, the initial state as Werner state [26]

$$\rho_0 = p|\psi^-\rangle\langle\psi^-| + \frac{1-p}{4}I,$$

where $|\psi^-\rangle = (|01\rangle - \langle 10|)/\sqrt{2}$ is a maximally entangled state and p $(0 < p < 1)$ indicates the purity of the initial states. When $p = 0$, the Werner state become totally mixed states, while for $p = 1$ it is well-known Bell-state.

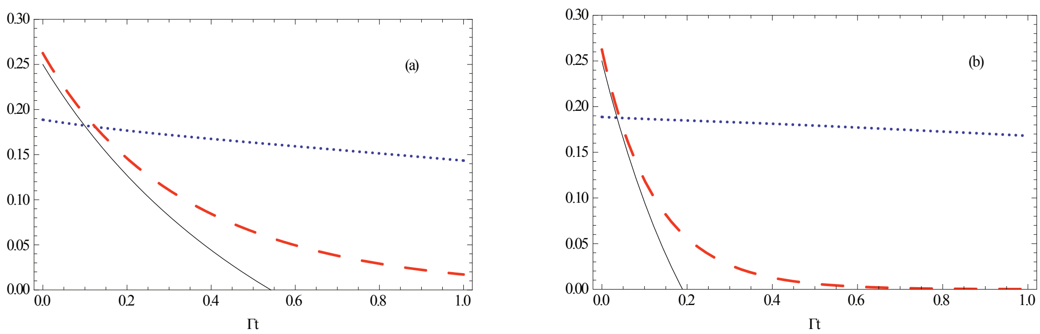

Figure 4: Dissipative dynamics of QD (dashed line), CC(dotted line) and QE (solid line) under one-side decoherence for ($p = 0.5$) (a) $r = 0.5$, (b) $r = 1$.

As shown in Fig. 4, QD, CC and QE monotonously decrease with time, CC and QD present no sudden changes, and the order keeps all the timeąACC drops with the time more slowly when is smaller . In this case we also say that this initial state is robust associated with CC and QD, while not with respect to QE. Moreover, the dynamics of QD, CC and QE is not influenced by dipole-dipole interaction, this stems from the fact that the Werner states are the eigenstates of the coupling Hamiltonian $H=\Omega(\sigma_+^1\sigma_-^2+\sigma_-^1\sigma_+^2)$.

4 Conclusion

In this paper, we have studied the dynamics of two qubits in two-side and one- side decoherence models (both or only one qubit) interacting independently with the multi-mode squeezed vacuum field baths. We have discussed mainly the effects of the squeeze parameter r and the dipole-dipole interaction Ω on QD, CC and QE. The result is as follows.

The sudden change of QD, CC and sudden death of entanglement are found. QD and QE decay faster with the increasing of r. The large squeeze parameter r can contribute to the entanglement death sooner and the time of sudden change earlier. The corresponding death time under two-side decoherence has been given.

For initial non-eigenstates of the coupling Hamiltonian, the dipole-dipole interaction Ω leads to the oscillation of quantum discord and concurrence. For large r, QD shows no evident oscillation after long time evolution, and remains to be non-zero value. Instead, concurrence falls abruptly to zero, i.e., the ESD occurs. The oscillations of QD and QE disappear due to the fact that large dominates over Ω, consequently the effects of Ω could be neglected at present.

For one-side decoherence model, the dynamics behaviors of CC, QD and QE are very similar with that present in two-side decoherence model, However, in one-side decoherence QD, CC and QE decay slowly under same initial conditions ,and CC decays more slowly with increasing r in one-side decoherence.

In conclusion, quantum discord evidently behaves differently from the concurrence under same conditions. Even when ESD occurs, the value of quantum discord keeps nonzero. This means that the absence of entanglement does not necessarily indicate the absence of quantum correlations, then quantum discord is more robust than the entanglementąAmeaning that quantum computers based on this kind of quantum correlation, differently from those based on entanglement, are more resistant to external environment.

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Appendix 1

Solution of the quantum-Liouville equation under two-side decoherence is

$$a=\frac{x^2}{4}\left(c_{2r}^{-2}+2(a_0-d_0)c_{2r}^{-1}+1-2(b_0+c_0)\right)-\frac{x}{2}\left(c_{2r}^{-2}-(b_0+c_0+2d_0)c_{2r}^{-1}+d_0-a_0\right)+\frac{(c_{2r}^{-1}-1)^2}{4},$$

$$b=-\frac{x^2}{4}\Big(c_{2r}^{-2}+2(a_0-d_0)c_{2r}^{-1}+1-2(b_0+c_0)\Big)$$
$$+\frac{x}{2}\Big(c_{2r}^{-2}+(a_0-d_0)c_{2r}^{-1}+\cos(2\Omega t)(b_0-c_0)-2\sin(2\Omega t)\mathrm{Im}(\nu_0)\Big)-\frac{(c_{2r}^{-1}-1)}{4},$$
$$c=-\frac{x^2}{4}\Big(c_{2r}^{-2}+2(a_0-d_0)c_{2r}^{-1}+1-2(b_0+c_0)\Big)$$
$$+\frac{x}{2}\Big(c_{2r}^{-2}+(a_0-d_0)c_{2r}^{-1}-\cos(2\Omega t)(b_0-c_0)+2\sin(2\Omega t)\mathrm{Im}(\nu_0)\Big)-\frac{(c_{2r}^{-1}-1)}{4},$$
$$d=\frac{x^2}{4}\Big(c_{2r}^{-2}+2(a_0-d_0)c_{2r}^{-1}+1-2(b_0+c_0)-\frac{x}{2}\Big(c_{2r}^{-2}+(1+a_0-d_0)c_{2r}^{-1}+a_0-d_0\Big)+\frac{(c_{2r}^{-1}+1)^2}{4},$$
$$\mu=\mathrm{Re}(\mu)+i\mathrm{Im}(\mu),\qquad \nu=\mathrm{Re}(\nu)+i\mathrm{Im}(\nu),$$
$$\mathrm{Re}(\mu)=x\Big(\sin\phi\,\mathrm{Re}(i\mu_0e^{-i\phi})+\frac{y\cos\phi}{2}\mathrm{Re}(\nu_0+e^{i\phi}\mu_0)+\frac{\cos\phi}{2y}\mathrm{Re}(-\nu_0+e^{-i\phi}\mu_0)\Big),$$
$$\mathrm{Im}(\mu)=x\Big(-\cos\phi\,\mathrm{Re}(i\mu_0e^{-i\phi})+\frac{y\sin\phi}{2}\mathrm{Re}(\nu_0+e^{i\phi}\mu_0)+\frac{\sin\phi}{2y}\mathrm{Re}(-\nu_0+e^{-i\phi}\mu_0)\Big),$$
$$\mathrm{Re}(\nu)=\frac{x}{2}\Big(y]Re(\nu_0+e^{i\phi}\mu_0)+\frac{1}{y}\mathrm{Re}(\nu_0-e^{-i\phi}\mu_0)\Big),$$
$$\mathrm{Im}(\nu)=\frac{x}{2}\Big(2\cos(2\Omega t)\mathrm{Im}(\nu_0)+\sin(2\Omega t)(b_0-c_0)\Big),$$

with $y=e^{-\Gamma_{s_{2,t}}}$, $y=e^{-s_{2,t}}$, $s_r=\sinh(r)$, $c_r=\cosh(r)$.

Appendix 2

Solution of the quantum- Liouville equation under one-side decoherence:

$$a=\frac{1}{2}e^{-t\Gamma\cosh(2r)}\Big(c_0(-1+e^{t\Gamma\cosh(2r)})+a_0(1+e^{t\Gamma\cosh(2r)})-(a_0+c_0)(-1+e^{t\Gamma\cosh(2r)})\mathrm{sech}(2r)\Big),$$
$$b=\frac{1}{2}e^{-t\Gamma\cosh(2r)}\Big(c_0(-1+a_0+c_0)(-1+e^{t\Gamma\cosh(2r)})+d_0(-1+e^{t\Gamma\cosh(2r)})$$
$$+b_0(1+e^{t\Gamma\cosh(2r)})\Big)\cosh(2r)\mathrm{sech}(2r),$$
$$c=\frac{1}{2}e^{-t\Gamma\cosh(2r)}\Big(a_0(-1+e^{t\Gamma\cosh(2r)})+c_0(1+e^t\Gamma\cosh(2r))$$
$$+(a_0+c_0)(-1+e^t\Gamma\cosh(2r))\mathrm{sech}(2r)\Big),$$
$$d=\frac{1}{2}e^{-t\Gamma\cosh(2r)}(-1+a_0+c_0)(1-e^t\Gamma\cosh(2r))$$
$$+\Big(b_0(-1+e^t\Gamma\cosh(2r))+d_0(1+e^t\Gamma\cosh(2r))\Big)\cosh(2r)\mathrm{sech}(2r),$$
$$\mu=\frac{1}{2}e^{\frac{1}{2}\Gamma(\cosh(2h)+\sinh(2r))t}\Big(\mathrm{Re}(\mu_0)(1+e^\Gamma\sinh(2r)t)+\mathrm{Re}(\nu_0)(1-e^\Gamma\sinh(2r)t)$$
$$+i\frac{1}{2}e^{\frac{1}{2}\Gamma(\cosh(2r)-\sinh(2r))t}\Big(\mathrm{Im}(\nu_0)(1-e^{-\Gamma\sinh(2r)t})+\mathrm{Im}(\mu_0)(1+e^\Gamma\sinh(2r)t)\Big),$$

$$\nu = \frac{1}{2}e^{\frac{1}{2}\Gamma(\cosh(2h)+\sinh(2r))t}\Big(\mathrm{Re}(\mu_0)(1+e^{\Gamma}\sinh(2r)t)+\mathrm{Re}(\nu_0)(1-e^{\Gamma}\sinh(2r)t)$$
$$+i\frac{1}{2}e^{\frac{1}{2}\Gamma(\cosh(2r)-\sinh(2r))t}\Big(\mathrm{Im}(\nu_0)(1-e^{-\Gamma\sinh(2r)t})+\mathrm{Im}(\mu_0)(1+e^{\Gamma}\sinh(2r)t)\Big).$$

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