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Preface
Special issue on Adaptive and Multilevel Methods in Computational Electromagnetics

In numerical analysis attention to problems arising in electromagnetic field computation had been scant for a long time, dwarfed by research efforts devoted to areas like computational fluid mechanics and numerical methods for elasticity. By the mid 1990s the situation was poised to change prompted by the invention of a new class of finite elements, the edge elements, tailored to the spatial discretization of electromagnetic fields.

A dramatic surge in mathematical research activities connected with computational electromagnetics followed, boosted by the fundamental insight that Maxwell's equations do not just give rise to mere standard elliptic boundary value problems for field components. Rather, they possess a special structure that must be preserved in the course of discretization. The spread of this understanding brought "mimetic" discretization schemes into focus and promoted "exact sequences", "co-chains" and "discrete differential forms" to familiar concepts in numerical analysis.

Grasping the role of algebraic properties of discrete fields paved the way for adapting many powerful techniques developed for the analysis and efficient solution of 2nd-order elliptic boundary value problems to the computation of electromagnetic fields. Prominent examples are multigrid methods, domain decomposition preconditioners, and adaptive finite element methods. They form the focus of this special issue of the Journal of Computational Mathematics.

The goal is to provide snapshots of current research on various issues connected with edge element discretization of electromagnetic field problems. The papers represent several important directions of research in a field of numerical analysis that has matured by now, but is still rapidly evolving. Contributed by experts in the field, they present recent results, but, implicitly, also give a survey of current knowledge. Some, also raise further issues worth pursuing. This renders this special issue a valuable resource for anyone who is conducting or intends to launch research in finite element techniques for computational electromagnetics.

Let us briefly review the different contributions that make up this special issue: The first by L.-Q. Zhong, S. Shu, G. Wittum, and J.-C. Xu, demonstrates the use of duality techniques for the a-priori error analysis of Maxwell boundary value problems in frequency domain. It highlights the clever use of decompositions in order to cope with solenoidal and irrotational field components. The second article by R. Hiptmair and W.-Y. Zheng develops the convergence theory of local multigrid for magneto-quasistatic field problems. It provides a detailed survey of techniques relevant for local multigrid analysis and establishes asymptotic optimality of geometric multigrid with hybrid smoothing also in the case of locally refined meshes. A fast iterative solution method is also presented in the following contribution by T. Kolev and P. Vassilevski. They give a detailed discussion of the rationale, aspects of (parallel) implementation, and performance of the so-called auxiliary space preconditioning strategy for discrete magneto-quasistatic boundary value problems. These are also treated in the fourth article by M. Bebendorf and J. Ostrowski, but their approach to preconditioning is radically different and employs the so-called hierarchical matrices, an algebraic generalization of clustering, aka multipole, techniques. The last two papers of this issue deal with adaptive finite element schemes. The contribution by M. Clemens, J. Lang, D. Teleaga, and G. Wimmer addresses the design of an algorithm for fully space-time adaptive magnetic field computations. The paper by R.H.W. Hoppe and J. Schöberl, examines the edge element discretization of a magneto-quasistatic model problem and the convergence of a an adaptive scheme employing residual based a-posteriori error estimation.

It goes without saying that the sheer scope of the field of finite element discretization and fast adaptive solution methods for electromagnetic field problems rules out exhaustive coverage in this special volume. We should point out that several important developments could not be included. For instance, papers on discontinuous Galerkin methods, weighted regularization approaches, least squares methods, and *hp*-finite elements for electromagnetics are missing in this issue. Yet, many fundamental ideas elaborated in the articles of this issue also govern the design of these other methods.

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