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Implementations for remotely preparing single- and two-particle states in a four-level system

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Abstract. We propose two schemes to realize remote state preparation (RSP). The first scheme is designed to prepare an arbitrary single-particle state in a four-level system with the aid of one bipartite maximally entangled channel, which is then generalized to the second scheme, i.e., RSP for a two-particle state in a four-level system. During the two preparations, one single-particle projective measurement and one two-particle projective measurement are performed respectively. Our results show that the preparation for single-particle or two-particle states can be remotely realized with at least 25% successful probability and unit fidelity. Furthermore, with respect to two special ensembles of the prepared states, i.e., real and equatorial-like, the successful probability can be pushed up to 100%. Hence our probabilistic schemes become deterministic ones.

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Key words: remote state preparation, four-level system, projective measurement, successful probability

1 Introduction

Quantum entanglement brings on an amazing application in quantum computation and information, namely, quantum teleportation (QT). As we know, QT was originally presented by Bennett *et al*. [1] in 1993, and essentially plays a central role in quantum-information processing [2–9]. QT is used to taking on a task of transmitting an unknown quantum state from a sender (say, Alice) to a distant receiver (say, Bob) with the assistance of quantum and classical resources. Being very similar to QT, remote state preparation (RSP) firstly proposed by Lo [10] is also devoted to remote transmission for quantum states, and is usually reckoned as "teleporting a known state". As a matter of fact, both of them require the help

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of quantum entanglement set up previously and classical information communication. The two creative and promising methods of quantum-information processing in essence reveal the interchangeability of different resources in quantum mechanics. Nevertheless, there are a few remarkable differences between QT and RSP as follows: (i) in QT, whether the sender knows the quantum state to be teleported is disregarded at all. Contrarily, the preparer of the state is entirely aware of the state to be prepared in RSP; (ii) The state to be delivered initially inhabits concrete particles in QT, while this never takes place in QT; (iii) Classical communication cost (CCC) needed in QT is different from that in RSP. Bennett *et al.* [11] have showed that the asymptotic classical communication cost of RSP is one bit per qubit-half that of teleportation. Nonetheless, noteworthily, its price is that the success probability (SP) of state preparation is less than one (probabilistic), but that of teleporting a state can reach one in QT (deterministic). Furthermore, the RSP protocol presented by Pati [12] showed that it requires only one classical bit (cbit) for conclusively preparing a single-qubit state chosen from equatorial or polar great circles on a Bloch sphere, while in standard teleportation two cbits are indispensable. In this sense, RSP is more economical than QT for the special states. Afterward, enlighten by those pioneering works [8–10], many authors concentrate on RSP so much, and have already put forward a large number of RSP proposals [13–29]. Such as, the low-entanglement RSP [13], the optimal RSP [14], the oblivious RSP [15], the RSP without oblivious conditions [16], the generalized RSP [17], the faithful RSP [18], the RSP for multi-parties [19], the joint RSP [20], the RSP for qubit states [21–24], the RSP for qutrit states [25] and the continuous variable RSP in phase space [26] have been presented theoretically. On the other hand, some RSP schemes have already been realized experimentally [27–32], e.g., Peng *et al.* presented a RSP scheme with the technique of NMR (nuclear magnetic resonance) [27], Xiang *et al.* [28] and Peters *et al.* [29] proposed other two RSP schemes using spontaneous parametric down-conversion.

The multilevel quantum systems are promised to be more compact and efficient in both coding and manipulating quantum information [33]. To our knowledge, recently, teleportation for multipartite and multilevel quantum states have been investigated extensively. By contrast, up to now there just have existed a few schemes [34, 35] for implementing RSP in multi-level systems. In fact, Refs. [34, 35] care more on RSP of qudit states in real Hilbert space and the equatorial qudits, and neither of which has investigated RSP of the ensembles of qudits in complex space. As a result, both of them cannot be readily generalized to the case for RSP of an arbitrary single-particle state. Alternatively, we firstly propose such a scheme, which is used for realizing the RSP for a single-particle state in a four-level system, and then generalize to the case of a two-particle state. In the former, we employ one bipartite maximally entangled state as quantum channel. During the preparation, it is required to carry out one single-particle projective measurement and one appropriate unitary operation. In the latter, two bipartite maximally entangled states are taken as quantum channel, and one two-particle projective measurement and one appropriate unitary operation are performed for the preparation. Generally speaking, the two RSP schemes can be faithfully achieved with the same SP as at least 25%. Furthermore, we find the SP can be increased to 100% in the case of two special ensembles. One will see this later.

This paper is organized as follows. In the next section, we will investigate the RSP for a single-particle state in a four-level system. Besides, we work out the SP and CCC. In Section 3, the RSP for a two-particle state is generalized. At the same time, the SP and CCC are also figured out. In Section 4, some comparisons between standard QT [36] and our protocols are made on resource consumptions, and finally a concise conclusion is given.

2 Remote preparation of a single-particle state in a four-level system based on one bipartite maximally entangled state

In this section, we present a RSP scheme for preparing an arbitrary single-particle state in a four-level system, which employs one bipartite maximally entangled state as quantum channel. Now let us start to elucidate it in detail. Provided that there are two legitimate participators, say Alice and Bob. Alice is the state preparer while Bob is her ministrant in a distant site. Alice wants to prepare an arbitrary single-particle state in Bob's place. In general, a single-particle state in a four-level system can be described as

$$|\Phi\rangle=\zeta_0|0\rangle+\zeta_1 e^{i\theta_1}|1\rangle+\zeta_2 e^{i\theta_2}|2\rangle+\zeta_3 e^{i\theta_3}|3\rangle, \tag{1}$$

where ζ_0, ζ_1, ζ_2 and ζ_3 are real and positive numbers, and satisfy the normalized condition. Besides the parameters θ_1, θ_2 and θ_3 are real. Note that, the state does not initially inhabit any particle at all, though in our scheme it is a precondition that the preparer Alice knows the state completely. Additionally, the ministrant Bob does not know the state at all. Alice and Bob share a two-particle maximally entangled state

$$|\Omega\rangle_{12}=\frac{1}{2}(|00\rangle+|11\rangle+|22\rangle+|33\rangle)_{12}, \tag{2}$$

where the particle 1 is hold by Alice while the particle 2 by Bob. This two-particle state is severed as the quantum channel in this scheme. To prepare the desired state $|\Phi\rangle$, the preparer Alice firstly performs a single-particle projective measurement on her particle 1 under a set of complete orthogonal basis vectors $\{|\varphi_i\rangle\}$, which consist of computational bases $\{|0\rangle, |1\rangle, |2\rangle, |3\rangle\}$ and can be expressed as

$$\left.\begin{array}{l}|\varphi_1\rangle=\zeta_0|0\rangle+\zeta_1 e^{-i\theta_1}|1\rangle+\zeta_2 e^{-i\theta_2}|2\rangle+\zeta_3 e^{i\theta_3}|3\rangle\\|\varphi_2\rangle=\zeta_1|0\rangle+\zeta_0 e^{-i\theta_1}|1\rangle+\zeta_3 e^{-i\theta_2}|2\rangle+\zeta_2 e^{i\theta_3}|3\rangle\\|\varphi_3\rangle=\zeta_2|0\rangle+\zeta_3 e^{-i\theta_1}|1\rangle+\zeta_0 e^{-i\theta_2}|2\rangle+\zeta_1 e^{i\theta_3}|3\rangle\\|\varphi_4\rangle=\zeta_3|0\rangle+\zeta_2 e^{-i\theta_1}|1\rangle+\zeta_1 e^{-i\theta_2}|2\rangle+\zeta_0 e^{i\theta_3}|3\rangle\end{array}\right\}. \tag{3}$$

In terms of the above four measuring basis vectors, it is obvious that the initially entangled state $|\Omega\rangle_{12}$ used as quantum channel can be expanded as

$$|\Omega\rangle_{12}=\sum_{i=1}^{4}|\varphi_i\rangle_1\bigotimes|\psi_i\rangle_2, \tag{4}$$

Table 1: Alice's measurement outcome (AMO), the classical bits from Alice to inform Bob of her measurement outcomes (CBs), the collapsed states of the particle 2 (CSs) and Bob's corresponding unitary operation on his particle 2 (U_2).

AMO	CBs	CSs	U_2
$\|\varphi_2\rangle_1$	01	$(\zeta_1\|0\rangle-\zeta_0\|1\rangle+\zeta_3\|2\rangle-\zeta_2\|3\rangle)_2$	U^1
$\|\varphi_3\rangle_1$	10	$(\zeta_2\|0\rangle-\zeta_3\|1\rangle+\zeta_0\|2\rangle-\zeta_1\|3\rangle)_2$	U^2
$\|\varphi_4\rangle_1$	11	$(\zeta_3\|0\rangle-\zeta_2\|1\rangle+\zeta_1\|2\rangle-\zeta_0\|3\rangle)_2$	U^3

where $|\psi_i\rangle_2 \equiv_1 \langle\varphi_i|\Omega\rangle_{12}$ ($i=1$, 2, 3, 4). Alice's measurement outcome should be one of the four states defined as the Eq. (3) and each will occur with equal probability (i.e., 25%). From the Eq. (4), once Alice measures $|\varphi_i\rangle_1$ ($i=1$, 2, 3, 4) and subsequently publishes the outcome to Bob through classical channel (i.e., sending cbits), Bob exactly knows his particle 2 has collapsed into the counterpart $|\psi_i\rangle_2$. By the way, both Alice and Bob in priori make an agreement that the outcome $|\varphi_1\rangle_1$ corresponds to cbits '00', $|\varphi_2\rangle_1$ to '01', $|\varphi_3\rangle_1$ to '10', and $|\varphi_4\rangle_1$ to '11'. If Alice's measurement outcome is the state $|\varphi_1\rangle_1$, Alice sends the cbits '00' to Bob via classical channel. Upon the classical message, Bob realizes his particle 2 has collapsed into

$$|\psi_1\rangle_2 =_1 \langle\varphi_1|\Omega\rangle_{12} = \frac{1}{2}\Big(\zeta_0|0\rangle+\zeta_1 e^{i\theta_1}|1\rangle+\zeta_2 e^{i\theta_2}|2\rangle+\zeta_3 e^{i\theta_3}|3\rangle\Big)_2 \equiv \frac{1}{2}|\Phi\rangle_2. \tag{5}$$

It needs to be noted that the coefficient $\frac{1}{2}$ in Eq. (5) only implies the occurrence probability of the outcome $|\varphi_1\rangle_1$ (i.e., $|\frac{1}{2}|^2=25\%$), and even never affects the status of $|\Phi\rangle_2$. That is the particle 2 has been in the desired state $|\Phi\rangle_2$. In other words, the RSP succeeds in this situation. For clarity, the corresponding quantum circuit has been shown as Fig. 1(a). And one can realize that the RSP can be accomplished with SP of 25% and unit fidelity, and the CCC is 2 cbits.

Of course, it is also possible for Alice to get the state $|\varphi_2\rangle_1$, $|\varphi_3\rangle_1$ or $|\varphi_4\rangle_1$ after her measurement. If so, the state of particle 2 will collapse to $(\zeta_1|0\rangle-\zeta_0 e^{i\theta_1}|1\rangle+\zeta_3 e^{i\theta_2}|2\rangle-\zeta_2 e^{i\theta_3}|3\rangle)_2$, $(\zeta_2|0\rangle-\zeta_3 e^{i\theta_1}|1\rangle-\zeta_0 e^{i\theta_2}|2\rangle+\zeta_2 e^{i\theta_1}|3\rangle)_2$ or $(\zeta_3|0\rangle+\zeta_2 e^{i\theta_1}|1\rangle-\zeta_1 e^{i\theta_2}|2\rangle-\zeta_0 e^{i\theta_3}|3\rangle)_2$. In contrast to the case of the former outcome, for the latter three outcomes it seems that Bob is not able to convert the collapsed states into the state $|\Phi\rangle$. Naturally, Alice needs not to send any classical message to Bob anymore. If so, it is clear that the total SP of RSP is at least 25% and the CCC is equal to $2\times\frac{1}{4}=0.5$ cbits on average. Apparently, the SP of 25% is relatively prettily low compared with that of the standard QT [34]. As is known, it is always a hot topic that how to realize high-efficiency transmissions for quantum information with finite quantum and classical resources. Henceforth, we turn to care on the issues, i.e., how to realize high-efficiency transmissions of the quantum state in present scheme and what the additional conditions are. After our extensive investigations, we find that when the prepared states are two special ensembles, the SP can be pushed up to 100%. In what follows, we will depict them in detail, i.e., real and equatorial-like [12].

Case I. The single-particle state to be prepared is in real Hilbert Space.

In this ensemble, the coefficients of the prepared state $|\Phi\rangle$ are real. This indicates that the relations $\theta_1 = \theta_2 = \theta_3 = 0$ are hold. After Alice's measurement, possible collapsed states of the particle 2 are listed in Table 1. Luckily, for each collapsed state Bob can perform an appropriate unitary operation to convert it into the state to be prepared. We have summarized them in Table 1. The unitary operations needed to perform on Bob's particles in Table 1 are

$$U^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad U^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad U^3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix},$$

$$U^4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad U^5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad U^6 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

$$U^7 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Take an example, if Alice's measurement outcome is $|\varphi_2\rangle_1$, Alice informs Bob of her outcome via classical channel (i.e. sending the classical bits '01' to Bob). Thus, Bob exactly realizes his particle 2 is in the state

$$|\psi_2\rangle_2 = (\zeta_1|0\rangle + \zeta_3|2\rangle - \zeta_2|3\rangle)_2 = U^1|\Phi\rangle_2, \tag{6}$$

where U^1 is a local unitary operation in a four-dimension Hilbert space. That is the collapsed state of Bob's particle 2 can be converted into $|\Phi\rangle$ to be prepared after being performed the operation U^1. Similarly, if Alice gets other measurement outcomes and notifies Bob via two classical messages, then Bob can perform an appropriate unitary operation to reconstruct the desired state. Here we do not depict them one by one anymore. As a summary, Bob's corresponding unitary operations to Alice's measurement outcomes are listed in Table 1. Meanwhile, one can easily see that the total SP of RSP equals to $(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4})\times 100\% = 100\%$ and the CCC should be $\frac{1}{4}\times(2+2+2+2) = 2$ cbits on average. This shows that the RSP is deterministic for this ensemble state.

Case II. The single-qubit state to be prepared is equatorial-like.

For the prepared state, its coefficients meet the equalities $|\zeta_0|=|\zeta_1|=|\zeta_2|=|\zeta_3|=1/2$, and the whole preparation process is similar as that in the Case I. For convenience, we don't depict it once more. The possible measured states of Alice's particle 1, the classical bits from Alice to inform Bob of her measurement outcomes, the correspondingly collapsed states of the particle 2 and Bob's corresponding unitary operation on his particle 2 already have been listed in Table 2.

From the Table 2, one can work out that the total SP of RSP is 100% and the CCC is $\frac{1}{4}\times(2+2+2+2)=2$ cbits on average. This displays that the RSP is also deterministic for this ensemble state.

3 Remote preparation of a two-particle state in a four-level system based on two bipartite maximally entangled States

Next let us elucidate the RSP scheme to prepare a two-particle state in a four-level system, which uses two bipartite maximally entangled states as quantum channel. For clearness, we firstly present its quantum circuit in Fig. 1(b). The scheme contains two legitimate participators, say Alice and Bob. Alice is the state preparer while Bob is her ministrant in a remote place. Alice would like to prepare a two-particle state in Bob's site. The state is

$$|\Psi\rangle=\zeta_0|00\rangle+\zeta_1 e^{i\eta_1}|11\rangle+\zeta_2 e^{i\eta_2}|22\rangle+\zeta_3 e^{i\eta_3}|33\rangle, \tag{7}$$

where ζ_0, ζ_1, ζ_2 and ζ_3 are real and positive numbers, and satisfy the normalized condition. Besides the parameters η_1, η_2 and η_3 are real. Alice and Bob share two bipartite maximally entangled states

$$|\Xi\rangle_{12}=\frac{1}{2}\Big(|00\rangle+|11\rangle+|22\rangle+|33\rangle\Big)_{12},\ |\Xi\rangle_{34}=\frac{1}{2}\Big(|00\rangle+|11\rangle+|22\rangle+|33\rangle\Big)_{34}. \tag{8}$$

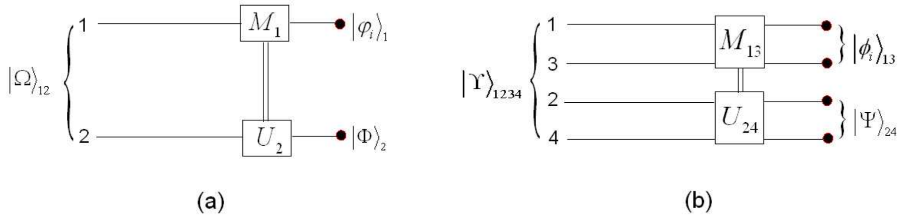

Figure 1: (a) Quantum circuit for implementing single-particle RSP. Element M_1 denotes a single-particle projective measurement on the particle 1 and element U_2 denotes a unitary operation on the particle 2. (b) Quantum circuit for implementing two-particle RSP. Element M_{13} denotes a two-particle projective measurement on the particle pair (1, 3) and element U_{24} denotes a unitary operation on the particle pair (2, 4).

Table 2: Same as Table 1, except with different U_2.

AMO	CBs	CSs	U_2
$\lvert\varphi_2\rangle_1$	01	$\frac{1}{2}(\lvert 0\rangle - e^{i\theta_1}\lvert 1\rangle + e^{i\theta_2}\lvert 2\rangle - e^{i\theta_3}\lvert 3\rangle)_2$	U^4
$\lvert\varphi_3\rangle_1$	10	$\frac{1}{2}(\lvert 0\rangle - e^{i\theta_1}\lvert 1\rangle - e^{i\theta_2}\lvert 2\rangle + e^{i\theta_3}\lvert 3\rangle)_2$	U^5
$\lvert\varphi_4\rangle_1$	11	$\frac{1}{2}(\lvert 0\rangle + e^{i\theta_1}\lvert 1\rangle - e^{i\theta_2}\lvert 2\rangle - e^{i\theta_3}\lvert 3\rangle)_2$	U^6

The particle pair (1, 3) is hold by Alice while the particle pair (2, 4) by Bob. The joint state of two particle pairs, $|\gamma\rangle_{1234} = |\Xi\rangle_{12} \bigotimes |\Xi\rangle_{34}$, is employed as the quantum channel. To prepare the desired state $|\Phi\rangle$, at first step, Alice performs a two-particle projective measurement on her particle pair (1,3) under a set of basis vectors, which consists of computational basis $\{|00\rangle$, $|01\rangle$, $|02\rangle$, $|03\rangle$, $|10\rangle$, $|11\rangle$, $|12\rangle$, $|13\rangle$, $|20\rangle$, $|21\rangle$, $|22\rangle$, $|23\rangle$, $|30\rangle$, $|31\rangle$, $|32\rangle$, $|33\rangle\}$ and can be described as the following forms

$$|\phi_1\rangle = \zeta_0|00\rangle + \zeta_1 e^{i\eta_1}|11\rangle + \zeta_2 e^{i\eta_2}|22\rangle + \zeta_3 e^{i\eta_3}|33\rangle \tag{9a}$$
$$|\phi_2\rangle = \zeta_1|00\rangle - \zeta_0 e^{i\eta_1}|11\rangle + \zeta_3 e^{i\eta_2}|22\rangle - \zeta_2 e^{i\eta_3}|33\rangle \tag{9b}$$
$$|\phi_3\rangle = \zeta_2|00\rangle - \zeta_3 e^{i\eta_1}|11\rangle - \zeta_0 e^{i\eta_2}|22\rangle + \zeta_1 e^{i\eta_3}|33\rangle \tag{9c}$$
$$|\phi_4\rangle = \zeta_3|00\rangle + \zeta_2 e^{i\eta_1}|11\rangle - \zeta_1 e^{i\eta_2}|22\rangle - \zeta_0 e^{i\eta_3}|33\rangle \tag{9d}$$
$$|\phi_5\rangle = \zeta_0|01\rangle + \zeta_1 e^{i\eta_1}|12\rangle + \zeta_2 e^{i\eta_2}|23\rangle + \zeta_3 e^{i\eta_3}|30\rangle \tag{9e}$$
$$|\phi_6\rangle = \zeta_1|01\rangle - \zeta_0 e^{i\eta_1}|12\rangle + \zeta_3 e^{i\eta_2}|23\rangle - \zeta_2 e^{i\eta_2}|30\rangle \tag{9f}$$
$$|\phi_7\rangle = \zeta_2|01\rangle - \zeta_3 e^{i\eta_1}|12\rangle - \zeta_0 e^{i\eta_2}|23\rangle + \zeta_1 e^{i\eta_2}|30\rangle \tag{9g}$$
$$|\phi_8\rangle = \zeta_3|01\rangle + \zeta_2 e^{i\eta_1}|12\rangle - \zeta_1 e^{i\eta_2}|23\rangle - \zeta_0 e^{i\eta_2}|30\rangle \tag{9h}$$
$$|\phi_9\rangle = \zeta_0|02\rangle + \zeta_1 e^{i\eta_1}|13\rangle + \zeta_2 e^{i\eta_2}|20\rangle + \zeta_3 e^{i\eta_2}|31\rangle \tag{9i}$$
$$|\phi_{10}\rangle = \zeta_1|02\rangle - \zeta_0 e^{i\eta_1}|13\rangle + \zeta_3 e^{i\eta_2}|20\rangle - \zeta_2 e^{i\eta_2}|31\rangle \tag{9j}$$
$$|\phi_{11}\rangle = \zeta_2|02\rangle - \zeta_3 e^{i\eta_1}|13\rangle - \zeta_0 e^{i\eta_2}|20\rangle + \zeta_1 e^{i\eta_2}|31\rangle \tag{9k}$$
$$|\phi_{12}\rangle = \zeta_3|02\rangle + \zeta_1 e^{i\eta_1}|13\rangle - \zeta_1 e^{i\eta_2}|20\rangle - \zeta_0 e^{i\eta_2}|31\rangle \tag{9l}$$
$$|\phi_{13}\rangle = \zeta_0|10\rangle + \zeta_1 e^{i\eta_1}|21\rangle + \zeta_2 e^{i\eta_2}|32\rangle + \zeta_3 e^{i\eta_2}|03\rangle \tag{9m}$$
$$|\phi_{14}\rangle = \zeta_1|10\rangle - \zeta_0 e^{i\eta_1}|21\rangle + \zeta_3 e^{i\eta_2}|32\rangle - \zeta_2 e^{i\eta_2}|03\rangle \tag{9n}$$
$$|\phi_{15}\rangle = \zeta_2|10\rangle - \zeta_3 e^{i\eta_1}|21\rangle - \zeta_0 e^{i\eta_2}|32\rangle + \zeta_1 e^{i\eta_2}|03\rangle \tag{9o}$$
$$|\phi_{16}\rangle = \zeta_3|10\rangle + \zeta_2 e^{i\eta_1}|21\rangle - \zeta_1 e^{i\eta_2}|32\rangle - \zeta_0 e^{i\eta_2}|03\rangle \tag{9p}$$

Since the composite state $|\gamma\rangle_{1234}$ can be rewritten as

$$|\gamma\rangle_{1234} = \sum_{i=l}^{16} |\phi_i\rangle_{13} \bigotimes |\vartheta_i\rangle_{24}, \tag{10}$$

where $|\vartheta_i\rangle_{24} \equiv_{13} \langle\phi_i|\gamma\rangle_{1234}$ ($i=1, \cdots, 16$). It is obvious that Alice's measurement outcome should be one of the sixteen states defined in Eq. (9) and each will occur with equal probability (i.e., 1/16). By the way, Alice and Bob in priori agree that the outcome $|\phi_1\rangle_{13}$ corresponds to cbits '0000', $|\phi_2\rangle_{13}$ to '0001', $|\phi_3\rangle_{13}$ to '0010', $|\phi_4\rangle_{13}$ to '0011', $|\phi_5\rangle_{13}$ to '0100', $|\phi_6\rangle_{13}$ to '0101', $|\phi_7\rangle_{13}$ to '0110', $|\phi_8\rangle_{13}$ to '0111', $|\phi_9\rangle_{13}$ to '1000', $|\phi_{10}\rangle_{13}$ to '1001', $|\phi_{11}\rangle_{13}$ to '1010', $|\phi_{12}\rangle_{13}$ to '1011', $|\phi_{13}\rangle_{13}$ to '1100', $|\phi_{14}\rangle_{13}$ to '1101', $|\phi_{15}\rangle_{13}$ to '1110', and $|\phi_{16}\rangle_{13}$ to '1111'. By applying the same analysis methods as mentioned in the first scheme, we judge when Alice's measurement outcome is $|\phi_1\rangle_{13}$ (or $|\phi_5\rangle_{13}$, $|\phi_9\rangle_{13}$, $|\phi_{13}\rangle_{13}$), Bob can convert the correspondingly collapsed state of his particles $|\vartheta_1\rangle_{24}$ (or $|\vartheta_5\rangle_{24}$, $|\vartheta_9\rangle_{24}$, $|\vartheta_{13}\rangle_{24}$) into the desired state $|\Psi\rangle_{24}$ by performing a unitary operation U_{24} on his particle pair (2,4). Here, we don't describe them one by one and omit the specific processes. Furthermore, we generalize the collapsed states of the particle pair (2, 4), the classical bits from Alice to inform Bob of her measurement outcomes and Bob's corresponding unitary operation on his particle pair (2, 4) into Table 3.

Table 3: Same as Table 1 except that U_{24} is denoted as a local unitary operation on particle pair (2, 4).

AMO	CBs	CSs	U_{24}
$\|\phi_5\rangle_{13}$	0100	$\frac{1}{4}(\zeta_0\|01\rangle+\zeta_1 e^{i\eta_1}\|12\rangle+\zeta_2 e^{i\eta_2}\|23\rangle+\zeta_3 e^{i\eta_3}\|30\rangle)_{24}$	$I_2\otimes(U^7)_4$
$\|\phi_9\rangle_{13}$	1000	$\frac{1}{4}(\zeta_0\|02\rangle+\zeta_1 e^{i\eta_1}\|13\rangle+\zeta_2 e^{i\eta_2}\|20\rangle+\zeta_3 e^{i\eta_3}\|31\rangle)_{24}$	$I_2\otimes(U^7)_4^{\otimes 2}$
$\|\phi_{13}\rangle_{13}$	1100	$\frac{1}{4}(\zeta_0\|10\rangle+\zeta_1 e^{i\eta_1}\|21\rangle+\zeta_2 e^{i\eta_2}\|32\rangle+\zeta_3 e^{i\eta_3}\|03\rangle)_{24}$	$(U^7)_2\otimes I_4$

From Table 3, one can easily work out that the SP is $(\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16})\times 100\%=25\%$ as well, and the CCC is $(4+4+4+4)\times\frac{1}{16}=1$ cbit on average.

Of course, it is also possible for Alice to get one of the other twelve states $|\phi_2\rangle_{13}$, $|\phi_3\rangle_{13}$, $|\phi_4\rangle_{13}$, $|\phi_6\rangle_{13}$, $|\phi_7\rangle_{13}$, $|\phi_8\rangle_{13}$, $|\phi_{10}\rangle_{13}$, $|\phi_{11}\rangle_{13}$, $|\phi_{12}\rangle_{13}$, $|\phi_{14}\rangle_{13}$, $|\phi_{15}\rangle_{13}$ and $|\phi_{16}\rangle_{13}$ after her measurement. If so, it seems that Bob can not transform his particles into the desired state. Naturally, Alice needs not to send any classical message to Bob anymore. As a result, RSP fails in those cases. However, as for the two special ensemble states mentioned before are concerned, the situation is completely changed. We find whatever Alice's measurement outcome is Bob always can convert the collapsed state of his particle pair (2, 4) into the desired state. That is the RSP can be faithfully and conclusively achieved in the case of two special ensembles. As a result, the total SP can be greatly pushed up from 25% to 100%. In what follows, we will concisely describe them one by one.

Case A. The two-particle state to be prepared is in real Hilbert Space.

In this ensemble, the coefficients of the prepared state $|\Psi\rangle$ are real, i.e., $\eta_1=\eta_2=\eta_3=0$ are hold. The collapsed states of the particle pair (2, 4) are listed in Table 4. For each collapsed state Bob can employ an appropriate unitary operation to convert it to the prepared state $|\Psi\rangle$.

Table 4: Same as Table 3.

AMO	CBs	CSs	U_{24}
$\|\phi_2\rangle_{13}$	0001	$\frac{1}{4}(\zeta_1\|00\rangle-\zeta_0\|11\rangle+\zeta_3\|22\rangle-\zeta_2\|33\rangle)_{24}$	$(U^1)_2\bigotimes I_4$
$\|\phi_3\rangle_{13}$	0010	$\frac{1}{4}(\zeta_2\|00\rangle-\zeta_3\|11\rangle-\zeta_0\|22\rangle+\zeta_1\|33\rangle)_{24}$	$(U^2)_2\bigotimes I_4$
$\|\phi_4\rangle_{13}$	0011	$\frac{1}{4}(\zeta_3\|00\rangle+\zeta_2\|11\rangle-\zeta_1\|22\rangle-\zeta_0\|33\rangle)_{24}$	$(U^3)_2\bigotimes I_4$
$\|\phi_6\rangle_{13}$	0101	$\frac{1}{4}(\zeta_1\|01\rangle-\zeta_0\|12\rangle+\zeta_3\|23\rangle-\zeta_2\|30\rangle)_{24}$	$(U^1)_2\bigotimes(U^7)_4$
$\|\phi_7\rangle_{13}$	0110	$\frac{1}{4}(\zeta_2\|01\rangle-\zeta_3\|12\rangle-\zeta_0\|23\rangle+\zeta_1\|30\rangle)_{24}$	$(U^2)_2\bigotimes(U^7)_4$
$\|\phi_8\rangle_{13}$	0111	$\frac{1}{4}(\zeta_3\|01\rangle+\zeta_2\|12\rangle-\zeta_1\|23\rangle-\zeta_0\|30\rangle)_{24}$	$(U^3)_2\bigotimes(U^7)_4$
$\|\phi_{10}\rangle_{13}$	0101	$\frac{1}{4}(\zeta_1\|02\rangle-\zeta_0\|13\rangle+\zeta_3\|20\rangle-\zeta_2\|31\rangle)_{24}$	$(U^1)_2\bigotimes(U^7)_4^{\bigotimes 2}$
$\|\phi_{11}\rangle_{13}$	0101	$\frac{1}{4}(\zeta_2\|02\rangle-\zeta_3\|13\rangle-\zeta_0\|20\rangle+\zeta_1\|31\rangle)_{24}$	$(U^2)_2\bigotimes(U^7)_4^{\bigotimes 2}$
$\|\phi_{12}\rangle_{13}$	1011	$\frac{1}{4}(\zeta_3\|02\rangle+\zeta_2\|13\rangle-\zeta_1\|20\rangle-\zeta_0\|31\rangle)_{24}$	$(U^3)_2\bigotimes(U^7)_4^{\bigotimes 2}$
$\|\phi_{14}\rangle_{13}$	1101	$\frac{1}{4}(\zeta_1\|10\rangle-\zeta_0\|21\rangle+\zeta_3\|32\rangle-\zeta_2\|03\rangle)_{24}$	$(U^7)_2\bigotimes(U^1)_4$
$\|\phi_{15}\rangle_{13}$	1110	$\frac{1}{4}(\zeta_2\|10\rangle-\zeta_3\|21\rangle-\zeta_0\|32\rangle+\zeta_1\|03\rangle)_{24}$	$(U^7)_2\bigotimes(U^2)_4$
$\|\phi_{16}\rangle_{13}$	1101	$\frac{1}{4}(\zeta_3\|10\rangle+\zeta_2\|21\rangle-\zeta_1\|32\rangle-\zeta_0\|03\rangle)_{24}$	$(U^7)_2\bigotimes(U^3)_4$

We have summarized them in Table 4. Here we do not depict them one by one anymore. As a summary, Bob's corresponding unitary operations to Alice's measurement results have been listed in Table 4. From Table 4, one can easily work out that the total SP of RSP is 100% and the CCC is 4 cbits on average as well. This shows that the RSP is also deterministic for this ensemble state.

Case B. The two-particle state to be prepared is equatorial-like.

In this case, the coefficients of the prepared state meet the equalities $|\zeta_0|=|\zeta_1|=|\zeta_2|=|\zeta_3|=\frac{1}{2}$, and the whole preparation process is similar as that in the Case A. For simplification, we don't describe it here. As a summary, the possible measured states of the particle pair (1, 3), the classical bits from Alice to inform Bob of her measurement outcomes, the collapsed states of the particle pair (2, 4) and Bob's corresponding unitary operation on his particle pair (2, 4) already have been listed in Table 5. From the table, one can easily work out that the total SP of RSP is 100% and the CCC is 4 bits on average. This displays that the RSP is also deterministic for this ensemble state.

Table 5: Same as Table 3.

AMO	CBs	CSs	U_{24}
$\vert\phi_2\rangle_{13}$	0001	$\frac{1}{8}(\vert 00\rangle - e^{i\eta_1}\vert 11\rangle + e^{i\eta_2}\vert 22\rangle - e^{i\eta_3}\vert 33\rangle)_{24}$	$(U^4)_2 \bigotimes I_4$
$\vert\phi_3\rangle_{13}$	0010	$\frac{1}{8}(\vert 00\rangle - e^{i\eta_1}\vert 11\rangle - e^{i\eta_2}\vert 22\rangle + e^{i\eta_3}\vert 33\rangle)_{24}$	$(U^5)_2 \bigotimes I_4$
$\vert\phi_4\rangle_{13}$	0011	$\frac{1}{8}(\vert 00\rangle + e^{i\eta_1}\vert 11\rangle - e^{i\eta_2}\vert 22\rangle - e^{i\eta_3}\vert 33\rangle)_{24}$	$(U^6)_2 \bigotimes I_4$
$\vert\phi_6\rangle_{13}$	0101	$\frac{1}{8}(\vert 01\rangle - e^{i\eta_1}\vert 12\rangle + e^{i\eta_2}\vert 23\rangle - e^{i\eta_3}\vert 30\rangle)_{24}$	$(U^4)_2 \bigotimes (U^7)_4$
$\vert\phi_7\rangle_{13}$	0110	$\frac{1}{8}(\vert 01\rangle - e^{i\eta_1}\vert 12\rangle - e^{i\eta_2}\vert 23\rangle + e^{i\eta_3}\vert 30\rangle)_{24}$	$(U^5)_2 \bigotimes (U^7)_4$
$\vert\phi_8\rangle_{13}$	0111	$\frac{1}{8}(\vert 01\rangle + e^{i\eta_1}\vert 12\rangle - e^{i\eta_2}\vert 23\rangle - e^{i\eta_3}\vert 30\rangle)_{24}$	$(U^6)_2 \bigotimes (U^7)_4$
$\vert\phi_{10}\rangle_{13}$	0101	$\frac{1}{8}(\vert 02\rangle - e^{i\eta_1}\vert 13\rangle + e^{i\eta_2}\vert 20\rangle - e^{i\eta_3}\vert 31\rangle)_{24}$	$(U^4)_2 \bigotimes (U^7)_4^{\bigotimes 2}$
$\vert\phi_{11}\rangle_{13}$	0101	$\frac{1}{8}(\vert 02\rangle - e^{i\eta_1}\vert 13\rangle - e^{i\eta_2}\vert 20\rangle + e^{i\eta_3}\vert 31\rangle)_{24}$	$(U^5)_2 \bigotimes (U^7)_4^{\bigotimes 2}$
$\vert\phi_{12}\rangle_{13}$	1011	$\frac{1}{8}(\vert 02\rangle + e^{i\eta_1}\vert 13\rangle - e^{i\eta_2}\vert 20\rangle - e^{i\eta_3}\vert 31\rangle)_{24}$	$(U^6)_2 \bigotimes (U^7)_4^{\bigotimes 2}$
$\vert\phi_{14}\rangle_{13}$	1101	$\frac{1}{8}(\vert 10\rangle - e^{i\eta_1}\vert 21\rangle + e^{i\eta_2}\vert 32\rangle - e^{i\eta_3}\vert 03\rangle)_{24}$	$(U^7)_2 \bigotimes (U^4)_4$
$\vert\phi_{15}\rangle_{13}$	1110	$\frac{1}{8}(\vert 10\rangle - e^{i\eta_1}\vert 21\rangle - e^{i\eta_2}\vert 32\rangle + e^{i\eta_3}\vert 03\rangle)_{24}$	$(U^7)_2 \bigotimes (U^5)_4$
$\vert\phi_{16}\rangle_{13}$	1101	$\frac{1}{8}(\vert 10\rangle + e^{i\eta_1}\vert 21\rangle - e^{i\eta_2}\vert 32\rangle - e^{i\eta_3}\vert 03\rangle)_{24}$	$(U^7)_2 \bigotimes (U^6)_4$

4 Discussion and conclusions

In this paper we have put forward two schemes for remotely preparing single- and two-particle states in a four-level system respectively. With the help of maximally entangled quantum channels and classical communication, the two schemes can be realized with the same SP of at least 25% and unit fidelity. What is of importance, the SP of RSP can be greatly enhanced to 100% with respect to two special ensembles, i.e., real and equatorial-like. Now let us compare our schemes with the standard QT scheme [36] in quantum resource consumptions and operation complexity aspects. With respect to transmitting this kind of single-particle states, in standard QT [36] three particles are indispensable and one two-particle projective measurement needs to perform; Contrarily, two particles are enough for the preparation in our RSP scheme, and one single-particle projective measurement needs to be operated in our scheme, in others words, quantum resource consumptions of our scheme are reduced and operation complexity is greatly degraded compared with the standard QT [36]. Similarly, for delivering the two-particle states, in standard QT [36] six particles are indispensable and two two-particle projective measurements need to be performed; Contrarily, three particles, half of the consumption in the former, are enough for the transmission in our RSP scheme, and one two-particle projective measurement needs to be operated in our scheme. That is to say, quantum resource consumptions of our scheme are reduced and operation complexity is greatly degraded compared with the former. Besides, as for as classical communication cost is concerned, the requirement of our first scheme is 2-cbit for transmitting the two special ensembles mentioned in Section 2, less than 4-cbit required in the standard QT. And the requirement of our second scheme is 4-cbit for transmitting the two special ensembles referred in Section 3, less than 8-cbit required in the standard QT. Thus, it is in this sense that our schemes are more economic and optimal for preparing those special ensembles compared with the standard QT.

In summary, we have put forward two schemes for remotely preparing single- and two-particle states in a four-level system respectively. With the help of maximally entangled quantum channels and classical communication, the two schemes can be realized with the same SP of at least 25% and unit fidelity. What is of importance, the SP of RSP can be greatly enhanced to 100% with respect to two special ensembles, i.e., real and equatorial-like. During our two RSP processes, single-particle and two-particle projective measurements are indispensable and crucial. To date, single- and two-particle projective measurements have been explored theoretically [37–40] and realized in a few experiments [41,42]. Moreover, the experimental realization of four-level qudits has been proposed via biphotons [43]. Thereby we expect our schemes might be realized experimentally in near future, and in principle lots of physical systems, such as cavity quantum electrodynamics (CQED) [44], trapped ions [45], NMR [46], quantum dots [47], and superconducting quantum interference devices (SQUID) [48], can be as candidates for implementing our proposals. Furthermore, our schemes may be available for extending the further investigations concerning on RSP for multi-particle states in four- or higher-level systems.

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