

## UNIQUENESS OF SOLUTIONS FOR SEMICONDUCTOR EQUATIONS WITH AVALANCHE TERM\*

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**Abstract** In this paper, we consider the initial and mixed boundary value problems for the semiconductor equations with avalanche term, the uniqueness of the weak solution for the semiconductor equation has been proved.

**Key Words** Semiconductor equations; avalanche term; weak solution; uniqueness.

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### 1. Introduction

Let  $G$  be a bounded domain in  $R^n$ ,  $1 \leq n \leq 3$ . Set  $Q_T = (0, T) \times G$ . Suppose that  $\partial G = \Gamma_D \cup \Gamma_N$ , where  $\Gamma_D$  and  $\Gamma_N$  are pairwise disjoint and  $\Gamma_D$  is closed and possesses positive surface measure. Moreover,  $\nu(x_0)$  denotes the outer unit normal at  $x_0 \in \partial G$ . In this paper we study the following system of nonlinear partial differential equations that describes the transport of electrons and holes in a semiconductor device

$$\frac{\partial u_i}{\partial t} - \operatorname{div} J_i = -R(u_1, u_2) + \alpha_1(\nabla \psi)|J_1| + \alpha_2(\nabla \psi)|J_2|, \quad i = 1, 2 \quad (1.1)$$

$$-\nabla \cdot (a \nabla \psi) = f + u_2 - u_1 \quad (1.2)$$

with boundary conditions

$$(u_i, \psi)|_{\Gamma_D} = (\bar{u}_i, \bar{\psi}), \quad \frac{\partial u_i}{\partial \nu} \Big|_{\Gamma_N} = \frac{\partial \psi}{\partial \nu} \Big|_{\Gamma_N} = 0 \quad (1.3)$$

and initial conditions

$$u_i(0, x) = u_{0i}(x), \quad x \in G \quad (1.4)$$

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The unknown functions  $u_1, u_2$  and  $\psi$  denote the free electron carrier concentration, the free hole carrier concentration and the electrostatic potential.  $f$  is the net density of ionized impurities.  $R(u_1, u_2) = r(u_1, u_2)(u_1 u_2 - n_i^2)$ ,  $J_i = D_i \nabla u_i + q_i u_i M_i \nabla \psi$ ,  $q_1 = -1$ ,  $q_2 = 1$ ,  $D_i = D_i(x, \nabla \psi)$ ,  $M_i = M_i(x, \nabla \psi)$ . The coefficients  $\alpha_1$  and  $\alpha_2$  represent the ionization rates for electrons and holes, respectively. The term  $\alpha_1(\nabla \psi)|J_1| + \alpha_2(\nabla \psi)|J_2|$  models the generation of charged particles due to impact ionization (avalanche generation of electrons and holes). In [1, 2], the authors proved the local existence of a weak solutions to (1.1)-(1.4) ( $R = 0, n = 3, 2$ ). In [3], the existence of weak solutions is proved for space dimensions=1, 2, 3, and the uniqueness of solutions is showed in the case of one space dimension. The aim of this paper is to prove the uniqueness of solutions to (1.1)-(1.4) in the case  $n$  ( $n \leq 3$ ) space dimension.

## 2. Notations and Assumption

Using the standard notation we denote by  $H^1(G)$  the Sobolev Space. A norm in a Banach space  $E$  is denoted by  $\|\cdot\|_E$ . The norm in the space  $L^p$ ,  $p \geq 1$  is, for short denoted by  $\|\cdot\|_p$  both for the space  $L^p(G)$  and  $L^p(G; R^n)$ ,  $\|\cdot\| = \|\cdot\|_2$ . We introduce some function spaces  $L_+^q(G) = \{v : v \in L^q(G), v \geq 0 \text{ a.e., in } G\}$ ,  $Y = \{v : v \in H^1(G), v|_{\Gamma_D} = 0\}$ ,  $Y^*$  be dual space of  $Y$ . We denote by  $(\cdot, \cdot)$  the scalar product in  $L^2(G)$  and  $\langle \cdot, \cdot \rangle$  the duality pairing between  $Y^*$  and  $Y$ . In this paper we shall work on the following assumption that

$$(H_1) \bar{u}_i \in H^1(G) \cap L_+^\infty(G), \bar{\psi} \in H^1(G) \cap L^\infty(G);$$

$$(H_2) f = f(x) \in L^3(G), n_i^2 = n_i^2(x) \in L_+^\infty(G), n_1 = n_2;$$

(H<sub>3</sub>)  $a$  is a positive constant; the diffusion coefficients  $D_i = D_i(x, y)$  and the mobilities  $M_i = M_i(x, y)$  satisfy the following: (i)  $D_i$  and  $M_i$  are measurable in  $x \in G$ , continuous in  $y \in R^n$ , and there are positive constants  $\bar{D}_i$  and  $d_i$  such that  $d_i \leq D_i(x, y) \leq \bar{D}_i$  for all  $(x, y) \in G \times R^n$ ; (ii)  $M_i$  ( $i = 1, 2$ ) are of the form  $M_i(x, y) = \mu_i + B_i(x, y)$ ,  $(x, y) \in G \times R^n$ ; where  $\mu_i$  ( $i = 1, 2$ ) are nonnegative constants, and there exists a constant  $B_0$  such that the functions  $B_i$  satisfy

$$|B_i(x, y)y| \leq B_0, \quad (x, y) \in G \times R^n$$

$$(H_4) r : R_+^2 \rightarrow R_+ \text{ is Lipschitzian;}$$

$$(H_4)' R \text{ is the Shockley-Read-Hall term}$$

$$R(u_1, u_2) = \frac{b}{r_0 + r_1 u_1 + r_2 u_2} (u_1 u_2 - n_i^2),$$
 where  $b \geq 0, r_j > 0$  ( $j = 0, 1, 2$ ) are positive constants;

$$(H_5) u_{0i} \in L_+^\infty(G), i = 1, 2;$$

$$(H_6) \alpha_i(y) \in C(R^n), 0 \leq \alpha_i(y) \leq \alpha_{0i} = \text{const} < +\infty, y \in R^n;$$

(H<sub>7</sub>) Let (H<sub>4</sub>) hold, and  $\rho_0(u_1 + u_2) \leq r(u_1, u_2) \leq \rho_1(1 + u_1 + u_2)$ ,  $u_i \in R_+$ ,  $\rho_0, \rho_1$  are positive constants;

(H<sub>8</sub>) Let (H<sub>3</sub>), (H<sub>6</sub>) hold, and  $D_i(x, y)$  are positive constants  $D_{0i}, B_i(x, y)y, \alpha_i(y)$  satisfies globally Lipschitz condition.

### 3. Existence of a Weak Solution to the Problem (1.1)–(1.4)

Using similar methods of the proof in reference [1–4], we can obtain

**Theorem 3.1** *Let the hypotheses (H<sub>1</sub>)–(H<sub>6</sub>) be satisfied, and (i)  $\mu_1, \mu_2 > 0$  (H<sub>7</sub>) hold or (ii)  $\mu_1 = \mu_2 = 0$ . Then in the problem (1.1)–(1.4) there exists a weak solution  $(u_i, \psi)$ , such that for every  $T > 0$*

$$\begin{aligned} u_i - \bar{u}_i &\in L^2(0, T; Y), \psi - \bar{\psi} \in C([0, T]; Y) \\ u_i - \bar{u}_i &\in C([0, T]; L^2(G)), u_i(0, x) = u_{0i}(x) \end{aligned} \quad (3.1)$$

$$\begin{aligned} u_i &\in L^p(Q_T), \quad (2 \leq p < 10) \quad u_i \nabla \psi \in L^2(Q_T), \quad \psi \in L^\infty(0, T; L^\infty(G)) \\ u_{it} &\in L^2(0, T; Y^*) \end{aligned} \quad (3.2)$$

and

$$\begin{aligned} \int_0^T (u_{it}, \varphi_i) dt + \int_{Q_T} (D_i \nabla u_i + q_i u_i M_i \nabla \psi) \nabla \varphi_i dx dt \\ = \int_{Q_T} (r(u_1, u_2)(n_i^2 - u_1 u_2) + \alpha_1 (\nabla \psi) |J_1| + \alpha_2 (\nabla \psi) |J_2|) \varphi_i dx dt \end{aligned} \quad (3.3)$$

for every  $\varphi_i \in L^2(0, T; Y)$ ,

$$a \int_G \nabla \psi \cdot \nabla \varphi dx = \int_G (f + u_2 - u_1) \varphi dx, \quad \text{for every } \varphi \in Y, t \in [0, T] \quad (3.4)$$

**Theorem 3.2** *Let the hypotheses (H<sub>1</sub>)–(H<sub>6</sub>) be satisfied, and  $\mu_1, \mu_2 > 0$ , (H<sub>4</sub>)' hold. Then there exists a  $T_1 > 0$ , such that in the problem (1.1)–(1.4) there exists a weak solution  $(u_i, \psi)$  on  $[0, T_1] \times G$ .*

### 4. Uniqueness of Weak Solutions

**Theorem 4.1** *Under the assumptions of Theorem 3.1 or Theorem 3.2, (H<sub>8</sub>) holds. Then the weak solution of the problem (1.1)–(1.4) is unique.*

**Proof** Suppose that  $(u_i, \psi)$  and  $(\hat{u}_i, \hat{\psi})$  be two different solutions to the problem (1.1)–(1.4) satisfying (3.1)–(3.4). Then

$$a \int_G \nabla(\hat{\psi} - \psi) \cdot \nabla \varphi dx = \int_G [(\hat{u}_2 - u_2) - (\hat{u}_1 - u_1)] \varphi dx \quad (4.1)$$

for every  $\varphi \in Y, t \in [0, T]$ ,

$$\|\nabla(\hat{\psi} - \psi)(t)\| \leq C \Sigma \|(\hat{u}_i - u_i)(t)\| \quad (4.2)$$

$$\|(\hat{\psi} - \psi)(t)\|_\infty \leq C \Sigma \|(\hat{u}_i - u_i)(t)\| \quad (4.3)$$

$$\frac{1}{2} \Sigma \|(\hat{u}_i - u_i)(t)\|^2 + \Sigma D_{0i} \int_{Q_i} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau$$

$$\begin{aligned}
&= \Sigma(-q_i) \int_{Q_t} [(\hat{u}_i(\mu_i + B_i(x, \nabla\psi))\nabla\hat{\psi} - u_i(\mu_i + B_i(x, \nabla\psi))\psi) \cdot \nabla(\hat{u}_i - u_i)] dx d\tau \\
&\quad + \Sigma \int_{Q_t} (R(u_1, u_2) - R(\hat{u}_1, \hat{u}_2))(\hat{u}_i - u_i) dx d\tau \\
&\quad + \Sigma \int_{Q_t} [\Sigma(\alpha_i(\nabla\hat{\psi})|\hat{J}_i| - \alpha_i(\nabla\psi)|J_i|)](\hat{u}_i - u_i) dx d\tau \\
&\leq \Sigma(-q_i)\mu_i \int_{Q_t} (\hat{u}_i - u_i)\nabla\psi \cdot \nabla(\hat{u}_i - u_i) dx d\tau \\
&\quad + \Sigma\mu_i \int_{Q_t} \hat{u}_i |\nabla(\hat{\psi} - \psi)| \cdot |\nabla(\hat{u}_i - u_i)| dx d\tau \\
&\quad + \Sigma B_0 \int_{Q_t} |\hat{u}_i - u_i| \cdot |\nabla(\hat{u}_i - u_i)| dx d\tau \\
&\quad + \Sigma L_1 \int_{Q_t} \hat{u}_i |\nabla(\hat{\psi} - \psi)| \cdot |\nabla(\hat{u}_i - u_i)| dx d\tau \\
&\quad + \Sigma C \int_{Q_t} (1 + u_1^2 + u_2^2 + \hat{u}_1^2 + \hat{u}_2^2) |\hat{u}_i - u_i|^2 dx d\tau \\
&\quad + \Sigma \int_{Q_t} \left\{ \Sigma\alpha_{0i}[D_{0i}|\nabla(\hat{u}_i - u_i)| + \mu_i(\hat{u}_i|\nabla(\hat{\psi} - \psi)| + |\hat{u}_i - u_i| \cdot |\nabla\psi|) \right. \\
&\quad \left. + B_0|\hat{u}_i - u_i| + L_1\hat{u}_i|\nabla(\hat{\psi} - \psi)| \right\} |\hat{u}_i - u_i| dx d\tau \\
&\quad + \Sigma \int_{Q_t} (\Sigma|J_i|)L_2(\nabla(\hat{\psi} - \psi)) \cdot |\hat{u}_i - u_i| dx d\tau \\
&= E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + E_7
\end{aligned} \tag{4.4}$$

$$E_2 \leq \varepsilon \Sigma \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau + K_1(\varepsilon) \Sigma \int_{Q_t} \hat{u}_i^2 |\nabla(\hat{\psi} - \psi)|^2 dx d\tau \tag{4.5}$$

$$E_3 \leq \varepsilon \Sigma \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau + K_2(\varepsilon) \Sigma \int_{Q_t} \|(\hat{u}_i - u_i)\|^2 dx d\tau \tag{4.6}$$

$$E_4 \leq \varepsilon \Sigma \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau + K_3(\varepsilon) \Sigma \int_{Q_t} \hat{u}_i^2 |\nabla(\hat{\psi} - \psi)|^2 dx d\tau \tag{4.7}$$

$$\begin{aligned}
E_5 &\leq C \Sigma \int_0^t \|(1 + u_1^2 + u_2^2 + \hat{u}_1^2 + \hat{u}_2^2)\|_3 \|\hat{u}_i - u_i\| \|\hat{u}_i - u_i\|_6 d\tau \\
&\leq \varepsilon \Sigma \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau \\
&\quad + K_4(\varepsilon) \int_{Q_t} \|(1 + u_1^2 + u_2^2 + \hat{u}_1^2 + \hat{u}_2^2)\|_3^2 \|\hat{u}_i - u_i\|^2 d\tau
\end{aligned} \tag{4.8}$$

$$\begin{aligned}
E_6 &\leq \varepsilon \Sigma \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau + \varepsilon \Sigma \int_{Q_t} |\hat{u}_i - u_i|^2 |\nabla\psi|^2 d\tau \\
&\quad + K_5(\varepsilon) \Sigma \int_{Q_t} |(\hat{u}_i - u_i)|^2 dx d\tau + K_6(\varepsilon) \Sigma \int_{Q_t} \hat{u}_i^2 |\nabla(\hat{\psi} - \psi)|^2 dx d\tau
\end{aligned} \tag{4.9}$$

$$E_7 \leq L_2 \Sigma \int_0^t \left\| (\Sigma|J_i|) \right\| \cdot \left( \int_G |\nabla(\hat{\psi} - \psi)|^2 |\hat{u}_i - u_i|^2 dx \right)^{1/2} d\tau \tag{4.10}$$

Let  $\varphi = \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} \in Y$  in (3.4), ( $\delta > 0$ ), we obtain

$$\begin{aligned} & \int_{Q_t} (\hat{u}_i - u_i) \nabla \psi \cdot \frac{\nabla(\hat{u}_i - u_i)}{(1 + \delta(\hat{u}_i - u_i)^2)^2} dx d\tau \\ &= \frac{1}{2} \int_{Q_t} \nabla \psi \cdot \nabla \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} dx d\tau \\ &= \frac{1}{2a} \int_{Q_t} (f + u_2 - u_1) \cdot \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} dx d\tau \end{aligned} \quad (4.11)$$

and taking  $\delta \rightarrow 0$  in (4.11), we obtain

$$\begin{aligned} E_1 &\leq \Sigma \mu_i \left| \int_{Q_t} (\hat{u}_i - u_i) \nabla \psi \cdot \nabla(\hat{u}_i - u_i) dx d\tau \right| \\ &= \Sigma \frac{\mu_i}{2a} \left| \int_{Q_t} (f + u_2 - u_1) (\hat{u}_i - u_i)^2 dx d\tau \right| \\ &\leq \Sigma \frac{\mu_i}{2a} \int_{Q_t} \left( \| |f| + u_2 + u_1 \|_3 \|(\hat{u}_i - u_i)\| \cdot \|(\hat{u}_i - u_i)\|_6 d\tau \right) \\ &\leq \varepsilon \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau \\ &\quad + K_7(\varepsilon) \Sigma \int_0^t \| |f| + u_2 + u_1 \|_3^2 \|(\hat{u}_i - u_i)(\tau)\|^2 d\tau \end{aligned} \quad (4.12)$$

Let  $\varphi = \frac{\hat{u}_i^2}{1 + \delta \hat{u}_i^2} (\hat{\psi} - \psi) \in Y$  in (4.1), it is easy to see

$$\begin{aligned} \int_{Q_t} \frac{\hat{u}_i^2}{1 + \delta \hat{u}_i^2} |\nabla(\hat{\psi} - \psi)|^2 dx d\tau &= \frac{1}{a} \int_{Q_t} [(\hat{u}_2 - u_2) - (\hat{u}_1 - u_1)] \cdot \frac{\hat{u}_i^2}{1 + \delta \hat{u}_i^2} (\hat{\psi} - \psi) dx d\tau \\ &\quad - \int_{Q_t} \nabla(\hat{\psi} - \psi) \cdot \frac{2u_i \nabla u_i}{(1 + \delta \hat{u}_i^2)^2} (\hat{\psi} - \psi) dx d\tau \end{aligned} \quad (4.13)$$

taking  $\delta \rightarrow 0$  in (4.13), we obtain

$$\begin{aligned} \int_{Q_t} \hat{u}_i^2 |\nabla(\hat{\psi} - \psi)|^2 dx d\tau &\leq K_8 \int_0^t \|(\hat{\psi} - \psi)(\tau)\|_\infty (\Sigma \|(\hat{u}_i - u_i)\|_6) \|\hat{u}_i\| \cdot \|\hat{u}_i\|_3 d\tau \\ &\quad + K_9 \int_0^t \|(\hat{\psi} - \psi)(\tau)\|_\infty^2 \|\nabla \hat{u}_i\|^2 d\tau \\ &\leq (\sup \| \hat{u}_i(t) \|) K_{10} \int_0^t \|(\hat{\psi} - \psi)\|_\infty (\Sigma \|\nabla(\hat{u}_i - u_i)\|) \|\hat{u}_i\|_3 d\tau \\ &\quad + K_9 \int_0^t \|(\hat{\psi} - \psi)(\tau)\|_\infty^2 \|\nabla \hat{u}_i\|^2 d\tau \\ &\leq \varepsilon \Sigma \int_0^t \|\nabla(\hat{u}_i - u_i)(\tau)\|^2 d\tau + \int_0^t h_1(\tau) (\Sigma \|(\hat{u}_i - u_i)(\tau)\|^2) d\tau \end{aligned} \quad (4.14)$$

where  $h_1(\tau) \in L^1_+(0, T)$ .

Let  $\varphi = \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} \psi \in Y$  in (3.4), we obtain

$$\begin{aligned} \int_{Q_t} \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} |\psi|^2 dx d\tau &= \frac{1}{a} \int_{Q_t} (f + u_2 - u_1) \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} \psi dx d\tau \\ &\quad - \int_{Q_t} \nabla \psi \cdot \frac{2(\hat{u}_i - u_i) \nabla(\hat{u}_i - u_i)}{(1 + \delta(\hat{u}_i - u_i)^2)^2} \psi dx d\tau \end{aligned} \quad (4.15)$$

taking  $\delta \rightarrow 0$ , we obtain the estimate

$$\begin{aligned} \varepsilon \Sigma \int_{Q_t} (\hat{u}_i - u_i)^2 |\nabla \psi|^2 dx d\tau &\leq \varepsilon \Sigma \frac{2}{a} \int_0^t \|(|f| + u_2 + u_1)\|_3 \|\hat{u}_i - u_i\| \|\hat{u}_i - u_i\|_6 \|\psi(\tau)\|_\infty d\tau \\ &\quad + 4\varepsilon \Sigma \int_0^t \|\nabla(\hat{u}_i - u_i)\|^2 \|\psi(\tau)\|_\infty^2 d\tau \\ &\leq \varepsilon C \Sigma \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau + \int_0^t h_2(\tau) (\Sigma \|\hat{u}_i - u_i\|^2) d\tau \end{aligned} \quad (4.16)$$

where  $h_2(t) \in L^1_+(0, T)$ .

Let  $\varphi = \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} (\hat{\psi} - \psi) \in Y$ , in (4.1), we obtain

$$\begin{aligned} \int_G \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} |\nabla(\hat{\psi} - \psi)|^2 dx &= \frac{1}{a} \int_G [(\hat{u}_2 - u_2) - (\hat{u}_1 - u_1)] \cdot \frac{(\hat{u}_i - u_i)^2}{1 + \delta(\hat{u}_i - u_i)^2} (\hat{\psi} - \psi) dx \\ &\quad - \int_G \nabla(\hat{\psi} - \psi) \cdot \frac{2(\hat{u}_i - u_i) \nabla(\hat{u}_i - u_i)}{(1 + \delta(\hat{u}_i - u_i)^2)^2} (\hat{\psi} - \psi) dx \end{aligned} \quad (4.17)$$

taking  $\delta \rightarrow 0$  in (4.17), we obtain the estimate

$$\begin{aligned} \int_G (\hat{u}_i - u_i)^2 |\nabla(\hat{\psi} - \psi)|^2 dx &\leq \frac{2}{a} \|(\hat{\psi} - \psi)(t)\|_\infty (\Sigma \|\hat{u}_i - u_i\|_3) \|\hat{u}_i - u_i\| \|\hat{u}_i - u_i\|_6 \\ &\quad + 4 \|(\hat{\psi} - \psi)(t)\|_\infty^2 \|\nabla(\hat{u}_i - u_i)\|^2 \leq C (\Sigma \|\hat{u}_i - u_i\|^2) (\Sigma \|\nabla(\hat{u}_i - u_i)\|^2) \end{aligned} \quad (4.18)$$

Using (4.18) and (4.10), we obtain

$$E_7 \leq \varepsilon \Sigma \int_{Q_t} |\nabla(\hat{u}_i - u_i)|^2 dx d\tau + K_{11} \Sigma \int_0^t \|(|J_1| + |J_2|)\|^2 \|(\hat{u}_i - u_i)(\tau)\|^2 d\tau \quad (4.19)$$

From (4.4)–(4.10), (4.12), (4.14), (4.16), (4.19), we conclude

$$\Sigma \|(\hat{u}_i - u_i)(t)\|^2 \leq \int_0^t h(\tau) (\Sigma \|(\hat{u}_i - u_i)(\tau)\|^2) d\tau \quad (4.20)$$

where  $h(t) \in L^1_+(0, T)$ . In view of the Gronwall's inequality, we get  $\Sigma \|(\hat{u}_i - u_i)(t)\|^2 = 0$ , i.e.,  $\hat{u}_i = u_i$ . By (4.3), this implies that  $\hat{\psi} = \psi$ . Thus, there is at most one solution to the semiconductor equations.

We complete the proof of Theorem 4.1.

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