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THE FINITE ELEMENT METHOD FOR SEMILINEAR PARABOLIC EQUATIONS WITH DISCONTINUOUS COEFFICIENTS*1)

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Abstract

In this paper we investigate the existence, uniqueness and regularity of the solution of semilinear parabolic equations with coefficients that are discontinuous across the interface, some prior estimates are obtained. A net shape of the finite elements around the singular points was designed in [7] to solve the linear elliptic problems , by means of that net, we prove that the approximate solution has the same convergence rate as that without singularity.

Key words: Finite element, Semitinear parabolic equation, Discontinuous coefficients.

1. Introduction

Let x be points on plane R^2, and Ω be a polygonal domain, we denote the boundary of Ω by $\partial\Omega$. There in Ω are finite many broke lines which divide it into finite polygonal subdomains $\Omega_l, l = 1, \cdots, L$. The function $p(x) \in L^2(\Omega)$ is assumed to have bounded first derivatives in all subdomains Ω_l, while p is allowed to be discontinuous on the interfaces $\partial\Omega_i \cap \partial\Omega_j$. And there exists a positive constant τ such that

$$p(x) \geq \tau, \quad \forall x \in \Omega.$$

We adopt the usual notations of the Sobolev spaces in this paper, that is, denote by $H^s(\Omega)$ and $H_0^s(\Omega)$ the spaces and $\|\cdot\|_s$ the norms, $|\cdot|_s$ the semi norms.

We define a linear operator A by

$$Au = \bigtriangledown(p(x) \bigtriangledown u), \quad D(A) = \{u \in H_0^1(\Omega), Au \in L^2(\Omega)\},$$

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where $\bigtriangledown$ is gradient operator. Let $f(x) \in L^2(\Omega)$, $u_0 \in D(A^2), T_0$ be a positive constant. In this paper we study the following nonlinear initial value problem

$$\begin{cases} \dfrac{\partial u}{\partial t} = Au - \phi(u) + f(x) & \text{in} \quad \Omega \times (0, T_0] \\ u(t,x) = 0 & texton \quad \partial\Omega \times [0, T_0] \\ u(0,x) = u_0(x) & \text{in} \quad \Omega \end{cases} \tag{1.1}$$

where $\phi \in C^1(R)$ and we assume further that there exist positive constants λ_1, λ_2 and positive integer k such that

$$0 < \phi'(u) \leq \lambda_1 \mid u \mid^k + \lambda_2.$$

In section 2 we first get the existence and uniqueness of (1.1), then we investigate the regularity of the solution, some prior estimates are obtained. In section 3 we present the finite element method which is suitable for (1.1), some error estimates for interpolation operator of finite element space are gotten. In section 4 we obtain the error estimate for the finite element solution.

2. Existence, Uniqueness and Regularity

By the usual approach[1−3] it is easy to prove that A is the infinitesemal generator of an analytic semigroup $T(t)$ on $L^2(\Omega)$. As in [1], [2] and [3], for $0 \leq \alpha \leq 1$, we introduce operators A^α which are fractional powers of A, we denote the domain of A^α by $D(A^\alpha)$, $D(A^\alpha)$ equipped with the norm $\|u\|_\alpha = \|A^\alpha u\|_{L^2(\Omega)}$ is a Banach space which we denote by X_α.

By Gagliardo-Nirenberg inequality[1−3], we have

$$X_\alpha \subset L^{4k}(\Omega) \quad \text{when} \quad 1 - \frac{1}{2k} < \alpha \leq 1. \tag{2.1}$$

and the imbeddings are continuous.

Analogous to [1], [2] and [3], by the contraction mapping theorem, it is easy to know that (1.1) has a unique local solution $u \in C([0, t_1]; X_\alpha)$, where $1 - \dfrac{1}{2k} < \alpha < 1, t_1$ is a positive constant depending on u_0.

Similar to [4], we can prove that there is a unique $u^* \in D(A)$ satisfying

$$Au^* - \phi(u^*) + f(x) = 0.$$

Now we consider the following initial problem

$$\begin{cases} \dfrac{\partial v}{\partial t} = Av - (\phi(v + u^*) - \phi(u^*)) & \text{in} \quad \Omega \times (0, T_0] \\ v(t,x) = 0 & \text{on} \quad \partial\Omega \times [0, T_0] \\ v(0,x) = u_0(x) - u^*(x) & \text{in} \quad \Omega \end{cases} \tag{2.2}$$

Since as long as the solution exists, $t \to \int_\Omega \mid v(t,x) \mid^{2l} dx$ is nonincreasing for all positive integer l, it follows from (2.1) and Gronwall's inequality that there exists a constant M_1 independent of t such that

$$\|v(t,x)\|_\alpha \leq M_1 \tag{2.3}$$

which implies that the unique solution of (2.2) exists for all $t \geq 0$. In other words we have gotten the global existence of (1.1).

On the other hand, by (2.3) we can get

$$\|F(t,u(t)) - F(s,u(s))\|_{L^2(\Omega)} \leq L\|u(t)-u(s)\|_\alpha, \tag{2.4}$$

where $F(t,u(t)) = -\phi(u(t)) + f(x), L$ is a positive constant independent of $t \in R$. Notice that

$$\|(T(h)-I)A^\alpha T(t)u_0\|_{L^2(\Omega)} \leq \int_0^h \|T(\tau+t)\| \cdot \|A^{\alpha-1}\| \cdot \|A^2 u_0\|_{L^2(\Omega)} d\tau \leq M_2 h,$$

Where M_2 can be chosen to be independent of t. Similar to [1], we have

Theorem 1. *Let $u \in C([0,T_0]; X_\alpha)$ be the solution of* (1.1), $1-\dfrac{1}{2k} < \alpha < 1$. *Then for $0 < \beta < 1-\alpha$, we have*

$$F(t,u(t)) = -\phi(u(t)) + f(x) \in C^\beta([0,T_0]; L^2(\Omega)). \tag{2.5}$$

Furthermore,

$$Au \in C([0,T_0]; L^2(\Omega)), \quad u \in C^1([0,T_0]; L^2(\Omega)). \tag{2.6}$$

From Theorem 1, by [5] and [6], we can obtain the regularity of the solution of (1.1). u may posses singularities at the following points: the crosspoints of interfaces, the turning points of interfaces, the crosspoints of interfaces with $\partial\Omega$ and the points on $\partial\Omega$ with interior angles greater than π. The first - order derivatives of the solution near these points, which will be generally known as singular points, may be unbounded. Denoting the singular points by $x^{(m)}, m = 1, \cdots, M$. We define $\Omega^{(m)} = \{x \in \Omega, | x - x^{(m)} | \leq \rho\}$, where $|\cdot|$ is Euclidean norms in R^2, we can choose $\rho \leq 1$ such that $\Omega^{(m)}$ does not contain any other singular point. Let (ρ_m, θ_m) be the polar coordinates centered at $x^{(m)}$, $\Omega^* = \Omega \setminus \cup_{m=1}^M \overline{\Omega}^{(m)}$, then we have

Theorem 2. *The solution u of* (1.1) *has the followingdecompsition:*

$$u(t,x) = \sum_{m=1}^M \sum_{j\in J^{(m)}} \sigma_j^{(m)}(t)\rho_m^{\sqrt{\lambda_j{}^{(m)}}} \chi(\rho_m)\Theta_j^{(m)}(\theta_m) + u^*(t,x) \tag{2.7}$$

where $\sigma_j^{(m)}(t)$ are constants independent of r_m and θ_m, $\chi \in C^\infty(0,\rho)$ with

$$\chi(r) = \begin{cases} 1, & \textit{for } r < \rho/3 \\ 0, & \textit{for } r > 2\rho/3. \end{cases}$$

$\{\lambda_j^{(m)}, \Theta_j^{(m)}\}_{j\geq 1}$ *is the complete eigensystem of the corresponding Sturm-Liouville eigenvalue problem* (2.9), $J^{(m)} = \{j$ *is a positive integer* $| \lambda_j{}^{(m)} \in (0,1)\}$, $u^* \in H^2$ *on each* Ω_l, *moreover, we have*

$$\sum_{m=1}^M \sum_{j\in J^{(m)}} \left(\sigma_j^{(m)}(t)\right)^2 + \sum_{l=1}^L \|u^*(t,x)\|_{2,\Omega_l}^2 \leq C\|Au\|^2_{C([0,T_0];L^2(\Omega))} \tag{2.8}$$

In $\Omega^{(m)}$, let $\mu \in L^\infty(\Omega^{(m)})$ be the piecewise constant function, that represents the discontinuities of $p(x)$ at $x^{(m)}$:

$$\mu \mid_{\Omega^{(m)} \cap \Omega_i} := \mu_i := p \mid_{\Omega_i} (x^{(m)})$$

Let $\{\rho_m \le \rho,\ \theta_m = \theta^{(\beta)}\}$, $1 \le \beta \le N_0$ be the interfaces where the function μ is discontinuous. When $x^{(m)} \in \Omega$, the Sturm-Liouville eigenvalue problem is

$$\begin{cases} \mu\Theta'' + \mu\lambda\Theta = 0, & \theta_m \ne \theta^{(\beta)} \\ \Theta(0) = \Theta(2\pi) & \\ \Theta(\theta^{(\beta)} - 0) = \Theta(\theta^{(\beta)} + 0) & \\ \mu(\theta^{(\beta)} - 0)\Theta'(\theta^{(\beta)} - 0) = \mu(\theta^{(\beta)} + 0)\Theta'(\theta^{(\beta)} + 0), & 1 \le \beta \le N_0. \end{cases} \tag{2.9}$$

When $x^{(m)} \in \partial\Omega$, (2.9) is replaced by

$$\begin{cases} \mu\Theta'' + \mu\lambda\Theta = 0, & \theta_m \ne \theta^{(\beta)} \\ \Theta(0) = \Theta(\omega) = 0 & \\ \Theta(\theta^{(\beta)} - 0) = \Theta(\theta^{(\beta)} + 0) & \\ \mu(\theta^{(\beta)} - 0)\Theta'(\theta^{(\beta)} - 0) = \mu(\theta^{(\beta)} + 0)\Theta'(\theta^{(\beta)} + 0), & 1 \le \beta \le N_0. \end{cases}$$

where ω is the interior angle of Ω at $x^{(m)}$. Similar to [7], by a direct computation, we get

$$\text{when} \qquad x^{(m)} \in \Omega, \quad \min_{j \in J^{(m)}} \lambda_j{}^{(m)} \ge \frac{\min \mu_i}{\max \mu_i},$$
$$\text{when} \qquad x^{(m)} \in \partial\Omega, \quad \min_{j \in J^{(m)}} \lambda_j{}^{(m)} \ge \frac{\min \mu_i}{\max \mu_i} (\frac{\pi}{\omega})^2$$

3. Finite Element Method

The presence of singularities generally leads to nonoptimal convergence rates by the usual finite element method. Without destorying the usual scheme of finite element method, taking into account the singularities as precisely as possible allows a better approximation of solution. More precisely we discretize the domain Ω by a special mesh subdivision $\mathcal{J}$ which defined in the following.

For the sake of simplicity, we make assumption A): the included angles at singular points $x^{(m)}$ between interfaces $\le \pi/3$. (otherwise, we can draw fictitious interface).

In $\Omega^{(m)}$, we choose $\alpha_0 \le min_{j \in J^{(m)}} \sqrt{\lambda_j{}^{(m)}}$, $R = \rho^{2\alpha_0/3}$.

$$r_0 = 0; \quad r_i = (R\frac{i}{n})^{\frac{3}{2\alpha_0}}, \quad i = 1, 2, \cdots, n. \tag{3.1}$$

We define $G_i = \{x \in \Omega, \mid x - x^{(m)} \mid < r_i\}$, i.e., $G_i = \{x \in \Omega, \rho_m < r_i\}$, it is obvious that $G_n = \Omega^{(m)}$. We denote the inscribed polygon in G_i which vertexes are the intersection points between ∂G_i and the interfaces by E_i. Let $\Lambda_1 = G_1$, $\Lambda_i = G_i \setminus G_{i-1}$, $i = 2, \cdots, n$. $D_1 = E_1, D_i = E_i \setminus E_{i-1}$, $A_i = D_i \cap \Lambda_{i-1}, i = 2, \cdots, n$.

In Ω, we make a triangulation $\mathcal{J}$ such that in every $\Omega^{(m)}$ it satisfies the following assumptions:

I) In each $D_i, \mathcal{J}$ is a quasi uniform triangulation, and there exists constants C_1 and C_2 independent of i such that for $\forall \Delta \in D_i$, the diameter d_Δ satisfies

$$C_2(r_i - r_{i-1}) \le d_\Delta \le C_1(r_i - r_{i-1}) \tag{2.2}$$

II) The intersection of every triangle element $\Delta \in \mathcal{J}$ and the interface is either empty, the edge of Δ or the vertex of Δ.

In the above, for convenience the right upper index $^{(m)}$ for $G_i, E_i, \Lambda_i, D_i, A_i$ have been omited, i.e., we should denote G_i, E_i, Λ_i, D_i and A_i by $G_i^{(m)}, E_i^{(m)}, \Lambda_i^{(m)}, D_i^{(m)}$ and $A_i^{(m)}$ respectively.

In $\Omega^{**} = \Omega \setminus \cup_{m=1}^{M} E_n^{(m)}, \mathcal{J}$ is a conventional quasi uniform triangulation which also satisfies the assumption II), it is required that these triangulations conform to each other, that is, the nodes of them coincide on $\partial E_n^{(m)}$.

Let $V_h \in H_0^1(\Omega)$ be the the linear conforming finite element space associate with $\mathcal{J}$. The semidiscrete approximate problem correspondind to (1.1) is

$$\begin{cases} \text{find } u_h \in V_h \text{ such that} \\ \Big(\dfrac{\partial u_h}{\partial t}, v\Big) + a(u_h, v) + (\phi(u_h), v) = (f, v) \quad \forall v \in V_h, \end{cases} \tag{3.2}$$

where $(u,v) = \int_\Omega uv dx$, $a(u,v) = \int_\Omega p(x) \bigtriangledown u \bigtriangledown v dx$.

Let Δt be the steplength of t, N be the integer part of $\dfrac{T_0}{\Delta t}$. Let Π be the V_h-interpolation operator[8]. We select $u_h^0 = \Pi u_0$, and discrete (3.2) using implicit difference scheme in time, then

$$\begin{cases} \text{find } u_h^{n+1} \in V_h \text{ such that} \\ \Big(\dfrac{u_h^{n+1} - u_h^n}{\Delta t}, v\Big) + a(u_h^{n+1}, v) + (\phi(u_h^{n+1}), v) = (f, v) \quad \forall v \in V_h. \end{cases} \tag{3.3}$$

The finite element solution of (1.1) is gotten by algorithm (3.3). We first state the following Lemma 1 which proved in [7].

Lemma 1. *For $v = \rho_m^d \chi(\rho_m)\Theta(\theta_m)$, $\alpha_0 \le d \le 1$, where Θ is the solution of (2.9), we have*

$$\|v - \Pi v\|_{0,E_n} \le C\frac{1}{n^2}, \quad \|v - \Pi v\|_{1,E_n} \le C\frac{1}{n},$$

where C is independent of n.

Since

$$r_n - r_{n-1} \le C\frac{1}{n}, \tag{3.4}$$

it follows from [8] and Theorem 2 that

Lemma 2. Let $u^*(t,x)$ be the function which defined in Theorem 2. Then

$$\|u^*(t,x) - \Pi u^*(t,x)\|_{0,E_n} \le C\frac{1}{n^2}, \quad \|u^*(t,x) - \Pi u^*(t,x)\|_{1,E_n} \le C\frac{1}{n},$$

where C is independent of n and t.

By Theorem 2, combining Lemma 1 and Lemma 2 yields

Lemma 3. *In $\Omega^{(m)}$, for the solution $u(t,x)$ of (1.1), we have*

$$\|u(t,x)-\Pi u(t,x)\|_{0,E_n}\le C\frac{1}{n^2},\quad \|u(t,x)-\Pi u(t,x)\|_{1,E_n}\le C\frac{1}{n},$$

where C is independent of n and t.

For the solution $u(t,x)$ of (1.1), noting the definition of function χ and assumption A), we get $u(t,x)=u^*(t,x)$ in Ω^{**}, then by [8], it follows from Theorem 2 and (3.4) that

Lemma 4. *Let $u(t,x)$ be the solution of (1.1). Then*

$$\|u(t,x)-\Pi u(t,x)\|_{0,\Omega^{**}}\le C\frac{1}{n^2},\quad \|u(t,x)-\Pi u(t,x)\|_{1,\Omega^{**}}\le C\frac{1}{n},$$

where C is independent of n and t.

We define

$$h=\frac{1}{n}. \tag{3.5}$$

By Lemma 3 and Lemma 4, we have

Theorem 3. *For the solution $u(t,x)$ of (1.1), there exist constants C_1,C_2 independent of h and t such that*

$$\|u(t,x)-\Pi u(t,x)\|_{0,\Omega}\le Ch^2,\quad \|u(t,x)-\Pi u(t,x)\|_{1,\Omega}\le Ch.$$

4. Error Estimate of the Finite Element Solution

We define $u^n=u(n\Delta t,x)$, $w^n=u^n-\Pi u^n$, $e^n=u^n-u_h^n$. It follows from (1.1) and (3.2) that

$$\begin{aligned}\Big(\frac{e^{n+1}-e^n}{\Delta t},e^{n+1}\Big)+a(e^{n+1},e^{n+1})&=\Big(\frac{e^{n+1}-e^n}{\Delta t},w^{n+1}\Big)+a(e^{n+1},w^{n+1})\\&\quad+\Big(\frac{u^{n+1}-u^n}{\Delta t}-\frac{\partial u^{n+1}}{\partial t},\Pi u^{n+1}-u_h^{n+1}\Big)\\&\quad+(\phi(u_h^{n+1})-\phi(u^{n+1}),\Pi u^{n+1}-u_h^{n+1})\le\sum_{j=1}^4 P_n^j,\end{aligned}\tag{4.1}$$

where

$$\begin{aligned}P_n^1&=\Big(\frac{e^{n+1}-e^n}{\Delta t},w^{n+1}\Big),\quad P_n^2=a(e^{n+1},w^{n+1}),\\P_n^3&=\Big(\frac{u^{n+1}-u^n}{\Delta t}-\frac{\partial u^{n+1}}{\partial t},\Pi u^{n+1}-u_h^{n+1}\Big),\\P_n^4&=(\phi(\Pi u^{n+1})-\phi(u^{n+1}),\Pi u^{n+1}-u_h^{n+1}).\end{aligned}$$

(4.1) yields that

$$\frac{1}{2}\|e^{m+1}\|_0^2+\frac{1}{2}\sum_{n=0}^m\|e^{n+1}-e^n\|_0^2+\sum_{n=0}^m a(e^{n+1},e^{n+1})\Delta t$$

$$\leq \frac{1}{2}\|e^0\|_0^2 + \sum_{j=1}^{4}\sum_{n=0}^{m} P_n^j \Delta t \tag{4.2}$$

where $m = 1, 2, \cdots, N$. We estimate each of the four terms separately. From Theorem 3 we have

$$\sum_{n=0}^{m} P_n^1 \Delta t \leq Ch^2 \sum_{n=0}^{m} \|e^{n+1} - e^n\|_0 \leq \frac{1}{2}\sum_{n=0}^{m} \|e^{n+1} - e^n\|_0^2 + C\frac{h^4}{\Delta t}, \tag{4.3}$$

and

$$\sum_{n=0}^{m} P_n^2 \Delta t \leq \frac{1}{8}\sum_{n=0}^{m} a(e^{n+1}, e^{n+1})\Delta t + Ch^2. \tag{4.4}$$

In the following we denote $u(t,x)$ by $u(t)$ for simplicity. By Theorem 1 and Theorem 3,

$$\Big(\frac{\partial u(t)}{\partial t} - \frac{\partial u^{n+1}}{\partial t}, \Pi u^{n+1} - u^{n+1}\Big) \leq Ch^2. \tag{4.5}$$

We define $Q(t, t^{n+1}) = (F(t, u(t)) - F(t^{n+1}, u(t^{n+1})), e^{n+1})$.

When $t \in [n\Delta t, (n+1)\Delta t]$, by (1.1) and Theorem 1,

$$\begin{aligned}
\Big(\frac{\partial u(t)}{\partial t} - \frac{\partial u^{n+1}}{\partial t}, e^{n+1}\Big) &= -a(u(t) - u(t^{n+1}), e^{n+1}) + Q(t, t^{n+1}) \\
&\leq \frac{1}{8}a(e^{n+1}, e^{n+1}) + Ca(u(t) - u(t^{n+1}), u(t) - u(t^{n+1})) + Q(t, t^{n+1}) \\
&\leq \frac{1}{8}a(e^{n+1}, e^{n+1}) + C\|u(t) - u(t^{n+1})\|_0 + Q(t, t^{n+1}) \\
&\leq \frac{1}{8}a(e^{n+1}, e^{n+1}) + C\Delta t + C\Delta t^{\beta}\|e^{n+1}\|_0.
\end{aligned} \tag{4.6}$$

(4.5) and (4.6) yield that

$$\begin{aligned}
P_n^3 =& \frac{1}{\Delta t}\int_{n\Delta t}^{(n+1)\Delta t} \Big(\frac{\partial u(t)}{\partial t} - \frac{\partial u^{n+1}}{\partial t}, \Pi u^{n+1} - u^{n+1}\Big)dt \\
&+ \frac{1}{\Delta t}\int_{n\Delta t}^{(n+1)\Delta t} \frac{\partial u(t)}{\partial t} - \frac{\partial u^{n+1}}{\partial t}, e^{n+1}\Big)dt \\
\leq& Ch^2 + \frac{1}{8}a(e^{n+1}, e^{n+1}) + C\Delta t + C\Delta t^{\beta}\|e^{n+1}\|_0,
\end{aligned}$$

this implies that

$$\sum_{n=0}^{m} P_n^3 \Delta t \leq \frac{1}{4}\sum_{n=0}^{m} a(e^{n+1}, e^{n+1})\Delta t + Ch^2 + C\Delta t^{\beta}. \tag{4.7}$$

From Theorem 1 and Theorem 3, there exist positive constant C_1, C_2 independent of n such that

$$\|u^{n+1}\|_1 \leq C_1, \quad \|u^{n+1} - \Pi u^{n+1}\|_1 \leq C_2 h,$$

hence

$$\|\phi(u^{n+1}) - \phi(\Pi u^{n+1})\|_0 \leq C(\|u^{n+1}\|_{L^{4k}}^k + \|\Pi u^{n+1}\|_{L^{4k}}^k + 1)\|u^{n+1} - \Pi u^{n+1}\|_{L^4}$$

$$\leq C(\|u^{n+1}\|_1^k + \|\Pi u^{n+1}\|_1^k + 1)\|u^{n+1} - \Pi u^{n+1}\|_1 \leq Ch,$$

which yields that

$$\begin{aligned} P_n^4 =& (\phi(\Pi u^{n+1}) - \phi(u^{n+1}), \Pi u^{n+1} - u^{n+1}) + (\phi(\Pi u^{n+1}) - \phi(u^{n+1}), u^{n+1} - u_h^{n+1}) \\ \leq& Ch^3 + Ch\|e^{n+1}\|_0, \end{aligned} \tag{4.8}$$

where we used Theorem 3. It follows from (4.8) that

$$\sum_{n=0}^{m} P_n^4 \Delta t \leq Ch^2 + \frac{1}{8}\sum_{n=0}^{m} a(e^{n+1}, e^{n+1})\Delta t. \tag{4.9}$$

Combining (4.2) with (4.3),(4.4),(4.7) and (4.9) we have

$$\|e^{m+1}\|_0^2 + \sum_{n=0}^{m} a(e^{n+1}, e^{n+1})\Delta t \leq C(h^2 + \frac{h^4}{\Delta t} + \Delta t^{\beta}). \tag{4.10}$$

The main result of this paper comes next:

Theorem 4. *Let u and u_h^n be the solution of* (1.1) *and* (3.3) *respectively. Then we have*

$$\max_n \|u^n - u_h^n\|_0^2 + \sum_{n=0}^{N-1} \|u^{n+1} - u_h^{n+1}\|_1^2 \Delta t \leq C(h^2 + \frac{h^4}{\Delta t} + \Delta t^{\beta}),$$

where $0 < \beta < \dfrac{1}{2k}$, C is a constant independent of $h, \Delta t$ and n.

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