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A METHOD FOR SOLVING THE INVERSE SCATTERING PROBLEM FOR SHAPE AND IMPEDANCE*

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Abstract

The inverse problem considered in this paper is to determine the shape and the impedance of an obstacle from a knowledge of the time-harmonic incident field and the phase and amplitude of the far field pattern of the scattered wave in two-dimension. Single-layer potential is used to approach the scattered waves. An approximation method is presented and the convergence of the proposed method is established. Numerical examples are given to show that this method is both accurate and easy to use.

Mathematics subject classification: 65N38, 65N30.
Key words: Impedance boundary condition, Helmholtz equation, Inverse scattering, Convergence.

1. Introduction

The inverse scattering problem for time-harmonic acoustic waves in two-dimension has been considered for various boundary conditions in a series of papers [1–7]. Among these problems, we are interested in numerical methods for determining the shape and the impedance of an obstacle from the knowledge of the incident field and the scattered field of the far field pattern.

Let D be a bounded, connected domain in the plane with boundary $\partial D \in C^2$ and let the incident field u^i be given by $u^i(x) = \exp[\mathrm{i}kx \cdot d]$ where $k > 0$ is the wave number and d is a fixed unit vector. If we denote the scattered field by u^s and define the total field u by $u = u^i + u^s$, then the direct scattering problem is to find a solution $u \in C^2(\mathbb{R}^2 \backslash \bar{D}) \cap C(\mathbb{R}^2 \backslash D)$ of the Helmholtz equation

$$\Delta_2 u + k^2 u = 0 \quad \text{in } \mathbb{R}^2 \backslash \bar{D}, \tag{1.1}$$

which satisfies the boundary condition

$$\frac{\partial u}{\partial \nu} + \mathrm{i}k\lambda(x)u = 0 \quad \text{on } \partial D; \tag{1.2}$$

and u^s satisfies the Sommerfeld radiation condition

$$\lim_{r\to\infty} \sqrt{r}\left\{\frac{\partial u^s}{\partial r} - \mathrm{i}ku^s\right\} = 0, \quad r = |x|, \tag{1.3}$$

uniformly in all directions $x/|x|$.

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Under the above conditions, it is easily shown [8] that u^s has the asymptotic behavior

$$u^s(x) = \frac{e^{\mathrm{i}k|x|}}{\sqrt{|x|}}\{u_\infty(\hat{x}) + \mathcal{O}(|x|^{-1})\}, \quad |x| \to \infty, \tag{1.4}$$

where u_∞ is known as the far field pattern of the scattered wave u^s. From Green's formula and the asymptotic behavior of the Hankel function $H_0^{(1)}$, we can easily show [8] that

$$u_\infty(\hat{x}) = \frac{e^{\mathrm{i}\pi/4}}{\sqrt{8\pi k}} \int_{\partial D} \left\{ u^s(y)\frac{\partial e^{-\mathrm{i}k\hat{x}\cdot y}}{\partial \nu(y)} - \frac{\partial u^s}{\partial \nu}(y) e^{-\mathrm{i}k\hat{x}\cdot y} \right\} ds(y), \tag{1.5}$$

for $\hat{x} = x/|x|$.

For the problem (1.1)-(1.3), there exists the following theorem.

Theorem 1.1. ([9]) *The exterior impedance boundary-value problem has at most one solution provided $Im(\lambda) \geq 0$ on ∂D. The solution u^s in $\mathbb{R}^2 \backslash D$ and each differentiation of u^s in $\mathbb{R}^2 \backslash \bar{D}$ depend continuously on the boundary data.*

Let $\Gamma \in C^2$ be a closed curve contained in D and assume that k^2 is not a Dirichlet eigenvalue of Laplacian in Γ. Let the single-layer potential

$$v(x) = \int_\Gamma \varphi(y)\Phi(x,y)ds(y), \quad \varphi \in L^2(\Gamma) \tag{1.6}$$

approach the scattered field u^s, where $\Phi(x,y) = \frac{\mathrm{i}}{4}H_0^{(1)}(k\,|x-y|)$ denotes the fundamental solution to the Helmholtz equation in two-dimension. From the asymptotic for $u(x)$:

$$u(x) = \frac{e^{\mathrm{i}k|x|}}{\sqrt{|x|}}\{u_\infty(\hat{x}) + \mathcal{O}(|x|^{-1})\}, \quad |x| \to \infty,$$

uniformly in all directions $\hat{x} = x/\,|x|$, and the asymptotic for the Hankel function:

$$H_0^{(1)}(r) = \sqrt{\frac{1}{\pi r}} e^{\mathrm{i}(r-\pi/4)}\Big(1 + o(\frac{1}{r})\Big),$$

we see that the far-field pattern of the potential (1.6) is given by

$$u_\infty(\hat{x}) = \frac{e^{-\mathrm{i}\pi/4}}{\sqrt{8\pi k}} \int_\Gamma \mathrm{i}e^{-\mathrm{i}k\hat{x}\cdot y}\varphi(y)ds(y). \tag{1.7}$$

To solve inverse obstacle scattering problems, we consider a numerical method to solve the inverse scattering problem for shape and impedance. Hence, for the given far-field pattern, we solve the integral equation

$$(F\varphi)(\hat{x}) = u_\infty(\hat{x}), \tag{1.8}$$

where $F : L^2(\Gamma) \to L^2(\Omega)$ is defined by

$$(F\varphi)(\hat{x}) := \frac{e^{-\mathrm{i}\pi/4}}{\sqrt{8\pi k}} \int_\Gamma \mathrm{i}e^{-\mathrm{i}k\hat{x}\cdot y}\varphi(y)ds(y), \quad \hat{x} \in \Omega. \tag{1.9}$$

Then we need to find the boundary Γ as the location where the boundary condition (1.2) is satisfied in a least square sense.

In comparison with [10], our reconstructions do not require the solution of the function u and its normal derivative $\partial u/\partial \nu$ at each iteration step. We only require the nonzero initials of φ, ρ, λ. Furthermore, this method does not contain the hyper-singular operator, which makes the computation easy.

2. Mathematical Analysis of the Inverse Scattering Problem

For the sake of simplicity, we confine our presentation to the case of star shaped domains, that is, we assume that the boundary Γ can be in the form

$$\Gamma : \{x(\theta) = (x_1(\theta),\ x_2(\theta))\},$$

where $x_1(\theta) = \rho(\theta)\cos\theta,\ x_2(\theta) = \rho(\theta)\sin\theta,\ 0 \le \theta \le 2\pi$ and ρ is 2π periodic positive C^2 function.

Now we need the following theorem:

Theorem 2.1. ([10]) *The far-field patterns corresponding to an infinite number of plane waves with distinct directions uniquely determine the shape and location of the scatterer D and the impedance function λ.*

We are now in a position to present the method. Eq. (1.8) is ill-posed, so we use the Tikhonov regularization method to solve this problem. More precisely, for the regularization parameter $\alpha > 0$, find the solution $\varphi_\alpha \in L^2(\Gamma)$ satisfying

$$\begin{aligned} &\|F\varphi_\alpha - u_\infty\|^2_{L^2(\Omega)} + \alpha\,\|\varphi_\alpha\|^2_{L^2(\Gamma)} \\ &= \inf_{\varphi\in L^2(\Gamma)} \left\{\|F\varphi - u_\infty\|^2_{L^2(\Omega)} + \alpha\,\|\varphi\|^2_{L^2(\Gamma)}\right\}. \end{aligned} \tag{2.1}$$

Define

$$U := \{\lambda : 0 \leqslant \lambda \leqslant M_1, |x(\theta) - y(\theta)| \leqslant M_2, x, y \in \partial D\},$$

where M_1 and M_2 are positive constants. From the Arzela-Ascoli Theorem, U is compact in $C(\partial D)$. The approach of the scattered wave is the single-layer potential

$$u^s_\alpha = \int_\Gamma \varphi_\alpha(y)\Phi(x,y)ds(y).$$

We should find $\rho(\theta)$ and $\lambda(\theta)$, which minimizes the impedance boundary condition

$$\inf_{(\rho\times\lambda)\in(C^2[0,2\pi]\times U)} \left\|\frac{\partial}{\partial\nu}(u^i(\rho(\theta)) + u^s_\alpha(\rho(\theta))) + \mathrm{i}k\lambda(\theta)(u^i(\rho(\theta)) + u^s_\alpha(\rho(\theta)))\right\|. \tag{2.2}$$

Define operator $S : L^2(\Gamma) \to L^2(\partial D)$

$$(S\varphi)(x) := \int_\Gamma \varphi(y)\Phi(x,y)ds(y), \quad x \in \partial D.$$

Then for the boundary $\rho(\theta)$ and impedance $\lambda(\theta)$, we can define the minimization problem

$$\begin{aligned} &\mu(\rho,\lambda,\varphi;\alpha) \\ &= \min_{(\rho\times\lambda\times\varphi)\in(C^2[0,2\pi]\times U\times L^2(\Gamma))} \Bigg\{ \|F\varphi - u_\infty\|^2_{L^2(\Gamma)} + \alpha\,\|\varphi\|^2_{L^2(\Gamma)} \\ &\quad + \left\|\frac{\partial}{\partial\nu}(u^i(\rho(\theta)) + (S(\rho(\theta)))\varphi) + \mathrm{i}k\lambda(\theta)(u^i(\rho(\theta)) + (S(\rho(\theta)))\varphi)\right\|^2_{L^2(\partial D)} \Bigg\}. \end{aligned} \tag{2.3}$$

3. Convergence Analysis

Definition 3.1. *Given the incident field u^i, a far field pattern u_∞ and a regularization parameter $\alpha > 0$, a pair $(\rho_0, \lambda_0) \in C^2[0, 2\pi] \times U$ is called admissible if there exists $\varphi_0 \in L^2(\Gamma)$ such that $(\varphi_0, \rho_0, \lambda_0)$ minimizes the expression in (2.3) over all $\varphi \in L^2(\Gamma), \rho \in C^2[0, 2\pi]$ and $\lambda \in U$. Namely, we have*

$$\mu(\rho_0, \lambda_0, \varphi_0; \alpha) = m(\alpha)$$

where

$$m(\alpha) := \inf_{(\rho,\lambda,\varphi)\in C^2[0,2\pi]\times U\times L^2(\Gamma)} \mu(\rho, \lambda, \varphi; \alpha).$$

Theorem 3.1. *For each $\alpha > 0$ there exists an optimal pair $(\rho_0, \lambda_0) \in C^2[0, 2\pi] \times U$.*

Proof. Let $(\varphi_n, \rho_n, \lambda_n)$ be a minimizing sequence in $L^2(\Gamma) \times C^2[0, 2\pi] \times U$, i.e.,

$$\lim_{n\to\infty} \mu(\varphi_n, \rho_n, \lambda_n; \alpha) = m(\alpha).$$

The sequence $\{\rho_n, \lambda_n\}$ lies in a compact set $C^2[0, 2\pi] \times U$, and hence there exists convergent subsequences. We can then assume that $\rho_n \to \rho_0, \lambda_n \to \lambda_0$ as $n \to \infty$. From

$$\alpha \|\varphi_n\| \le \mu(\varphi_n, \rho_n, \lambda_n; \alpha) \to m(\alpha), \quad n \to \infty,$$

and $\alpha > 0$, we conclude that the sequence $\{\varphi_n\}$ is bounded, i.e., $\|\varphi_n\|_{L^2(\Gamma)} \le c$ for all n and some constant c. Hence, we can assume that it converges weakly $\varphi_n \to \varphi_0 \in L^2(\Gamma)$ as $n \to \infty$.

Since $F : L^2(\Gamma) \to L^2(\partial D)$ and $S : L^2(\Gamma) \to L^2(\partial D)$ represent compact operators, it follows that

$$F\varphi_n \to F\varphi_0, S\varphi_n \to S\varphi_0, \quad n \to \infty.$$

This now implies

$$\begin{aligned}
\alpha \|\varphi_n\|^2_{L^2(\Gamma)} &= \mu(\varphi_n, \rho_n, \lambda_n; \alpha) - \|F\varphi_n - u_\infty\|^2_{L^2(\Gamma)} \\
&\quad - \left\| \frac{\partial}{\partial\nu}(u^i + S\varphi_n) + \mathrm{i}k\lambda_n(u^i + S\varphi_n) \right\|^2_{L^2(\partial D)} \\
&\to m(\alpha) - \|F\varphi_0 - u_\infty\|^2_{L^2(\Gamma)} \\
&\quad - \left\| \frac{\partial}{\partial\nu}(u^i + S\varphi_0) + \mathrm{i}k\lambda_0(u^i + S\varphi_0) \right\|^2_{L^2(\partial D)} \\
&\le \alpha \|\varphi_0\|^2_{L^2(\Gamma)} \quad \text{for } n \to \infty.
\end{aligned}$$

Since we already have weak convergence $\varphi_n \to \varphi_0$ as $n \to \infty$, it follows that

$$\lim_{n\to\infty} \|\varphi_n - \varphi_0\|^2_{L^2(\Gamma)} = \lim_{n\to\infty} (\|\varphi_n\|^2_{L^2(\Gamma)} - \|\varphi_0\|^2_{L^2(\Gamma)}) \le 0,$$

i.e., we also have convergence $\varphi_n \to \varphi_0$ as $n \to \infty$ in norm. Finally, from the continuity, we have

$$\mu(\varphi_0, \rho_0, \lambda_0; \alpha) = \lim_{n\to\infty} \mu(\varphi_n, \rho_n, \lambda_n; \alpha) = m(\alpha),$$

which completes the proof of the theorem. □

Theorem 3.2. *Let u_∞ be the far field pattern corresponding to the incident field u^i, $(\rho(\theta), \lambda(\theta)) \in C^2[0, 2\pi] \times U$. Then we have convergence of the cost functional*

$$\lim_{\alpha \to 0} m(\alpha) = 0. \tag{3.1}$$

Proof. See [10, Thm. 7].

Theorem 3.3. *If the condition of Theorem 3.2 is satisfied, $\alpha_n > 0, n = 1, 2, \cdots$ is a sequence converging to zero, $\{\rho_n, \lambda_n\}$ is the admissible solutions corresponding to it, then $\rho_n(\theta) \to \rho(\theta), \lambda_n(\theta) \to \lambda(\theta)$ as $n \to \infty$.*

Proof. The sequence $\{\rho_n, \lambda_n\}$ lies in a compact set $C^2[0, 2\pi] \times U$ and hence there exists a convergent subsequence, which we again denote by $\{\rho_n, \lambda_n\}$, and $\rho_n \to \rho^* \in C^2[0, 2\pi], \lambda_n \to \lambda^* \in U$. We want to show that $\rho^*(\theta) = \rho(\theta)$ and $\lambda^*(\theta) = \lambda(\theta)$.

Let u^* be the scattering waves corresponding to the boundary ρ^* and the impedance λ^*. That is, it satisfies the boundary condition

$$\frac{\partial}{\partial \nu}(u^*(\rho^*) + u^i(\rho^*)) + \mathrm{i}k\lambda^*(u^*(\rho^*) + u^i(\rho^*)) = 0, \quad \text{on } \partial D. \tag{3.2}$$

(ρ_n, λ_n) is the admissible solution corresponding to α_n, and by Definition 3.1, there exists $\varphi_n \in L^2(\Gamma)$ such that

$$\mu(\varphi_n, \rho_n, \lambda_n; \alpha) = m(\alpha_n).$$

By Theorem 3.2, these boundary data satisfy

$$\lim_{n \to \infty} \left\| \frac{\partial}{\partial \nu}(S(\rho_n)\varphi_n + u^i(\rho_n)) + \mathrm{i}k\lambda_n(S(\rho_n)\varphi_n + u^i(\rho_n)) \right\|_{L^2(\partial D)} = 0.$$

Now we have

$$\begin{aligned}
&\lim_{n \to \infty} \left\| (\frac{\partial}{\partial \nu} S(\rho_n)\varphi_n + \mathrm{i}k\lambda_n S(\rho_n)\varphi_n) - (\frac{\partial}{\partial \nu} u^*(\rho^*) + \mathrm{i}k\lambda^* u^*(\rho^*)) \right\|_{L^2(\partial D)} \\
= &\lim_{n \to \infty} \left\| (\frac{\partial}{\partial \nu} S(\rho_n)\varphi_n + \mathrm{i}k\lambda_n S(\rho_n)\varphi_n) + (\frac{\partial}{\partial \nu} u^i(\rho^*) + \mathrm{i}k\lambda^* u^i(\rho^*)) \right\|_{L^2(\partial D)} \\
= &\ 0.
\end{aligned}$$

From Theorem 1.1 and the far field pattern (1.7), the far field pattern $F\varphi_n$ of the acoustic single-layer potential

$$(S\varphi)(x) = \int_\Gamma \varphi(y)\Phi(x, y)ds(y)$$

converges to the far field pattern u^*_∞ of u^*.

On the other hand, from Theorem 3.2 and

$$\|F\varphi_n - u_\infty\|_{L^2(\Omega)} \to 0, \quad \left\| (\frac{\partial u^s_n}{\partial \nu} + \mathrm{i}k\lambda_n u^s_n) + (\frac{\partial u^i}{\partial \nu} + \mathrm{i}k\lambda u^i) \right\|_{L^2(\Omega)} \to 0,$$

both as $n \to \infty$, we have $u_\infty = u^*_\infty$, which implies that $u^s = u^*$. Notice that $u^i(\rho) = e^{\mathrm{i}k\rho\cos(\theta - \phi)}$, where ϕ denotes the incoming angle. So we have

$$\rho(\theta) = \rho^*(\theta), \lambda(\theta) = \lambda^*(\theta), \quad \theta \in [0, 2\pi].$$

This completes the proof of the theorem. □

4. Numerical Examples

In this section, we shall discuss the numerical implementation of the algorithm presented in the previous section. The data for the inverse problem is the far field pattern for a variety of incoming waves and choices of the wave number k. To obtain better results, the incoming waves are written as

$$u_N^i(x) = \sum_{p=1}^{N} e^{\mathrm{i}kx\cdot d_p}$$

and $u_\infty^N(\hat{x})$ is the far field pattern corresponding to it. For our examples, this data is generated by approximately solving the direct scattering problem (given the obstacle D). In order to generate the data, we refer to [8, 12].

The Newton's iteration is used to solve the optimization problem. To discretize the inverse problem (2.3), the integrals are approximated using the trapezoidal rule with $\theta_j = j\pi/n,\ j = 0, 1, \cdots, 2n-1$ and $\psi(\theta) = \varphi(y)$. By

$$\frac{\partial}{\partial\nu}(S\varphi) = (K' - I)\varphi,$$

where

$$(K'\varphi)(x) = 2\int_\Gamma \frac{\partial\Phi(x,y)}{\partial\nu(x)}\varphi(y)ds(y), \quad x \in \partial D,$$

the representation (2.3) can be written as

$$\begin{aligned}
&\mu(\psi,\rho,\lambda;\alpha)\\
=&\sum_{q=0}^{L-1}\left\|\frac{e^{-\mathrm{i}\pi/4}}{\sqrt{8\pi k}}\frac{\pi}{n}\sum_{j=0}^{2n-1}\mathrm{i}\,|x'(\theta_j)|\,e^{-\mathrm{i}k(\hat{x}_q)\cdot(x_1(\theta_j),x_2(\theta_j))}\psi(\theta_j) - u_\infty(\hat{x}_q)\right\|^2\\
&+\alpha\sum_{j=0}^{2n-1}\|\psi(\theta_j)\|^2 + \left\|\sum_{j=0}^{2n-1}\Big(\sum_{p=0}^{N-1}\frac{\partial e^{\mathrm{i}kx(\theta_j)\cdot d_p}}{\partial\nu} + (K'(\rho(\theta_j)) - I)\psi(\theta_j)\right.\\
&\left.+\,\mathrm{i}k\lambda(\theta_j)\Big(\sum_{p=0}^{N-1}e^{\mathrm{i}kx(\theta_j)\cdot d_p} + S(\rho(\theta_j))\psi(\theta_j)\Big)\Big)\right\|^2.
\end{aligned} \tag{4.1}$$

For the numerical method of the operators S, K', we will use the following interpolators quadrature rules

$$\int_0^{2\pi}\ln(4\sin^2\frac{\theta-\tau}{2})f(\tau)d\tau \approx \sum_{j=0}^{2n-1}R_j^{(n)}(\theta)f(\theta_j^{(n)}),$$

where

$$R_j^{(n)}(\theta) = -\frac{2\pi}{n}\sum_{m=1}^{n-1}\frac{1}{m}\cos m(\theta-\theta_j^{(n)}) - \frac{\pi}{n^2}\cos n(\theta-\theta_j^{(n)}).$$

In order to discretize the inverse problem (4.1), we approximate the functions ψ, ρ and λ by finite trigonometric series

$$\psi_\alpha(\theta) = \sum_{j=-n_1}^{n_1} g_j e^{\mathrm{i}j\theta}, \quad g_j \in \mathbb{C},$$

$$\rho_\alpha(\theta) = a_0^{(n_2)} + \sum_{j=1}^{n_2} (a_j^{(n_2)} \cos j\theta + b_j^{(n_2)} \sin j\theta), \quad a_j^{(n_2)},\ b_j^{(n_2)} \in \mathbb{R},$$

$$\lambda_\alpha(\theta) = a_0^{(n_3)} + \sum_{j=1}^{n_3} (a_j^{(n_3)} \cos j\theta + b_j^{(n_3)} \sin j\theta), \quad a_j^{(n_3)},\ b_j^{(n_3)} \in \mathbb{R}.$$

We now report on the examples we have computed. The approximate minimum occurs at $k = 1.0$ and the fixed unit vector

$$d_p = \begin{pmatrix} \cos(2\pi p/3) \\ \sin(2\pi p/3) \end{pmatrix}, \quad (p = 0, 1, 2).$$

In our examples, the full line denotes graph of ρ or λ, and the broken line denotes graph of ρ_α or λ_α.

Example 4.1. *The pinched ellipse.*

The exact figure is the pinched ellipse $\rho(\theta) = 1+\frac{1}{2}\cos 2\theta$ and the impedance is $\lambda(\theta) = 1+\sin^3(\theta)$. The parameters used for the problem: $n_0 = 6, m_0 = 64$. The number of incoming waves is 3. For the inverse problem: $n_1 = 8, n_2 = 4, n_3 = 4, L = 30, \alpha = 10^{-10}$. The numerical results of ρ_α and λ_α are shown in Fig. 4.1.

Example 4.2. *The garlic.*

The exact figure is the garlic $\rho(\theta) = 1 - \sin\theta\cos^2\theta$ and the impedance is $\lambda = 2 + \cos^3(\theta) + \sin^3(\theta)$. The parameters used for the direct problem are $n_0 = 6,\ m_0 = 64$. The number of incoming waves is 3. For the inverse problem, $n_1 = 8, n_2 = 4, n_3 = 4, L = 30, \alpha = 10^{-10}$. The numerical results of ρ_α and λ_α are shown in Fig. 4.3.

About the data involving noise we refer to [10]. In terms of the total field $u = S + u^i$ we introduce the operator G defined by

$$G : (\rho, \lambda)| \to (\nu \cdot \mathrm{grad} u + \mathrm{i}k\lambda u)|_\Gamma$$

or in a slight abuse of notation

$$G : (\rho, \lambda)| \to \nu \cdot (\mathrm{grad} u) \circ \rho + \mathrm{i}k(\lambda u) \circ \rho \quad \text{in } [0, 2\pi].$$

The perturbations $h(\theta)$ to the boundary is also starlike. The perturbations to the λ is μ. Then for the boundary condition we need to solve the minimization problem

$$\inf_{(\rho(\theta)\times\lambda(\theta))\in(C^2[0,2\pi]\times C[0,2\pi])} \left\| G(\rho,\lambda) + \frac{\partial G}{\partial \rho}(\rho,\lambda)h + \frac{\partial G}{\partial \lambda}(\rho,\lambda)\mu \right\|_{L^2(\partial D)},$$

where

$$\begin{aligned} \frac{\partial G}{\partial \rho}(\rho,\lambda)h = & -k^2 h_\nu u \circ \rho - \frac{\partial}{\partial \tau}(h_\nu u(\frac{\partial u}{\partial \tau}) \circ \rho) \\ & +(\mathrm{i}k\lambda - \frac{x'' \cdot \nu}{|x'|^2}) h_\nu u \frac{\partial u}{\partial \nu} \circ \rho + \mathrm{i}k h_\nu u(\frac{\partial \lambda}{\partial \nu} u) \circ \rho \end{aligned}$$

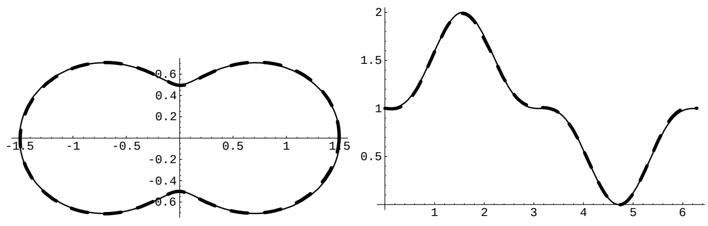

Fig. 4.1. Example 4.1: Reconstruction for the pinched ellipse and the impedance.

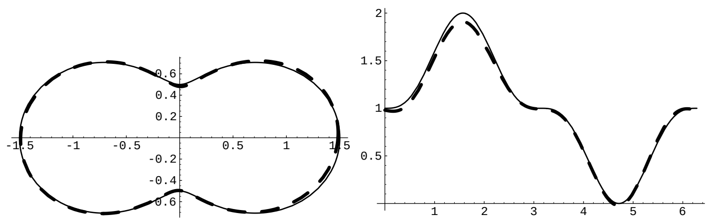

Fig. 4.2. Example 4.1: Reconstruction with 1% noise.

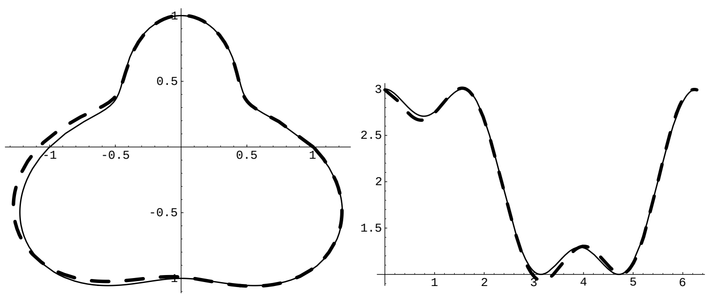

Fig. 4.3. Example 4.2: Reconstruction for the garlic and the impedance.

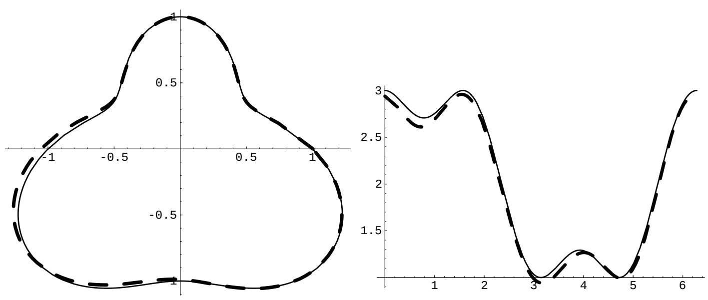

Fig. 4.4. Example 4.2: Reconstruction with 1% noise.

and

$$\frac{\partial G}{\partial \lambda}(\rho, \lambda)\mu = \mathrm{i}k(\mu u) \circ \rho,$$

both in $[0, 2\pi]$. For the above two examples, the numerical results of data involving noise are shown in Figs. 4.2 and 4.4.

Our reconstructions do not require the solution of the direct scattering problem at each iteration step. In compared with [11], on boundary ρ the 2-norm (2.2) is minimized and we express the unknown impedance function and boundary ρ by the trigonometric polynomials. So the reconstruction is insensitive to the initial guess. For both examples we used as initial guess a circle of radius 1.0 and a constant impedance $\lambda = 1.0$. The results are found very accurate.

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