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Relativistic thermodynamic properties of interacting Fermi gas in a strong magnetic field

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Abstract. Based on the Mean Field Theory, the relativistic thermodynamic potential function (RTPF) of interacting Fermi gases in a strong magnetic field is derived. On this basis, by taking the interacting term from the RTPF and using the thermodynamic relations, analytical expressions of the energy, the heat capacity and the chemical potential are calculated at low temperatures. And combined with numerical simulation analysis, the effect of interaction on the thermodynamic properties of the system is analyzed and the regulation mechanisms on the interaction impacts of both the magnetic field and the relativistic effect are discussed.

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Key words: interaction strong magnetic field, relativity effect, Fermi gas

1 Introduction

Since 1995, because of ultracold Fermionic Gas and Bose Gas being imprisoned in laboratory experiments, people are passionate in the ultracold quantum gas theory. Facts indicate, statistical properties of quantum gases are affected by the external potential, particle mass, interaction, scale of particle system, dimensional effect, relativity effect and other influencing factors. Currently, many articles have researched thermodynamic properties of quantum gases. For example, literatures [1-8] have showed important researching results about ultracold Fermi gases; Gupta et al. utilizing the method of quantum statistics and the thermodynamic theory, have discussed the magnetic susceptibility and system's statistic characters of Fermi gases within a strong magnetic field and low temperatures [9, 10]; Wang et al. have studied non-extensive relativistic system's thermodynamic properties and its stability [11,12]; Men et al. have studied the relativistic effects of the thermodynamic properties of Fermi gas in a strong magnetic field and the

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influential mechanisms of temperatures and the magnetic field on the relativistic effects of the system [13]; Schunck gave the experimental testify to the super-fluidity of ultracold Fermi gas, and has reported that atomic gas 6Li can introduce an everlasting and frictionless eddy motion during the process of forming Bose Einstein Condensation (BEC) [14]; Besides, Fan et al. have discussed the relativity effect of Fermi system in the case of strong magnetic field, and also analyzed its influence mechanism thoroughly [15]. However, systems researched are non-interaction. Namely, they neglected interactions between fermions, however, those interactions affect system properties significantly. This paper has took interactions in Fermi system into consideration making calculation closer to fact, explained the RTPF of the system in a strong magnetic field. Based on the RTPF, and using thermodynamic relations, analytical expressions of thermodynamic quality caused by interaction at low temperatures are computed; according to results of numerical simulation, the influence of weak interaction on the system's thermodynamic properties is analyzed, and the regulation mechanisms on the interaction's impacts of both magnetic field and relativity effect are also discussed.

2 The relativistic thermodynamic potential function of interacting Fermi gas in a strong magnetic field

Suppose that the research system is consisted of particles with spin quantum number equating to 1/2, rest mass m, and this system is within an uniform magnetic field with an intensity of B (only has z-direction). In view of the Dirac energy operator and the second order approximation of non-relativity effect, the energy operator of single particle is given

$$H=\frac{1}{2m}[\sigma(p-\frac{e}{c}A)]^2-\frac{p^4}{8m^3c^2}+n\alpha, \tag{1}$$

where, the first term is Pauli energy operator, the second relativistic modification of kinetic energy, and the last one coming from the Mean Field Theory represents interacting energy. In addition, n is particle-number density and $\alpha=4\pi a\hbar^2/m$ is the parameter of interactions. Here a is the s-wave scattering length. Defining the potential function $A_x=-By$, $A_y=A_z=0$, and according to Men et al. [16], when considering inter-particle interactions, the total energy of single fermion in a strong magnetic field can be written as

$$\varepsilon_p=\frac{p^2}{2m}+2n'\sigma B-\frac{p^4}{8m^3c^2}+n\alpha, \tag{2}$$

where, $\sigma=\frac{\hbar e}{2m}$ is Bohr magneton, and $n'=0,1,2,3,\cdots$ is the quantum number. Popularizing the literature [17], the system's thermodynamic potential function is given

$$\Omega=2\sigma B\left\{\frac{1}{2}f(\mu+\frac{p^4}{8m^3c^2}-n\alpha)+\sum_{n'=1}^{\infty}f(\mu+\frac{p^4}{8m^3c^2}-2n'\sigma B-n\alpha)\right\} \tag{3}$$

Applying the Poisson formula

$$\frac{1}{2}F(0)+\sum_{n'=1}^{\infty}F(n')=\int_0^{\infty}F(x)dx+2r_e\sum_{k=1}^{\infty}F(x)e^{2\pi ikx}dx, \tag{4}$$

to Eq. (3), then

$$\Omega=\Omega_0(\mu)+\frac{mTV}{\pi^2h^3}r_e\sum_{k=1}^{\infty}I_k, \tag{5}$$

where $\Omega_0(\mu)$ is the system's thermodynamic potential function without an external field, μ is the chemical potential of a free system, and

$$I_k=-2\sigma B\int_0^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu}{T}-\frac{p^2}{2mT}+\frac{p^4}{8m^3c^2T}-\frac{2\sigma xB}{T}-\frac{n\alpha}{T}})e^{2\pi ikx}dxdp. \tag{6}$$

If suppose that $\varepsilon=\frac{p^2}{2m}+2x\sigma B-\frac{p^4}{8m^3c^2}+n\alpha$, then Eq. (6) can be transformed to

$$I_k=-\int_0^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{-\frac{ik\pi p^2}{2m\sigma B}}e^{\frac{ik\pi p^4}{8m^3c^2\sigma B}}e^{-\frac{ik\pi n\alpha}{\sigma B}}d\varepsilon dp. \tag{7}$$

Considering two approximations,

$$e^{\frac{ik\pi p^4}{8m^3c^2\sigma B}}\approx 1+\frac{ik\pi p^4}{8m^3c^2\sigma B}, \quad \text{and} \quad e^{-\frac{ik\pi n\alpha}{\sigma B}}\approx 1-\frac{ik\pi n\alpha}{\sigma B},$$

Eq. (7) is further expressed as after integration

$$\begin{aligned}I_k=&-\int_{-\infty}^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{\frac{ik\pi p^2}{2m\sigma B}}(1-\frac{ik\pi n\alpha}{\sigma B}+\frac{ik\pi p^4}{8m^3c^2\sigma B}-\frac{ik\pi p^4}{8m^3c^2\sigma B}\cdot\frac{ik\pi n\alpha}{\sigma B})d\varepsilon dp\\ =&-\int_{-\infty}^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{\frac{ik\pi p^2}{2m\sigma B}}d\varepsilon dp-\frac{ik\pi}{8m^3c^2\sigma B}-\int_{-\infty}^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{\frac{ik\pi p^2}{2m\sigma B}}\\ &p^4d\varepsilon dp+\frac{ik\pi n\alpha}{\sigma B}\int_{-\infty}^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{\frac{ik\pi p^2}{2m\sigma B}}d\varepsilon dp\\ &+-\frac{ik\pi n\alpha}{\sigma B}\cdot\frac{ik\pi}{8m^3c^2\sigma B}\int_{-\infty}^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{\frac{ik\pi p^2}{2m\sigma B}}p^4d\varepsilon dp\\ =&I_k^N+I_k^R-\frac{ik\pi n\alpha}{\sigma B}(I_k^N+I_k^R)=I_k^N+I_k^R+I_k^F.\end{aligned} \tag{8}$$

In here, I_k^N is the term without consideration to the relativity effect, I_k^R the term due to relativity effect and I_k^F comes from mutual interactions between Fermions.

To simplify Eq. (8), suppose $I_k^F=I_{k1}^F+I_{k2}^F$, and define

$$I_{k1}^F=\frac{ik\pi n\alpha}{\sigma B}\int_{-\infty}^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{-\frac{ik\pi p^2}{2m\sigma B}}d\varepsilon dp, \tag{9}$$

and

$$I_{k2}^F = \frac{ik\pi n\alpha}{\sigma B}\int_{-\infty}^{\infty}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}e^{-\frac{ik\pi p^2}{2m\sigma B}}\frac{ik\pi p^4}{8m^3c^2\sigma B}d\varepsilon dp. \tag{10}$$

Since $\int_{-\infty}^{\infty}e^{-i\alpha p^2}dp = e^{-i\pi/4}\sqrt{\frac{\pi}{\alpha}}$, then $\int_{-\infty}^{\infty}e^{-\frac{ik\pi p^2}{2m\sigma B}}dp = e^{-i\pi/4}\sqrt{\frac{2m\sigma B}{k}}$, and $\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}} d\varepsilon = \frac{\sigma B}{ik\pi}\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})de^{\frac{ik\pi}{\sigma B}}$.

If integrations are performed two times and variable substitution $\frac{\varepsilon-\mu}{T}=\xi$ is used for the left part, then the integration can be computed as

$$\begin{aligned}&\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})e^{\frac{ik\pi\varepsilon}{\sigma B}}d\varepsilon\\ =&\frac{\sigma B}{ik\pi}\Big(-\int_0^{\infty}\log(1+e^{\frac{\mu-\varepsilon}{T}})\Big)-\frac{\sigma B}{ik\pi}\Big(\frac{e^{\mu/T}}{1+e^{\mu/T}}-e^{\frac{ik\pi}{\sigma B}}\frac{\pi^2kT}{\sigma B\sinh(\pi^2kT/\sigma B)}\Big).\end{aligned}$$

Therefore, the real part of I_{k1}^F is

$$r_e(I_{k1}^F) = n\alpha\sqrt{\frac{m\sigma B}{k}}\Big(-\log(1+e^{\mu/T})+\frac{\sigma B}{k\pi T}\frac{e^{\mu/T}}{1+e^{\mu/T}}+\frac{\sqrt{2}\pi\sin(\frac{k\pi\mu}{\sigma B}-\frac{\pi}{4})}{\sinh(\pi^2kT/\sigma B)}\Big). \tag{11}$$

According to Ref. [16],

$$\begin{aligned}I_{k2}^F =& n\alpha\Big(\frac{ik\pi}{8m^3c^2}\Big(\frac{2m}{\pi k}\Big)^{5/2}(\sigma B)^{3/2}\Gamma(\frac{5}{/}2)\Big)e^{\frac{-5\pi i}{4}}\\ &\times\Big(\log(1+e^{\mu/T})-\frac{\sigma B}{i\pi kT}\Big(\frac{e^{\mu/T}}{1+e^{\mu/T}}-e^{\frac{i\pi k\mu}{\sigma B}}\frac{\pi^2kT}{\sigma B\sinh(\frac{\pi^2kT}{\sigma B})}\Big)\Big)\end{aligned} \tag{12}$$

Calculate the real part for I_{k2}^F, and give

$$\begin{aligned}r_e(I_{k2}^F) =& n\alpha\frac{1}{8m^3c^2}\Big(\frac{2m}{\pi k}\Big)^{5/2}(\sigma B)^{3/2}\Gamma(\frac{5}{2})\\ &\times\Big(\frac{1}{\sqrt{2}}\log(1+e^{\mu/T})+\frac{\sigma B}{\sqrt{2}T}\frac{e^{\mu/T}}{1+e^{\mu/T}}+\frac{\pi^2k\cos\Big(\frac{k\pi\mu}{\sigma B}-\frac{5\pi}{4}\Big)}{\sinh(\frac{\pi^2kT}{\sigma B})}\Big).\end{aligned} \tag{13}$$

Finally,

$$\begin{aligned}r_e(I_{k2}^F) =& n\alpha\frac{1}{8m^3c^2}\Big(\frac{2m}{\pi k}\Big)^{5/2}(\sigma B)^{3/2}\Gamma(\frac{5}{2})\times\Big(\frac{1}{\sqrt{2}}\log(1+e^{\mu/T})+\frac{\sigma B}{\sqrt{2}T}\frac{e^{\mu/T}}{1+e^{\mu/T}}\\ &+\frac{\pi^2k\cos\Big(\frac{k\pi\mu}{\sigma B}-\frac{5\pi}{4}\Big)}{\sinh(\frac{\pi^2kT}{\sigma B})}\Big)+n\alpha\sqrt{\frac{m\sigma B}{k}}\Big(\log(1+e^{\mu/T})+\frac{\sigma B}{\pi kT}\frac{e^{\mu/T}}{1+e^{\mu/T}}\\ &+\frac{\sqrt{2}\pi\sin\Big(\frac{k\pi\mu}{\sigma B}-\frac{\pi}{4}\Big)}{\sinh(\frac{\pi^2kT}{\sigma B})}\Big)\end{aligned} \tag{14}$$

Considering the condition in a strong magnetic field, $KT \leq \sigma B \ll \mu$, approximations is shown

$$\log(1+e^{\frac{\mu}{T}}) = \frac{\mu}{T}, \text{ and } \frac{e^{\frac{\mu}{T}}}{1+e^{\frac{\mu}{T}}} = 1.$$

Substituting these two approximations into the expression (14), it is simplified to

$$\begin{aligned}\Omega^F(\mu) \approx & \frac{mTVn\alpha}{\pi^2 h^3}\Big(\sum_{k=1}^{\infty}\sqrt{\frac{m\sigma B}{k}}\Big(-\frac{\mu}{T}+\frac{\sqrt{2}\pi\sin\Big(\frac{k\pi\mu}{\sigma B}-\frac{\pi}{4}\Big)}{\sinh(\frac{\pi^2 kT}{\sigma B})}\Big)\Big) \\ & +\frac{1}{\sqrt{2mc^2}}(\frac{1}{\pi})^{\frac{5}{2}}(\sigma B)^{\frac{3}{2}}\Gamma(\frac{5}{2})\sum_{k=1}^{\infty}(\frac{1}{k})^{\frac{5}{2}}\Big(\frac{\mu}{\sqrt{2}T}+\frac{\pi^2 k\cos\Big(\frac{k\pi\mu}{\sigma B}-\frac{5\pi}{4}\Big)}{\sinh(\frac{\pi^2 kT}{\sigma B})}\Big)\Big)\end{aligned} \tag{15}$$

Consequently, compared to [16], the system's thermodynamic potential function is given

$$\Omega = \Omega_0(\mu)+\Omega^N(\mu)+\Omega^R(\mu)+\Omega^F(\mu). \tag{16}$$

Here, $\Omega^N(\mu)$ is the term because of the result without considering relativity effect within the magnetic field, $\Omega^R(\mu)$ due to relativity effect in magnetic field, and the term of $\Omega^F(\mu)$ derives from interaction.

3 Expressions of thermodynamic quality

Using thermodynamic relations

$$E = -T^2\frac{\partial}{\partial T}(\frac{\Omega}{T})_{V,N}, \quad C = \frac{\partial}{\partial T}(\frac{E}{T})_{V,N}, \quad \text{and} \quad \mu = \Big(\frac{\partial \Omega}{\partial N}\Big)_{V,N} \tag{17}$$

where, E, C, μ stands for the total energy, heat capacity, and chemical potential, respectively. In the case of low temperatures ($T \ll T_F$, where T_F is the Fermi temperature), for a free system, the influence of temperature on chemical potential can be neglected, and define $\mu = \varepsilon_F = \frac{h^2}{2m}(3\pi^2 n)^{2/3}$. Substituting Eq. (15) into Eq. (17), for an interacting Fermi system in a strong magnetic field at low temperatures, influence of interaction on the total energy, heat capacity and chemical potential are calculated respectively as

$$\begin{aligned}E^F = & -\frac{mVn\alpha T^2}{\pi^2 h^3}\Big\{\sum_{k=1}^{\infty}\sqrt{\frac{m\sigma B}{k}}\Big[\frac{\mu}{T^2}-\frac{\sqrt{2}\pi^3 k}{\sigma B}\sin\Big(\frac{k\pi\mu}{\sigma B}-\frac{\pi}{4}\Big)\frac{\cosh(\frac{\pi^2 kT}{\sigma B})}{\sinh^2(\frac{\pi^2 kT}{\sigma B})}\Big] \\ & +\frac{1}{\sqrt{2mc^2}}(\frac{1}{\pi})^{\frac{5}{2}}(\sigma B)^{\frac{3}{2}}\Gamma(\frac{5}{2})\sum_{k=1}^{\infty}(\frac{1}{k})^{\frac{5}{2}}\Big[-\frac{\mu}{\sqrt{2}T^2}-\pi^2 k\cos\Big(\frac{k\pi\mu}{\sigma B}-\frac{5}{4}\pi\Big) \\ & \times\frac{\frac{\pi^2 k}{\sigma B}\cosh(\frac{\pi^2 kT}{\sigma B})}{\sinh^2(\frac{\pi^2 kT}{\sigma B})}\Big]\Big\}\end{aligned} \tag{18}$$

$$
\begin{aligned}
C^F = & -\frac{mVn\alpha}{\pi^2 h^3}\Bigg\{\sum_{k=1}^{\infty}\sqrt{\frac{m\sigma B}{k}}\Bigg[\frac{\mu}{T^2}-\frac{\sqrt{2}\pi^3 k}{\sigma B}\sin\Big(\frac{k\pi\mu}{\sigma B}-\frac{\pi}{4}\Big) \\
& \times\Bigg(\frac{\cosh(\frac{\pi^2 kT}{\sigma B})+\frac{\pi^2 kT}{\sigma B}\sinh(\frac{\pi^2 kT}{\sigma B})}{\sinh^2(\frac{\pi^2 kT}{\sigma B})}-\frac{2\pi^2 kT\cosh^2(\frac{\pi^2 kT}{\sigma B})}{\sigma B\sinh^3(\frac{\pi^2 kT}{\sigma B})}\Bigg)\Bigg] \\
& +\frac{1}{\sqrt{2mc^2}}(\frac{1}{\pi})^{\frac{5}{2}}(\sigma B)^{\frac{3}{2}}\Gamma(\frac{5}{2})\sum_{k=1}^{\infty}(\frac{1}{k})^{\frac{5}{2}}\Big[-\frac{\mu}{\sqrt{2}T^2}-\pi^2 k\cos\Big(\frac{k\pi\mu}{\sigma B}-\frac{5}{4}\pi\Big) \\
& \times\frac{\pi^2 k}{\sigma B}\Bigg(\frac{\cosh(\frac{\pi^2 kT}{\sigma B})+\frac{\pi^2 kT}{\sigma B}\sinh(\frac{\pi^2 kT}{\sigma B})}{\sinh^2(\frac{\pi^2 kT}{\sigma B})}-\frac{2\pi^2 kT\cosh^2(\frac{\pi^2 kT}{\sigma B})}{\sigma B\sinh^3(\frac{\pi^2 kT}{\sigma B})}\Bigg)\Big]\Bigg\}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
\mu^F = & \frac{mTV\alpha}{\pi^3 h^3}\Bigg\{\sum_{k=1}^{\infty}\sqrt{\frac{m\sigma B}{k}}\Bigg[-\frac{\mu}{T}+\frac{\sqrt{2}\pi\sin\Big(\frac{k\pi\mu}{\sigma B}-\frac{\pi}{4}\Big)}{\sinh(\frac{\pi^2 kT}{\sigma B})}\Bigg] \\
& +\frac{1}{\sqrt{2mc^2}}(\frac{1}{\pi})^{\frac{5}{2}}(\sigma B)^{\frac{3}{2}}\Gamma(\frac{5}{2})\sum_{k=1}^{\infty}(\frac{1}{k})^{\frac{5}{2}}\Bigg[\frac{\mu}{\sqrt{2}T}+\frac{\pi^2 k\cos\Big(\frac{k\pi\mu}{\sigma B}-\frac{5\pi}{4}\Big)}{\sinh(\frac{\pi^2 kT}{\sigma B})}\Bigg]\Bigg\}
\end{aligned} \tag{20}
$$

4 Analysis and discussion

Since Ou and Chen [17] indicated, influences on the system's thermodynamic properties of relativity effect and magnetic field are independent, these influence mechanisms can be discussed separately. While, once considering weak interactions and according to Eqs. (18)-(20), interactions affect the system's properties and such impact is regulated by the magnetic field and the relativity effect.

Considering limitations from a strong magnetic field

$$KT \le \sigma B \ll \mu,$$

and from weak interactions

$$KT \le \sigma B \gg n\alpha,$$

based on Ref. [12], assume $n = 10^{24}$, $V = 1$, and keep the order of magnitude of α approaching to 10^{-52}, Figs. 1-11 are simulated through Mathematica.

4.1 The influence of interaction on thermodynamic properties

Compared to a system without interaction, Fig. 1 exhibits if the interaction between fermions is repulsive ($\alpha > 0$), the system's energy declines since $E^F < 0$; while, if the mutual force is attrahent ($\alpha < 0$), the system's energy increases according to $E^F > 0$. Regardless of repulsion or attraction, along with the increasing of temperature (within ultracold

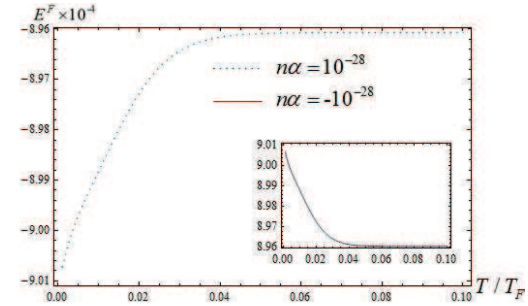

Figure 1: Influence of interaction on system's energy E^F.

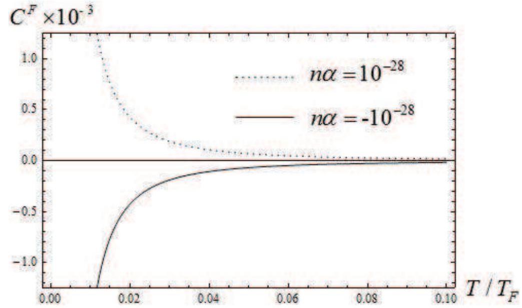

Figure 2: Influence of interaction on system's heat capacity C^F.

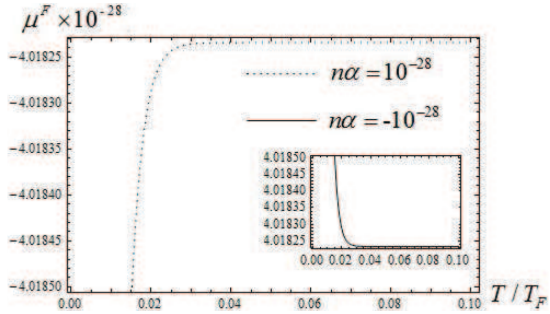

Figure 3: Influence of interaction on system's chemical potential μ^F.

temperature), interactions lead to the influence term of energy E^F decreasing rapidly; however, if the temperature excesses a certain value and continues to increase, E^F tends to saturation. Fig. 2 indicates when fermions exclude mutually, the system's chemical potential will augment; on the contrary, when attract, the chemical potential will decline. Generally, accompanying with temperature increasing, the effect of mutual action on the system's heat capacity decreases gradually to disappears. The Fig. 3 tells if fermions are repulsive ($\alpha > 0$), such interaction drives the system's chemical potential to decrease; if attrahent ($\alpha < 0$), this interaction will enhance the value system's chemical potential. What's more, the effect of such interaction on the system's chemical potential diminishes gradually to saturation along with the increasing of temperature.

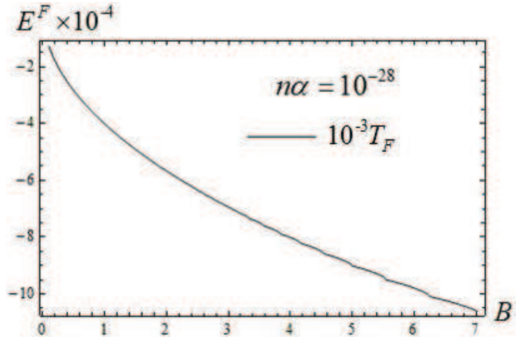

Figure 4: The regulation of strong magnetic field on interaction's influence on E^F under the same temperature.

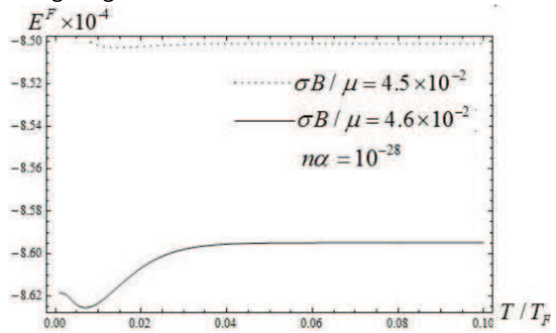

Figure 5: The regulation of strong magnetic field on interaction's influence on E^F under different temperature.

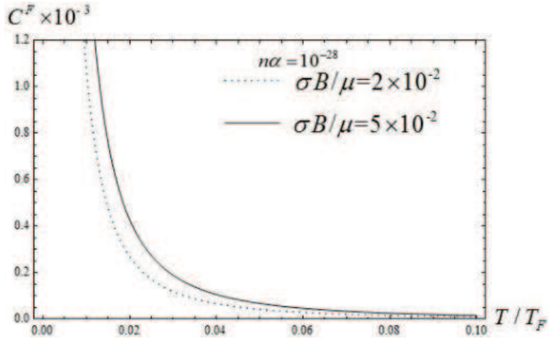

Figure 6: The regulation of strong magnetic field on interaction's influence on C^F under different temperature.

4.2 The regulation of strong magnetic field on interaction's influence

According to Figs. 4 and 5, in same temperatures, the amplifying of magnetic field results in the influence of interaction on energy increases; the Fig. 6 indicates the same regulation mechanism of the magnetic field on the heat capacity as on the energy. The Fig. 7 exhibits that, when in ultracold temperatures like $T=10^{-3}T_F$, the influence of interactions on chemical potential becomes more and more intense, and the oscillation of this physical quality also gradually amplifies during the increase of magnetic field intensity. However,

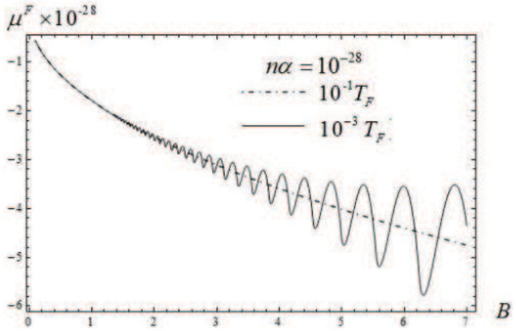

Figure 7: The regulation of strong magnetic field on interaction's influence on μ^F under different temperature.

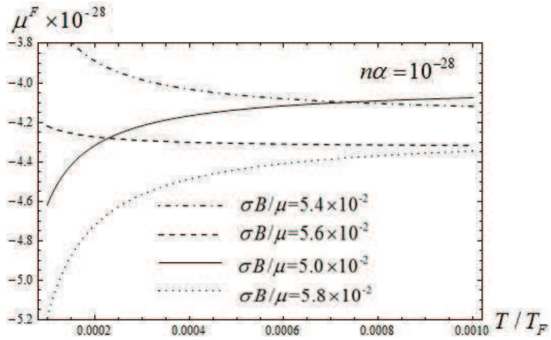

Figure 8: The regulation of strong magnetic field on interaction's influence on μ^F.

due to temperature rising up, when $T=10^{-1}T_F$, the influence of interactions on chemical potential does not oscillate when the magnetic field increase or the oscillation is invisible. For a different magnetic field, the effect of interactions on the system's chemical potential becomes complex with an increasing in the temperature, just as the Fig. 8 shows.

4.3 The regulation of relativity effect on interaction's influence

In the following, T_{F1} and T_{F2} stands for the corresponding Fermi temperature of particles with $m=7.1\times10^{-31}$kg and with $m=5.1\times10^{-31}$kg, respectively (here, the changing of the particle mass only shows a tendency, and the mass doesn't indicate any certain special fermion). The relation between both horizontal coordinates is $0.07T/T_{F2}\approx0.1T/T_{F1}$ in Fig. 10.

According to Figs. 9-11, in the case of the same temperature and magnetic field, the increasing of relativity effect could make interaction's impacts on system's properties aggrandize, including energy, heat capacity and chemical potential; when coming to a different value of relativity effect (mc^2), the impacts of interactions on heat capacity and chemical potential are weaken along with the increasing temperature, however, on energy is incremental first, then subdued.

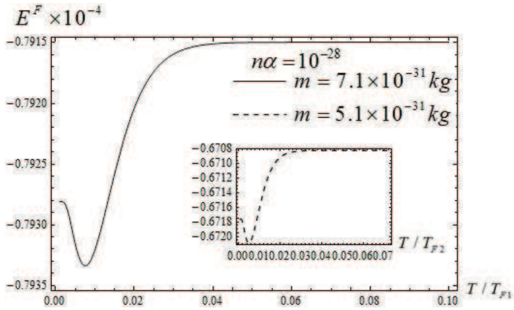

Figure 9: The regulation of relativity effect on interaction's influence on E^F.

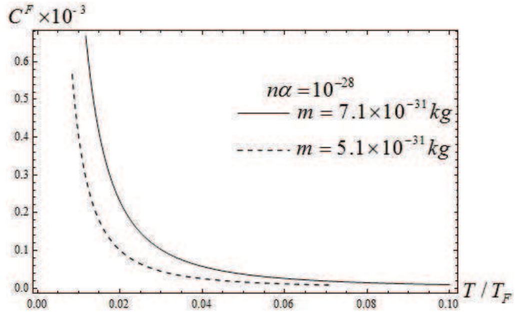

Figure 10: The regulation of relativity effect on interaction's influence on C^F.

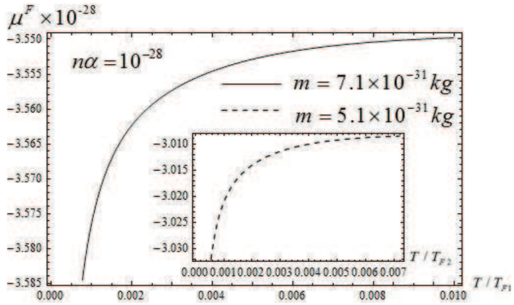

Figure 11: The regulation of relativity effect on interaction's influence on μ^F.

5 Conclusions

Based on the Mean Field Theory, the RTPF of interacting Fermi gas in a strong magnetic field is given. Using the thermodynamic relations, analytical expressions of statistical characteristic quality of corresponding to interactions under low temperature are obtained; using the method of numerical simulation, the influence of weak interactions on thermodynamic properties is analyzed, and the regulation mechanisms on the influence of interactions of both the magnetic field and relativity effect are discussed. Research exhibits, repulsive (attractive) interactions reduce (increase) the system's energy and chemical potential, however, increase (reduce) heat capacity; the intensifying magnetic field leads to fortification of the influence of interactions on thermodynamic properties, furthermore, the influence on chemical potential has the phenomenon of oscillation; the increase of relativity effect makes the impacts of interaction on system's thermodynamic properties including energy, heat capacity and chemical potential more obviously.

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