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Cooperative Sensing and Distributed Control of a Diffusion Process Using Centroidal Voronoi Tessellations

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Abstract. This paper considers how to use a group of robots to sense and control a diffusion process. The diffusion process is modeled by a partial differential equation (PDE), which is a both spatially and temporally variant system. The robots can serve as mobile sensors, actuators, or both. Centroidal Voronoi Tessellations based coverage control algorithm is proposed for the cooperative sensing task. For the diffusion control problem, this paper considers spraying control via a group of networked mobile robots equipped with chemical neutralizers, known as smart mobile sprayers or actuators, in a domain of interest having static mesh sensor network for concentration sensing. This paper also introduces the information sharing and consensus strategy when using centroidal Voronoi tessellations algorithm to control a diffusion process. The information is shared not only on where to spray but also on how much to spray among the mobile actuators. Benefits from using CVT and information consensus seeking for sensing and control of a diffusion process are demonstrated in simulation results.

AMS subject classifications: 94C15, 70E60

Key words: Consensus, centroidal Voronoi tessellations, diffusion process, distributed control, mobile actuator and sensor networks.

1. Introduction

Diffusion processes like chemical/radiation leaks, oil spills etc. can have a large impact on human health and natural environment. Nowadays, technological advances in networking and MEMS (Micro-Eletro-Mechanical Systems) make it possible to employ a large number of mobile/static sensors/actuators to observe the diffusion, locate the source, and even counter-react with the harmful pollutants when the mobile spray network is used. In the past decade, many researchers looked into this topic. A swarm of mobile robots

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*Corresponding author. *Email addresses:* `haiyang.chao@aggiemail.usu.edu` (H. Chao), `yangquan.chen@usu.edu` (Y. Q. Chen) are used to detect chemical plume source with gradient climbing [1]; a moving diffusion source can be identified based on the parameter estimation algorithm [2]; boundary estimation and following problem is considered [3]. However, only the source information is not enough for controlling a diffusion process. Centroidal Voronoi tessellations are introduced in coverage control of a static gradient field with mobile sensor networks [4–6] and extended to a diffusing and spaying scenario [7].

Actually, the monitoring and control of a diffusion process can be viewed as an optimal sensor/actuator placement problem in a distributed system [8]. Basically, a series of desired actuator positions are generated based on centroidal Voronoi tessellations and later integrated with PID controllers for neutralizing control based on Voronoi partitions. CVT algorithm provides a non-model-based method for coverage control and diffusion control using groups of vehicles. The CVT algorithm is robust and scalable [9, 10] and it can guarantee the groups asymptotically converging to the affected area even in multiple/mobile sources application [4].

Consensus is a common agreement reached by a group as a whole. The consensus can be made on robot formation, source location tracking, task assignment, and traffic control [11, 12, 14]. Although a group of mobile actuators are used for the diffusion control [7], the communication and information aspects are not taken care of. The mobile actuator only negotiates with its neighboring sensors, not neighboring actuators/sprayers, on how much to spray and where to go. As will be known in this paper, the information sharing and interaction among neighboring actuators/sprayers in a group can have a large impact on the coordinated movements of these actuators and the resulted control performance consequently. Since the actuators are sent out for the same task, consensus is needed on both where to spray and how much to spray. The mobile actuators need to get close to the polluted area but it is not efficient to cluster, or running together densely. On the other hand, the neutralizer spraying should also be balanced since the best energy saving way is to maximize the neutralizing ability of every actuator. A new consensus algorithm is introduced and integrated into the CVT algorithm to guarantee the actuator group to converge faster towards the affected area with an improved control performance.

The remaining part of this paper is organized as follows. In Section 2, the diffusion process is modeled by a PDE equation and the diffusion control problem is formulated. In Section 3, centroidal Voronoi tessellations based optimal actuator location algorithm is briefly introduced. Section 4 is devoted to introducing the information consensus into the CVT based optimal actuator location algorithm. Simulation results and comparisons with our previous CVT algorithm are presented Section 5. Finally, conclusions and future research directions are given in Section 6.

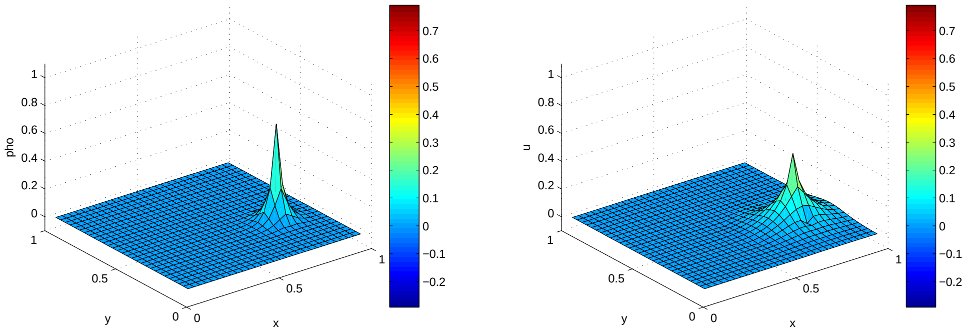
2. Mathematical modeling and problem formulation

In this section, the PDE mathematical model of a diffusion process is introduced and the neutralizing control problem is then formulated.

Suppose a diffusion process evolves in a convex polytope Ω: $\Omega \in \mathscr{R}^2$. $\rho(x, y) : \Omega \rightarrow \mathscr{R}_+$ is used to represent the pollutant concentration over Ω. The dynamic process can be modeled with the following partial differential equation (PDE):

$$\frac{\partial \rho}{\partial t} = k\left(\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2}\right) + f_d(x,y,t) + f_c(\tilde{\rho},x,y,t), \tag{2.1}$$

where k is a positive constant representing the diffusing rate; $f_d(x,y,t)$ shows the pollution source; $\tilde{\rho}$ is the measured sensor data; $f_c(\tilde{\rho},x,y,t)$ is the control input applied to the system which represents the effect of neutralizing chemical sent out by mobile actuators to counter-act the pollutants.



(a) $t=0.04$ (b) $t=0.2$

Figure 1: Surf plot of a diffusion process modeled by (2.1): $k=0.01$, $f_c=0$, $f_d=f_d(0.75,0.35,20e^{-t})$.

Assume n mobile actuators are sent to the field $f_c = f_{c_1} + \cdots + f_{c_n}$. $P = (p_1,\cdots,p_n)$ represent the locations of n actuators and $|\cdot|$ is the Euclidean distance. n actuators partition Ω into a collection of n Voronoi Diagrams $\mathscr{V} = \{V_1,\cdots,V_n\}$, $p_i \in V_i$, $V_i \cap V_j = \emptyset$ for $i \neq j$:

$$V_i = \{q \in \Omega \,|\, |q - z_i| < |q - z_j| \text{ for } j = 1,\cdots,n, j \neq i\}. \tag{2.2}$$

The control objectives are: (i) Control the diffusion of the pollution to a limited area; (ii) Neutralize the pollution as quickly as possible without making the area of interest overdosed.

To achieve the above requirements, the following evaluation equation needs to be minimized [4,7]:

$$\begin{aligned} &\min \ \mathscr{K}(P,\mathscr{V}) = \sum_{i=1}^{n} \int_{V_i} \rho(q)|q-p_i|^2 \mathrm{d}q \qquad \text{for } q \in \Omega, \\ &s.t. \quad |\dot{p}_i| < k_v, \quad |\ddot{p}_i| < k_a, \quad \sum_{i=1}^{n} \int u_{spray_i}(t)\mathrm{d}t < k_s, \end{aligned} \tag{2.3}$$

where $\dot{p}_i$ $\ddot{p}_i$ represent the first and second order dynamics of the actuator and $u_{spray_i}(t)$ is the neutralizing control input of the actuator i at time t.

Define the mass and centroid of region V_i as

$$M_{V_i} = \int_{V_i} \rho(q)\mathrm{d}q, \quad \bar{p}_i = \frac{\int_{V_i} q\rho(q)\mathrm{d}q}{\int_{V_i} \rho(q)\mathrm{d}q}.$$

To minimize $\mathscr{K}$, the distance $|q - p_i|$ should be small when the pollution concentration $\rho(q)$ is high. But it is not a wise strategy to drive all actuators very close to the pollution source, because the diffused pollutants far away from the source need also be neutralized quickly to minimize (2.3). A necessary condition to minimize $\mathscr{K}$ for coverage control in a static gradient field is that $\{p_i, V_i\}_{i=1}^n$ is a centroidal Voronoi tessellation of Ω [4].

$$\frac{\partial \mathscr{K}}{\partial p_i} = 2M_{V_i}(p_i - \bar{p}_i). \tag{2.4}$$

The CVT algorithm is further extended to a dynamical diffusion process [7]. It is based on a discrete version of (2.3) and the concentration information comes from the measurements of the static, low-cost mesh sensors. The diffusion control problem is converted to two subproblems: location optimization (where to go for actuators) and neutralizing control (how much to spray).

3. CVT-based dynamical actuator motion scheduling algorithm

In this section, CVT-based actuator motion planning algorithm is discussed in details.

The classic Lloyd's algorithm [6, 15] is an iterative algorithm to generate a centroidal Voronoi diagram from any set of generating points. It is modified to achieve coverage control [4] and diffusion control [7].

3.1. Motion planning for actuators with the first order dynamics

Assume that the sensors can be modeled by a first-order dynamical equation:

$$\dot{p}_i = u_i. \tag{3.1}$$

To minimize $\mathscr{K}$ in (2.3), the control input is set to be:

$$u_i = -k_p(p_i - \bar{p}_i), \tag{3.2}$$

where k_p is a positive gain and $\bar{p}_i$ is the mass centroid of V_i , and $\bar{p}_i$ is time-variant with diffusing.

3.2. Motion planning for actuators with the second order dynamics

If the second-order dynamical sensor model is used, similarly we have:

$$\ddot{p}_i = u_i. \tag{3.3}$$

To minimize $\mathscr{K}$ in (2.3), the control input is set to be:

$$u_i = -k_p(p_i - \bar{p}_i) - k_d\dot{p}_i, \tag{3.4}$$

where both k_p and k_d are positive constants.

The latter part of (3.4) $k_d\dot{p}_i$ is the viscous friction introduced [16], where k_d is the friction coefficient and $\dot{p}_i$ represents the velocity of the robot i. This part is used for eliminating the oscillatory behavior of robots [17] when the robot gets close to its destination. The viscous term guarantees the robot coming to a standstill final state even with no external force.

3.3. Neutralizing control

Proportional control is used for the neutralizing chemical releasing. The amount of chemicals each robot releases is proportional to the average pollutant concentration in the Voronoi cell belonging to that robot:

$$u_{spray_i}(t) = -k_{pr}\frac{\int_{\bar{V}_i} \rho(x,y)\mathrm{d}V}{\int_{\bar{V}_i} \mathrm{d}V}, \tag{3.5}$$

where $\bar{V_i} = V_i \cap C_i$, $C_i = \{q||q - p_i| < r_i\}$, r_i represents the sensing range of ith actuator and V_i is the Voronoi diagram of actuator i.

4. Information consensus in CVT-based diffusion control

In this section, we introduce information consensus and sharing to the CVT-based diffusion control. The control goal is to drive the actuators to the affected area and counteract the pollutants as quickly as possible.

4.1. Basic consensus algorithm

First we review the first-order consensus algorithms [11, 12, 14]. Let $p_i \in R^m$ be the information states of the i^{th} robot. For robots with single integrator dynamics given by

$$\dot{p}_i = u_i, \quad i = 1, \cdots, n, \tag{4.1}$$

where $u_i \in R^m$ is the control input, the following first-order consensus algorithm can be applied:

$$u_i = -\sum_{j=1}^{n} g_{ij}k_{ij}(p_i - p_j), \quad i = 1, \cdots, n, \tag{4.2}$$

where g_{ij} represents the set of robots whose information is available to robot i at time t, and k_{ij} is a positive weighting factor.

For the above consensus algorithm, consensus is said to be reached asymptotically among the n vehicles if $p_i(t) \to p_j(t)$, $\forall i \neq j$, as $t \to \infty$ for all $p_i(0)$. A classic rendezvous result is that the rendezvous state can be achieved if the information exchange graph has a spanning tree.

4.2. Requirements of diffusion control

The pollutant diffusion is both a temporal and a spatial evolution process. CVT method provides a spatial solution to partition the area into small Voronoi diagram and a final state of centroidal Voronoi tessellation can be achieved based on different weighted functions. However, the temporal characteristics is also a big challenge for extending CVT to dynamic diffusion control. There are several challenges to incorporate consensus with CVT-based diffusion control:

1. Converging Speed: To achieve a better control performance, the actuators should converge quickly to the affected area. But all actuators cannot detect the diffusion simultaneously due to the sensing limits. So, the consensus on the affected area needs to be introduced in such a way that the actuators far away from the diffusion source should move faster towards the area with high concentration.

2. Neutralizing Speed: The final control performance depends highly on how much and where the neutralizing materials are sprayed out. The total amount of the neutralizing material should be minimized given some final constrains on how much to spray totally.

3. Final State: CVT algorithm (3.2) or (3.4) can guarantee the actuator asymptotically converge to the diffusion source and form a centroidal Voronoi tessellation. But this is not enough for diffusion control since a diffusion process evolves with time.

4.3. Consensus-based CVT algorithm

Based on the above discussions, the new algorithm is proposed for the control of a diffusion process. Consensus algorithm is added on two parts: actuator motion control and actuator spraying/neutralizing control. The Consensus-based CVT algorithm is described as below:

1. Initial setting: actuator $p_i \in \{p_1, \cdots, p_n\}$, response time $t = 0$, concentration threshold k_a.
2. Compute Voronoi region $\bar{V}_i$.
3. Get the sensor data within the range r_s and compute centroid $\bar{p}_i$ and total pollutant in this region P_{total_i}.
4. Talk with neighboring actuators. If no diffusion ($\forall i, P_{total_i} < k_a$), go to 5); else apply corresponding control laws:
    (a) If actuator p_i is out of the affected region ($P_{total_i} < k_a$), make a consensus with neighbors on where is the affected area.
    (b) If actuator p_i is within the affected region ($P_{total_i} > k_a$), make a consensus with neighbors on how fast to spray.
    (c) Else, use CVT control law (3.2) or (3.4).
5. Stop since no pollution detected

In what follows, we will explain in detail on the two consensus algorithms for motion control and spraying control.

4.3.1. Consensus in actuator motion control

In the diffusion process, the actuators sense and react to the diffusion according to the distance from the source. Consensus is introduced here for faster converging speed. First, the affected area is defined as

$$A_j = \left\{q \in \Omega \middle| \ \rho(q) > k_a\right\} = \left\{q \in \Omega \middle| \ |q - d_j| < r_j(t)\right\}, \tag{4.3}$$

where d_j is the position of the jth diffusion source, k_a is a positive constant representing the concentration threshold, $r_j(t)$ represents the radius of the affected area. Here we assume there is no wind or other reasons affecting the diffusion process. The consensus to the affected area turns out to be a multi-leaders consensus problem. That is, the actuators out of affected area will follow the actuators already in the affected area.In other words, the diffusion-undetected actuators will follow the diffusion-detected actuators or rendezvous to them until they enter the affected area A_i. The difference with the common "Rendezvous Problem" is that here we want to rendezvous to an affected area instead of one point. This can be achieved with disconnected communication topology as in [14]:

$$u_i = -\sum_{j=1}^{n} g_{ij} k_{ij} (p_i - p_j), \quad i = 1, \cdots, n, \tag{4.4}$$

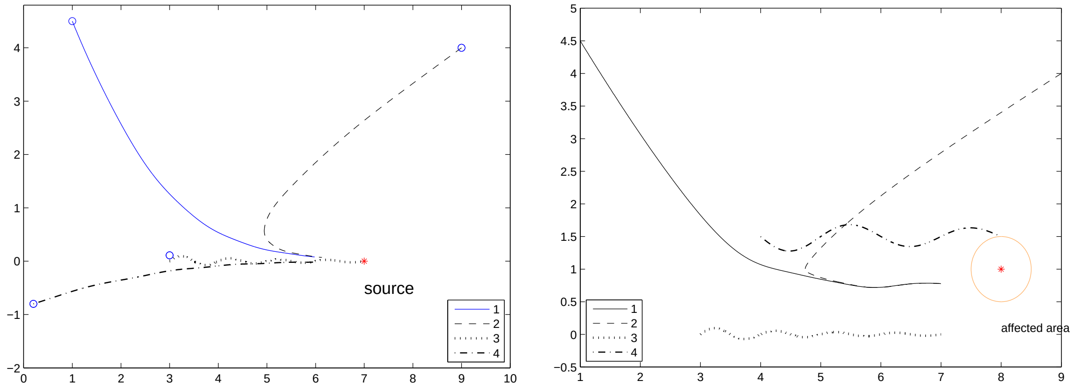
where $k_{ij} > 0$, $g_{ij} = 0$ and g_{ij} will be set to 1 if information flows from actuator j to i. In our case, it is mostly leader-follower case. The followers just need to rendezvous to the leaders which are already in the affected area.

Assuming that actuator j is out of the affected area at time t_d, we want to minimize $\mathscr{K}$

$$\begin{aligned} \frac{\partial \mathscr{K}}{\partial p_j} &= 2M_{V_i}(p_i - \bar{p_i}) \simeq 0, \\ M_{V_i} &\simeq 0. \end{aligned} \tag{4.5}$$

Based on plain CVT actuator motion planning, the actuator j will not react until $|p_i - \bar{p_i}| > \delta$. But the consensus algorithm introduces the information sharing among actuator so that the actuator out of affected area can react early and achieve a faster converging speed.

We set up an emulated scenario to show our idea. Suppose only one actuator (actuator #3) is close to the diffusion source and detect the diffusion very early Fig. 2(a). With CVT algorithm, the actuator #3 can drive to the affected area asymptotically. However, other actuators will not react to the diffusion quickly enough since it takes time for the pollutant to enter the area close to other actuators. With consensus algorithm, the actuator #3 can broadcast to the other actuators, or act as the leader of the group and lead all the others into the affected area. In Fig. 2(b), there are two actuators (#1, #4) which are close to the affected area. So, they will respond to both of the early arrivers and converge to the middle of actuator #1 and #4, which is also the affected area that needs to be controlled or sprayed. With this algorithm, consensus can be reached asymptotically for the n actuators since $p_i - d_j \to r_j(t)$, as $t \to \infty$ for all p_i.

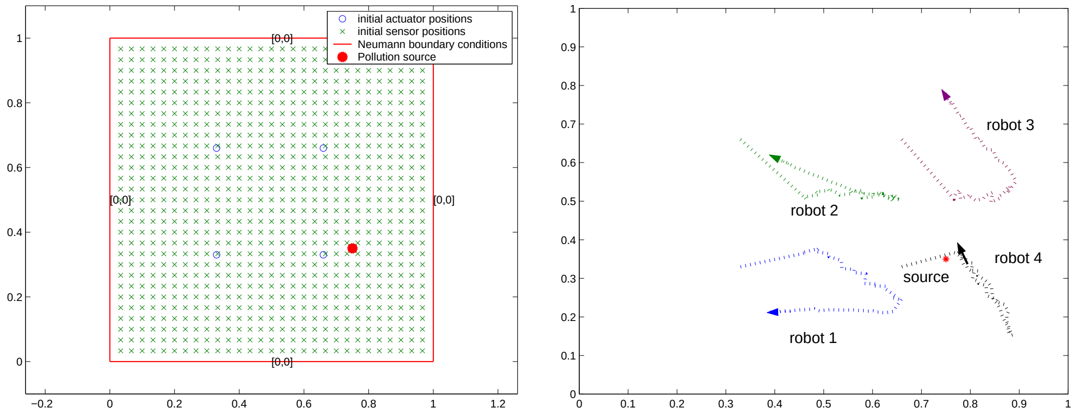


(a) (b)

Figure 2: Simulation: rendezvous to the affected area. (a): 1 leader (#3) and 3 followers (#1,#2, #4), (b): 2 leaders (#3, #4) and 2 followers (#1, #2.)

4.3.2. Consensus in actuator neutralizing control

The plain CVT algorithm in [7] introduces a spatial solution to the diffusion control problem. However, the neutralizing control part may not balance. Given a typical pollution/spraying control scenario using the plain CVT algorithm Fig. 3, we can observe from Fig. 4 that the actuator #4 sprays more neutralizing chemicals than the total sprayed by the other three, which is not an efficient way when employing more actuators.



(a) (b)

Figure 3: Plain CVT diffusion control. (a): Initial positions of actuators, (b): Trajectories of actuators.

In our present study, consensus is introduced to neutralizing control for maximizing the neutralizing ability of every actuator [13]. Consensus is said to be reached for the n actuators if u_{pr_i} is at the same order of magnitude or as close as possible, $\forall i \neq j$, as $t \to \infty$. CVT algorithm (3.2) or (3.4) can guarantee the actuator to converge to a final centroidal Voronoi tessellation as $t \to \infty$, but that is a scenario that can not happen in the

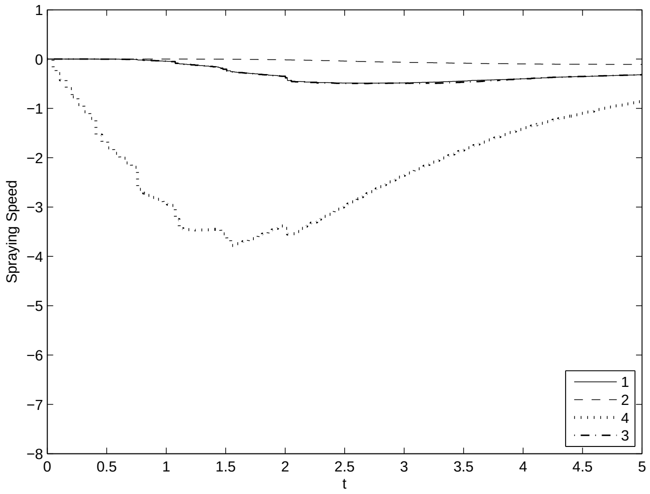


Figure 4: Spraying speed comparison for CVT.

diffusion evolving scenario. To achieve a better control performance, every actuator should be fully used in the neutralizing control. We wish to use the proposed consensus algorithm to avoid the situation that we could not send as many as possible mobile actuators to the most affected area.

To achieve this, the following spraying control input can be applied

$$u_i = -k_p(p_i - \bar{p}_i) - \sum_{j=1}^{N} g_{ij} k_{ij}(p_i - p_j), \tag{4.6}$$

where g_{ij} and k_{ij} have the same definitions as in (4.2). The first part $p_i - \bar{p}_i$ drives the actuator respond to the diffusing and the later part in (4.6) will drive the actuators closer to the actuator that has the highest P_{total_i}.

5. Simulation results

Two simulation examples are shown to demonstrate the effectiveness of the new algorithm. The first one has no constrain limits on how much to spray totally $k_s = \infty$. The second one illustrate how this constrains will affect the final control performance.

`Diff-MAS2D` [18] is used as the simulation platform for our implementation. The area concerned can be modeled by $\Omega = \{(x, y)|0 \le x \le 1, 0 \le y \le 1\}$. In (2.1) $k = 0.01$ and the boundary condition is given by

$$\frac{\partial u}{\partial n} = 0.$$

The stationary pollution source is modeled as a point disturbance f_d to the PDE system

(2.1) with its position at $(0.8, 0.2)$ and

$$f_d(t) = 20e^{-t}\Big|_{(x=0.8,y=0.2)}.$$

The mesh sensor network is assumed to provide the actuators with measurements on pollutant concentration. There are 29×29 sensors evenly distributed in a square area $(0,1)^2$ (a unit area) and four mobile actuators/robots that can release the neutralizing chemicals. The pollution source begins to diffuse at $t = 0$ to the area Ω and initially the mobile actuator robots are evenly distributed within the domain Ω (one by one square) at the following specific positions: for 2×2 grouping case, $(0.33, 0.33), (0.33, 0.66), (0.66, 0.33), (0.66, 0.66)$. The actuators and sensors get updates every $0.1s$. The dynamic model of actuator is assumed to be the first order. We will add more simulation results for the second order model in the final version.

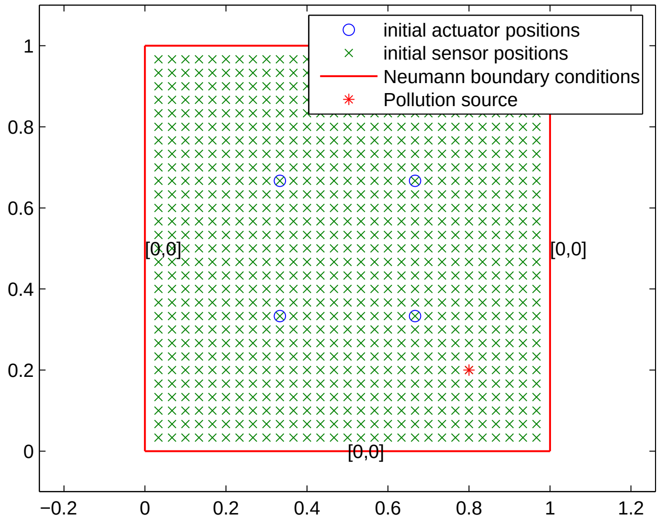


Figure 5: Initial layout of diffusion control.

Given the initial layout Fig. 5, we need to choose the corresponding control law and communication matrix. Let us consider the vector form of control input:

$$U = L_1 P - L_2 \bar{P}, \tag{5.1}$$

where $U = [u_1^T \cdots u_n^T]$, $P = [p_1^T \cdots p_n^T]$, $\bar{P} = [\bar{p_1}^T \cdots \bar{p_n}^T]$ are all vectors, L_1 is the control matrix determined by communication topology and corresponding control law.

In the beginning, the actuator #3 is relatively close to the diffusion process, and it will detect and react to the diffusing first. Then, it will broadcast this event to all the other three actuators. The communication topology shown in Fig. 6(a) and control matrixes L_1

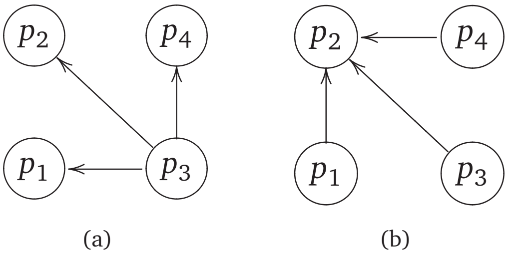


Figure 6: (a): p_3 alone broadcastsrajectory; (b): $p1, p_3$ and p_4 broadcast.

and L_2 are shown below:

$$\mathbf{L}_1 = \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \mathbf{L}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

After a certain time, actuator #1 and #4 also enter the affected area. The communication topology and control matrix are then changed:

$$\mathbf{L}_1 = \begin{pmatrix} -1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \mathbf{L}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

After all the four actuators have entered the affected area, the S_{total_i} are compared and converted to step 4c) for consensus on the amounts of neutralizing chemicals. The actuator trajectories are shown in Fig. 7.

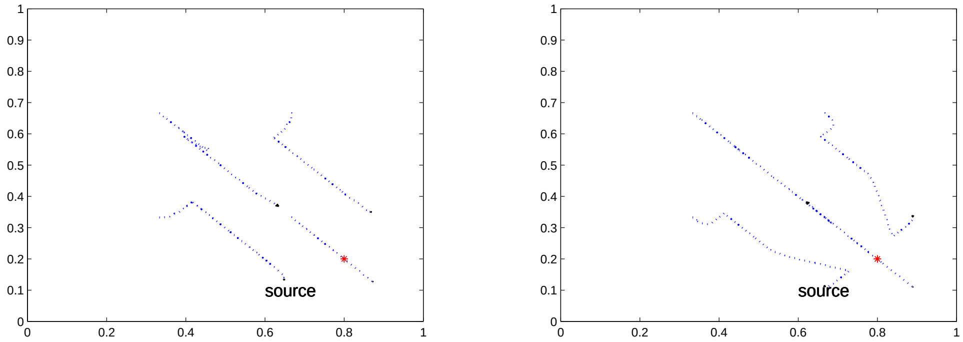
$$\mathbf{L}_1 = \begin{pmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad \mathbf{L}_2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Fig. 8 and Table 1 show the control performance comparison between plain CVT and consensus-based CVT, which shows a decrease in both the max and final total pollution value. The time actuators takes to arrive at the affected area can be compared in Fig. 9. Consensus-based CVT has a better control performance on the diffusion process over the plain CVT.

Table 1: Comparison of control performance.

Algorithm	P_{max}	t_{max}	P_{final}
CVT	12.9186	1.7980	1.9330
ConsensusCVT	12.7850	1.7420	1.5743
CVT (Spray Limits)	10.3318	2.3080	4.6901
Consensus (Spray limits)	12.7850	1.7420	2.9365

When controlling a diffusion process, another important factor is the constrains on the total neutralizing chemical sprayed (2.3). To make a comparison between consensus-based



(a) Trajectory of CVT (b) Trajectory of ConsensusCVT

Figure 7: Trajectory comparison between consensus-based CVT and plain CVT.

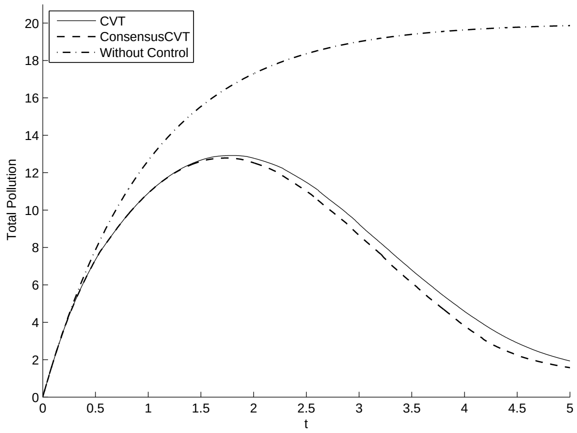
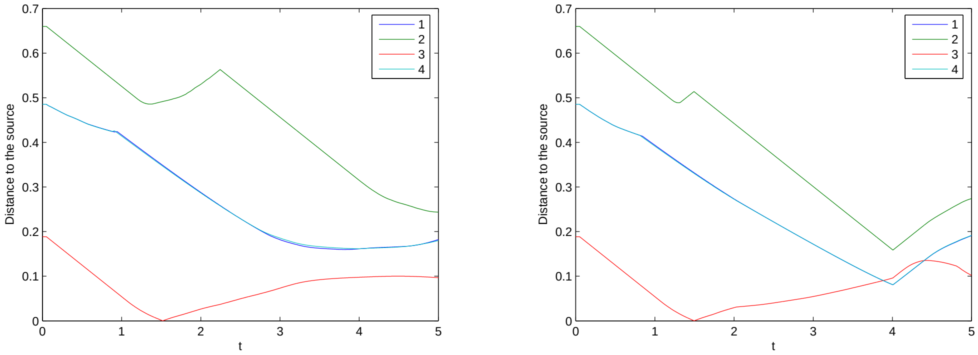


Figure 8: Comparison of total pollutants: plain CVT and consensus-CVT.



(a) Plain CVT (b) Consensus-CVT

Figure 9: Distance to the source.

CVT and the plain CVT, the total neutralizing amount is reduced to 70% of the preceding case. For consensus-based CVT, an saturation $[-2, 0]$ is added to guarantee the balance of spraying speed among actuators. The initial layout and all parameters are the same with the above simulation. The motion trajectories are shown in Fig. 10.

From Fig. 11 and Table 1, we can observe that although the maximal total pollutant is smaller, the final pollutant left using plain CVT is 4.6901, which is much more than that achieved via the consensus based CVT as low as 2.9365. So, this strategy is not so good because it does not make fully use of the neutralizing ability of all the 4 actuators.

In summary, the diffusion control problem is quite difficult because it evolves both spatially and temporally and PDEs are needed for modeling. There is still no good solution. Based on the presented simulation results, the following further discussions are presented in order:

1. Mobile Actuator Control Problem: One of the difficulties in diffusion control is that both actuator position and neutralizing speed need to be controlled. Especially, the neutralizing control strategy can have a large impact on the final control performance. Different control laws can be designed for various requirements. As shown in Figs. 11 and 12, CVT algorithm has smaller maximum pollutant values (see Table 2) but quickly sprays out the total neutralizing chemicals. Consensus CVT outperforms CVT in this aspect because it pays more attention to inter-actuator communication and tries to maintain a balance of neutralizer amount among actuators.

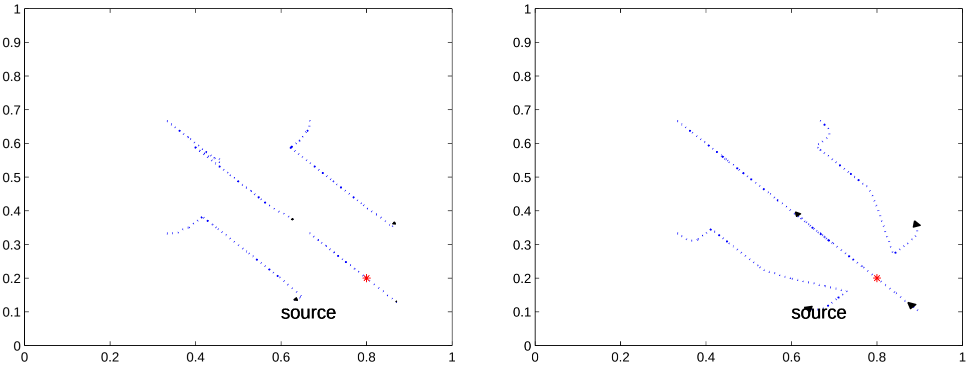
Table 2: Comparison of total neutralizing material.

Algorithm	S_1	S_2	S_3	S_4
CVT	4.25	0.53	9.01	4.18
ConsensusCVT	4.47	0.69	8.75	4.42
CVT (Spray limits)	3.89	0.31	7.01	4.02
Consensus (Spray limits)	4.67	0.70	7.00	4.61

2. CVT Advantages and Limitations: CVT algorithm is a non-model based method to control a diffusion process and it is easy to implement in a large-scale since it needs only the neighbor information. The diffusion source can be moving and can be multiple. However, CVT can only guarantee the slow converging to the source, as seen in Fig. 9. The final diffusion control performance depends a lot on the initial conditions like the starting points of actuators. The converging speed and computation burden are also limitations for CVT [5].

3. Communication Topology: this paper assumes that the actuator can get the sensor information within a certain distance of effectiveness and a full communication topology among actuators. But the simulation result is only based on some specific communication topologies. Further tests are needed for topology changing or switching while actuator moving and spraying.

4. 2D/3D Spatial Problem: CVT algorithm is a spatial solution to the diffusion control problem. With the availability of small and powerful robots and sensor network, these



(a) Plain CVT (b) Consensus-CVT

Figure 10: Actuator trajectories of consensus-based CVT and plain CVT.

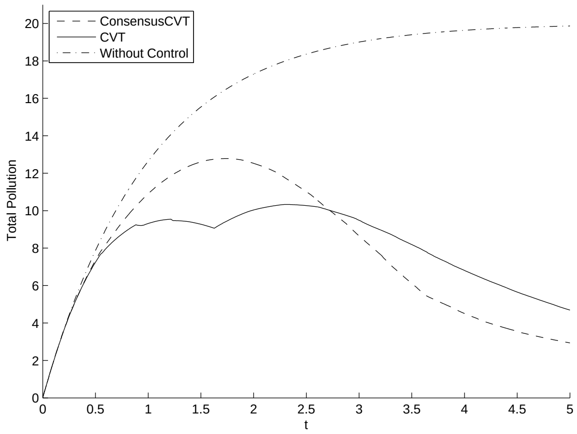
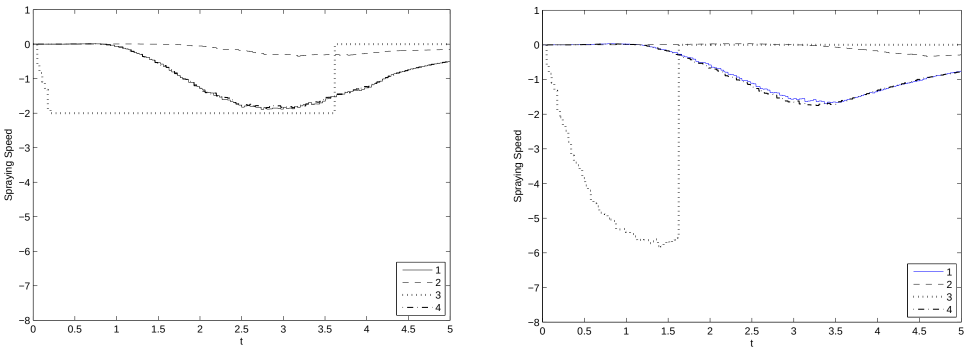


Figure 11: Comparison of total pollutants: plain CVT and consensus-CVT.



(a) Plain CVT (b) Consensus-CVT

Figure 12: Comparison of spraying speeds.

kind of spatial problem will sooner or later be solved. Unmanned aerial vehicles (UAVs) will be perfect platforms for this kind of experiments [19].

5. Experimental validation: more experiments on real mobile sensor and actuator networks will be interesting to validate the ideas in this paper. Similar works are undergoing [20].

6. Conclusion

In this paper, we propose to incorporate the information sharing and consensus strategy to the Centroidal Voronoi Tessellation based actuators motion planning for better control of a diffusing process. The new algorithm is tested with a first order dynamic model and its improvement has been demonstrated, especially under total spraying amount limit.

Further simulation results and comparisons should be made in the future using a second order actuator model. We will also further investigate the converging speed of Consensus-CVT and provide a universal proof for real applications and extend our research for pollution feedback control by using mobile sensors and take into account the sensor noise and unreliable communication induced uncertainties.

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