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Transient Waves Due to Mechanical Loads in Elasto-Thermo-Diffusive Solids

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Abstract. This paper deals with the study of transient waves in a homogeneous isotropic, solid halfspace with a permeating substance in the context of the theory of generalized elasto-thermodiffusion. The halfspace is assumed to be disturbed due to mechanical loads acting on its boundary. The model comprising of basic governing differential equations and boundary conditions has been solved by employing Laplace transform technique. Noting that the second sound effects are short lived, the small time approximations of solution for various physical quantities have been obtained and the results are discussed on the possible wave fronts. In case of continuous and periodic loads acting at the boundary, the displacement is found to be continuous at each wave front while it is discontinuous in case of impulsive load. The temperature and concentration fields are found to be discontinuous at all the wave fronts. The displacement, temperature change and concentration deviation due to impulsive, continuous and periodic mechanical loads have also been evaluated in the physical domain at all times by employing numerical inversion technique of integral transform. The computer simulated numerical results have been presented graphically in respect of displacement, temperature change and concentration deviation for brass. A significant effect of mass diffusion has been observed on the behaviour of mechanical and thermal waves.

AMS subject classifications: 76R50, 44A10, 34A45, 35L05

Key words: Diffusion, Laplace transform, small time approximations, wave fronts.

1 Introduction

In integrated circuit fabrication, the diffusion is used to introduce dopants in controlled amount into semiconductor substrate. It is used to form base and emitter in

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bipolar transistors, integrated resistors and source/drain in metal oxide semiconductor (MOS) transistor. Thermal diffusion utilizes transfer of heat across a thin liquid or gas to accomplish the process of isotope separation. Mechanical load allows us to estimate the strength of materials because it indicates where the limit of strength properties lie and can be taken to improve the practical properties. The phenomena find application in geophysics as most part of earth is solid and at elevated temperature atomic diffusion occur in many chemical and physical processes. Keeping in view the wide use of materials under high temperature in microelectronic industry, nuclear reactors, etc. the theoretical study of elasto-thermodiffusive material is an important task in solid mechanics.

The theory of thermoelasticity deals with effect of mechanical and thermal disturbance in an elastic body. Duhamel [1] and Neumann [2] introduced the theory of uncoupled thermoelasticity which inherited two defects namely (i) the mechanical state of elastic body have no effect on temperature and (ii) the heat equation being parabolic that predicts infinite speed of wave propagation which again contradict the physical facts. Biot [3] developed the coupled theory of thermoelasticity to eliminate the paradox inherent in classical uncoupled theory that elastic changes have no effect on the temperature and the heat equation, however, is of diffusion type predicting infinite speed of propagation. To account for the finite speed of wave propagation Lord and Shulman [4] introduced the theory of generalized thermoelasticity with one relaxation time in classical Fourier law of heat conduction ensuring finite speed of wave propagation of heat and elastic waves. The governing equation of motion and constitutive relations remain the same as those for coupled and uncoupled theories. This fact was further supported by the theory given by Green and Lindsay [5], who developed a temperature rate dependent thermoelasticity with entropy inequality of Green and Laws [6] by including temperature rate among the constitutive variables, without violating the classical Fourier law when body under consideration has a center of symmetry and it allows heat wave to travel with finite speed.

Nowacki [7–10] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [11], Olesiak and Pyryev [12] respectively discussed the theory of thermodiffusion and influence of cross effect arising due to coupling in the field of temperature, mass diffusion and strain in an elastic cylinder. Sherief et al. [13] derived the basic governing equations namely, equations of motion, heat conduction and mass diffusion for generalized elasto-thermodiffusive solid. Danilovskaya [14, 15] treated the problems of thermal shock on the surface of halfspace for the first time and obtained its analytical solution in dynamic uncoupled thermoelasticity. Sherief et al. [16] has studied problem of halfspace whose surface is rigidly fixed and subjected to effects of thermal shock in the context of generalized theory of thermoelasticity. Sherief et al. [17] has also studied halfspace problem in theory of generalized thermoelastic diffusion when surface of the halfspace is assumed traction free and subjected to time dependent thermal shock. Singh [18, 19] has investigated reflection of P and SV waves from free surface of elastic solid in context of generalized thermoelastic diffusion. Sharma [20] considered propagation of plane harmonic waves and studied their behaviour both at low and high frequency. Sharma et al. [21] studied the propagation of surface waves in a generalized elasto-thermodiffusive halfspace. Sharma et al. [22] have also obtained a general solution to the field equations of homogeneous isotropic, generalized thermoelastic diffusion with two relaxation times by using Fourier transform assuming that disturbance is harmonically time dependent.

As per knowledge of the authors, no systematic study of elasto-thermodiffusive waves in a one dimensional solid under loading environment; where thermal and mass concentration fields are allowed to change simultaneously is available in the literature. In order to explore the effects of various interacting fields on each other and keeping in view the applications of mass and thermal diffusion processes in industry, an attempt has been made to study the instant problem. The solutions for displacement, temperature change and mass concentration fields have been obtained in a generalized thermodiffusive solid subjected to impulsive, continuous and periodic loads by using Laplace transform technique. It has been noticed that the change of mass concentration field significantly affects the characteristics of mechanical and thermal waves in such materials. The model is solved numerically for brass and computer simulated results have been presented graphically.

2 Basic equations

The geometric and constitutive relations as well as basic governing equations for generalized thermodiffusive interaction in a homogeneous isotropic, elastic solid [13] are outlined below.

Strain-displacement relation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad i,j = 1,2,3, \tag{2.1}$$

where u_i are the components of displacement vector and e_{ij} is strain tensor.

Stress-Strain-Temperature-Concentration relations

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \beta_1 T\delta_{ij} - \beta_2 C\delta_{ij}, \tag{2.2a}$$

$$\rho T_0 S = \rho C_e T + \beta_1 T_0 e_{kk} + aT_0 C, \tag{2.2b}$$

$$P = -\beta_2 e_{kk} + bC - aT, \qquad i,j,k = 1,2,3, \tag{2.2c}$$

where

$$\beta_1 = (3\lambda + 2\mu)\alpha_T, \qquad \beta_2 = (3\lambda + 2\mu)\alpha_C,$$

are the material parameters. Here σ_{ij} is stress tensor, e_{kk} is dilatation; λ, μ are Lames parameters, α_T is coefficient of linear thermal expansion, α_C is coefficient of linear diffusion expansion, ρ is the density, C_e is the specific heat at constant strain, S is entropy per unit mass, a is thermodiffusive constant, b is diffusive constant, P is chemical potential per unit mass, T_0 is the temperature of the medium in its natural state,

$$T(x_i, t) = T_1(x_i, t) - T_0,$$

is the temperature change, such that

$$\Big|\frac{T_1 - T_0}{T_0}\Big| \ll 1,$$

and

$$C(x_i, t) = C_1(x_i, t) - C_0,$$

is the concentration change, C_0 being initial uniform concentration in the natural state and C_1 is the changed concentration. Eqs. (2.2a)-(2.2c) are also known as constitutive relations.

Equation of motion

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i} + \rho F_i = \rho \ddot{u}_i, \qquad i, j = 1, 2, 3. \tag{2.3}$$

Equation of heat conduction

$$K T_{,ii} - \rho C_e(\dot{T} + t_0 \ddot{T}) = \beta_1 T_0(\dot{e} + t_0 \ddot{e}) + a T_0(\dot{C} + t_0 \ddot{C}), \qquad i = 1, 2, 3, \tag{2.4}$$

where K is the thermal conductivity.

Equation of mass diffusion

$$C_{,ii} - \frac{1}{Db}(\dot{C} + t_1 \ddot{C}) = \frac{\beta_2}{b} e_{,ii} + \frac{a}{b} T_{,ii}, \qquad i = 1, 2, 3, \tag{2.5}$$

t_0, t_1 are thermal relaxation time parameters.

We assume that material parameters satisfy the inequalities

$$K > 0, \qquad D > 0, \qquad \lambda > 0, \qquad \rho > 0, \qquad C_e > 0, \tag{2.6a}$$
$$\mu > 0, \qquad T_0 > 0, \qquad t_0 > 0, \qquad t_1 > 0, \qquad C_0 > 0. \tag{2.6b}$$

3 Formulation of the problem

We shall consider a homogeneous isotropic, thermodiffusive, elastic solid occupying the halfspace at uniform initial temperature T_0 and initial concentration C_0 in the undisturbed state. We take any point on the surface of solid as origin of the Cartesian coordinate system O-xyz. Choose the plane perpendicular to x-direction. Hence from symmetry we have considered one dimensional case, in the x-direction where all physical quantities are functions of x and t. The surface of the halfspace is subjected to time dependent mechanical loads and also temperature as well as mass concentration are assumed to be known functions of time on it. All the considered functions are assumed to be bounded and vanish as $x\to\infty$. The basic governing Eqs. (2.3)-(2.5) of linear generalized elasto-thermodiffusion, in the absence of body forces and heat sources, become

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}-\beta_1\frac{\partial T}{\partial x}-\beta_2\frac{\partial C}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{3.1a}$$

$$K\frac{\partial^2 T}{\partial x^2}-\rho C_e\Big(\frac{\partial T}{\partial t}+t_0\frac{\partial T}{\partial t^2}\Big)=\beta_1 T_0\Big(\frac{\partial^2 u}{\partial x\partial t}+t_0\frac{\partial^3 u}{\partial x\partial t^2}\Big)+aT_0\Big(\frac{\partial C}{\partial t}+t_0\frac{\partial^2 C}{\partial t^2}\Big), \tag{3.1b}$$

$$\frac{\partial^2 C}{\partial x^2}-\frac{1}{Db}\Big(\frac{\partial C}{\partial t}+t_1\frac{\partial^2 C}{\partial t^2}\Big)=\frac{\beta_2}{b}\frac{\partial^3 u}{\partial x^3}+\frac{a}{b}\frac{\partial^2 T}{\partial x^2}, \tag{3.1c}$$

where

$$\vec{u}(x,t)=(u,0,0),$$

$T(x,t)$ and $C(x,t)$ are the displacement vector, temperature deviation and concentration change in the material respectively. To facilitate the solution we define the following dimensionless quantities

$$x'=\frac{\omega^* x}{c_L},\quad t'=\omega^* t,\quad u_i'=\frac{\rho\omega^* c_L u_i}{\beta_1 T_0},\quad T'=\frac{T}{T_0},\quad c_s^2=\frac{\mu}{\rho}, \tag{3.2a}$$

$$t_0'=\omega^* t_0,\quad t_1=\omega^* t_1,\quad \omega^*=\frac{C_e(\lambda+2\mu)}{K},\quad \varepsilon_T=\frac{T_0\beta_1^2}{\rho C_e(\lambda+2\mu)}, \tag{3.2b}$$

$$\bar{a}=\frac{aC_0}{\rho C_e},\quad \bar{\beta}=\frac{\beta_2 C_0}{\beta_1 T_0},\quad \bar{b}=\frac{aT_0}{bC_0},\quad \varepsilon_c=\frac{\beta_1\beta_2 T_0}{C_0 b(\lambda+2\mu)}, \tag{3.2c}$$

$$C'=\frac{C}{C_0},\quad \sigma_{xx}'=\frac{\sigma_{xx}}{\beta_1 T_0},\quad \bar{\omega}_b=\frac{c_l^2}{\omega^* Db},\quad c_L^2=\frac{(\lambda+2\mu)}{\rho}. \tag{3.2d}$$

Using non-dimensional quantities from Eq. (3.2) in Eqs. (3.1a)-(3.1c) and suppressing the primes for convenience, we obtain

$$u_{,xx}-T_{,x}-\bar{\beta}C_{,x}=\ddot{u}, \tag{3.3a}$$

$$T_{,xx}-(\dot{T}+t_0\ddot{T})-\varepsilon_T(\dot{u}_{,x}+t_0\ddot{u}_{,x})-\bar{a}(\dot{C}+t_0\ddot{C})=0, \tag{3.3b}$$

$$C_{,xx}-\bar{\omega}_b(\dot{C}+t_1\ddot{C})-\varepsilon_c u_{,xxx}-\bar{b}T_{,xx}=0. \tag{3.3c}$$

3.1 Initial regularity and boundary conditions

The halfspace is assumed to be undeformed and at rest as well as at uniform temperature T_0 initially. This leads to the initial and regularity conditions

$$u(x,0)=0=\dot{u}(x,0),\qquad T(x,0)=0=\dot{T}(x,0), \tag{3.4a}$$

$$C(x,0)=0=\dot{C}(x,0),\qquad \text{for all } x, \tag{3.4b}$$

$$u(x,t),T(x,t)C(x,t)\to 0,\quad \text{as } x\to\infty \text{ for all } t. \tag{3.4c}$$

The surface $x = 0$ of the halfspace is subjected to a time dependent mechanical load and hence, the non-dimensional boundary conditions assumed to be satisfied are

$$\sigma_{xx} = -\sigma_0 f(t), \quad T_{,x} + hT = 0, \quad C_{,x} + HC = 0, \tag{3.5}$$

where

$$\sigma'_{xx} = \frac{\sigma_{xx}}{\beta_1 T_0}, \qquad \sigma_{xx} = \frac{\partial u}{\partial x} - \bar{\beta} C - T,$$

is the non-dimensional stress (primes have been suppressed for convenience), $f(t)$ is a well behaved function of time, h and H are the surface heat and mass transfer coefficients respectively. Here $h \to 0$ refers to thermally insulated boundary and $h \to \infty$ corresponds to isothermal surface of the halfspace. Also $H \to 0$ pertains to impermeable surface and $H \to \infty$ refers to isoconcentrated boundary of the halfspace.

4 Formal solution of the problem

Applying the Laplace transform defined by

$$\bar{g}(x,p) = \int_0^\infty g(x,t) e^{-pt} dt, \tag{4.1}$$

wrt. time to Eqs. (3.3a)-(3.3c), we obtain

$$(D^2 - p^2)\bar{u} - D\bar{T} - \bar{\beta} D\bar{C} = 0, \tag{4.2a}$$
$$(D^2 - \tau_0 p^2)\bar{T} - \varepsilon_T \tau_0 p^2 D\bar{u} - \bar{a}\tau_0 p^2 \bar{C} = 0, \tag{4.2b}$$
$$(D^2 - \bar{\omega}_b p^2 \tau_1)\bar{C} - \varepsilon_c D^3 \bar{u} - \bar{b} D^2 \bar{T} = 0, \tag{4.2c}$$

where

$$\tau_0 = t_0 + p^{-1}, \qquad \tau_1 = t_1 + p^{-1}.$$

Solving the system of Eqs. (4.2) and using the regularity conditions (3.4c), we get

$$\bar{u}(x,p) = \sum_{i=1}^{3} B_i e^{-\lambda_i p x}, \tag{4.3a}$$

$$\bar{T} = \sum_{i=1}^{3} \frac{p}{\lambda_i} \bar{S}_i B_i e^{-\lambda_i p x}, \tag{4.3b}$$

$$\bar{C} = \sum_{i=1}^{3} p \lambda_i \bar{W}_i B_i e^{-\lambda_i p x}, \tag{4.3c}$$

where

$$\bar{W}_i = \frac{\bar{b}\{1 - \lambda_i^2(1+\varepsilon_a)\}}{\lambda_i^2(1+\bar{\beta}\bar{b}) - \bar{\omega}_b \tau_1}, \qquad \bar{S}_i = \{1 - \lambda_i^2(1 + \bar{\beta}\bar{W}_i)\}, \tag{4.4}$$

with $i = 1,2,3$. Here the quantities λ_i^2 ($i = 1,2,3$) are given by the cubic equation

$$\lambda^6 - p^2 A^* \lambda^4 + p^4 B^* \lambda^2 - p^6 C^* = 0, \tag{4.5}$$

where

$$\varepsilon_a = \frac{\varepsilon_C}{\bar{b}}, \tag{4.6a}$$

$$\sum \lambda_1^2 = A^* = \frac{1 + \bar{\omega}_b \tau_1 + \tau_0\{(1+\bar{a}\bar{b})(1+\varepsilon_a) + (1+\bar{\beta}\bar{b})(\varepsilon_T - \varepsilon_a)\}}{1 - \bar{\beta}\varepsilon_C}, \tag{4.6b}$$

$$\sum \lambda_1^2 \lambda_2^2 = B^* = \frac{\tau_0(1+\bar{a}\bar{b}) + \bar{\omega}_b \tau_1\{1 + t_0(1+\varepsilon_T)\}}{1 - \bar{\beta}\varepsilon_C}, \tag{4.6c}$$

$$\sum \lambda_1^2 \lambda_2^2 \lambda_3^2 = C^* = \frac{\bar{\omega}_b \tau_1 \tau_0}{1 - \bar{\beta}\varepsilon_C}. \tag{4.6d}$$

The Eq. (4.5) provides us three real roots for non zero values of t_0, t_1 correspond to three modes of wave propagation namely, thermal diffusive, mass diffusive and elastodiffusive mode. Upon applying integral transform to the boundary conditions (3.5) and using formal solutions (4.3) and solving the resulting equations, we obtain a system of three coupled Equations in three unknowns B_i ($i = 1,2,3$) as follows

$$B_1\lambda_2\lambda_3 + B_2\lambda_1\lambda_3 + B_3\lambda_1\lambda_2 = -\frac{\sigma_0 \bar{f}(p)}{p}\lambda_1\lambda_2\lambda_3, \tag{4.7a}$$

$$\bar{S}_1 B_1 \lambda_2\lambda_3 + \bar{S}_2\lambda_1\lambda_3 B_2 + \bar{S}_3\lambda_1\lambda_2 B_3 = 0, \tag{4.7b}$$

$$\bar{W}_1\lambda_1 B_1 + \bar{W}_2\lambda_2 B_2 + \bar{W}_3\lambda_3 B_3 = 0. \tag{4.7c}$$

Solving the above system of Eqs. (4.7), we obtain

$$B_i = \frac{\sigma_0 \bar{f}(p)\Delta_i}{p\Delta}, \qquad i = 1,2,3, \tag{4.8}$$

where

$$\Delta_1 = \lambda_1(\lambda_2^2 \bar{S}_3 \bar{W}_2 - \lambda_3^2 \bar{S}_2 \bar{W}_3), \tag{4.9a}$$

$$\Delta_2 = \lambda_2(\lambda_3^2 \bar{S}_1 \bar{W}_3 - \lambda_1^2 \bar{S}_3 \bar{W}_1), \tag{4.9b}$$

$$\Delta_3 = \lambda_3(\lambda_1^2 \bar{S}_2 \bar{W}_1 - \lambda_2^2 \bar{S}_1 \bar{W}_2), \tag{4.9c}$$

$$\Delta = \{\bar{W}_3\lambda_3^2(\bar{S}_2 - \bar{S}_1) + \bar{W}_1\lambda_1^2(\bar{S}_3 - \bar{S}_2) + \bar{W}_2\lambda_2^2(\bar{S}_1 - \bar{S}_3)\}. \tag{4.9d}$$

Here

$$\bar{f}(p) = \int_0^\infty f(t)e^{-pt}dt,$$

is the Laplace transformation of the function $f(t)$.

Substituting the values of B_i $(i=1,2,3)$ from (4.8) in Eqs. (4.3), we get

$$\bar{u}(x,p)=\sigma_0\bar{f}(p)\sum_{i=1}^{3}\frac{\Delta_i}{p\Delta}\exp(-\lambda_i px), \tag{4.10}$$

$$\bar{T}(x,p)=\sigma_0\bar{f}(p)\sum_{i=1}^{3}\frac{\bar{S}_i\Delta_i}{\lambda_i\Delta}\exp(-\lambda_i px), \tag{4.11}$$

$$\bar{C}(x,p)=\sigma_0\bar{f}(p)\sum_{i=1}^{3}\frac{\bar{W}_i\Delta_i\lambda_i}{\Delta}\exp(-\lambda_i px). \tag{4.12}$$

We take

$$f(t)=\begin{cases}\delta(t), & \text{for impulsive load,}\\ H(t), & \text{for continuous load,}\\ \cos\omega t, & \text{for periodic load.}\end{cases} \tag{4.13}$$

Taking Laplace transform of function (4.13), we obtain

$$\bar{f}(p)=\begin{cases}1, & \text{for impulsive load,}\\ \dfrac{1}{p}, & \text{for continuous load,}\\ \dfrac{p}{p^2+\omega^2}, & \text{for periodic load.}\end{cases} \tag{4.14}$$

4.1 Thermoelastic halfspace

In the absence of mass diffusion ($a=0=\beta_2\Rightarrow\varepsilon_C=0=\bar{b}$), we have

$$\bar{W}_i=\begin{cases}1, & i=3,\\ 0, & i=1,2,\end{cases}\qquad \bar{S}_i=\begin{cases}S_i^*, & i=1,2,\\ 0, & i=3,\end{cases} \tag{4.15}$$

so the Eq. (4.5) leads to

$$\lambda_1^{*2}+\lambda_2^{*2}=1+\tau_0(1+\varepsilon_T),\qquad \lambda_1^{*2}\lambda_2^{*2}=\tau_0,\qquad \lambda_3^{*2}=\varpi_b\tau_1. \tag{4.16}$$

The displacement and temperature fields in the transformed domain are given by

$$\bar{u}(x,p)=\frac{\sigma_0\bar{f}(p)}{(\bar{S}_2^*-\bar{S}_1^*)p}\Big(\lambda_2^*\bar{S}_1^*e^{-p\lambda_2^*x}-\lambda_1^*\bar{S}_2^*e^{-p\lambda_1^*x}\Big), \tag{4.17a}$$

$$\bar{T}(x,p)=\frac{\sigma_0\bar{S}_1^*\bar{S}_2^*\bar{f}(p)}{(\bar{S}_2^*-\bar{S}_1^*)p}\Big(e^{-p\lambda_1^*x}+e^{-p\lambda_2^*x}\Big), \tag{4.17b}$$

where

$$\bar{S}_i^*=1-\lambda_i^{*2},\qquad i=1,2.$$

5 Numerical inversion of Laplace transforms

Numerical integration is used to find answers for definite integrals that cannot be solved analytically. The function assumed to satisfy initial and regularity condition in addition to boundary conditions in order to obtain the solution of various problems,

$$\bar{g}(x,p)=\int_0^\infty g(x,t)e^{-pt}dt. \tag{5.1}$$

The inversion formula for Laplace transform is

$$g(t)=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\bar{g}(p)e^{pt}dp, \tag{5.2}$$

where γ is arbitrary real number greater than all real parts of all the singularities of $\bar{g}(p)$. The above integral can be solved by setting

$$p=\gamma+iy,$$

we get

$$g(t)=\frac{e^{\gamma t}}{2\pi}\int_{-\infty}^{+\infty}\bar{g}(\gamma+iy)e^{ity}dy. \tag{5.3}$$

Let

$$h(t)=g(t)e^{-\gamma t},$$

expanding the function $g(t)e^{-\gamma t}$ in a Fourier series in the interval $[0,2l]$ we obtain approximation formula as

$$g(t)=g_\infty(t)+E_D, \tag{5.4}$$

where

$$g_\infty(t)=\frac{C_0}{2}+\sum_{k=1}^{\infty}C_K, \qquad \text{for } \ 0\leqslant t\leqslant 2l.$$

Here

$$C_k=\frac{\exp(\gamma t)}{l}\,\mathrm{Re}\Big[\bar{g}\Big(\gamma+\frac{ik\pi}{l}\Big)\exp\Big(\frac{ik\pi t}{l}\Big)\Big],$$

and E_D is the discretisation error can be made arbitrary small by choosing γ large enough. As infinite series in Eq. (5.4) can be summed up to finite number (N) of terms, the approximate value of $g(t)$ becomes

$$g_N(t)=\frac{C_0}{2}+\sum_{k=1}^{N}C_K, \qquad \text{for all } \ 0\leqslant t\leqslant 2l. \tag{5.5}$$

Using this formula to evaluate $g(t)$ a new error called truncation error get introduced that must be added to discretisation error to produce the total approximation error. The Korrector method is used to reduce the discretisation error; while ε-algorithmic method is used to reduce the truncation error and hence to accelerate the convergence. The Korrector method is used to evaluate the function $g(t)$ as

$$g(t) = g_\infty(t) - \exp(-2\gamma l) g_\infty(2l + t) + E'_D,$$

where

$$|E'_D| < |E_{D.}|.$$

The approximate value of $g(t)$ becomes

$$g_{N_K}(t) = g_N(t) - \exp(-2\gamma l) g_{N'}(2l + 1), \tag{5.6}$$

where N is an integer such that $N' < N$. We shall now describe the ε-algorithm method that is used to accelerate the convergence of the series. Let N be an odd natural number and let

$$s_m = \sum_{K=1}^{N} C_K,$$

be the sum of partial sequences of Eq. (5.5). We define the ε sequence by

$$\varepsilon_{0,m} = 0, \qquad \varepsilon_{1,m} = s_m, \qquad \varepsilon_{n+1,m} = \varepsilon_{n-1,m+1} - \frac{1}{\varepsilon_{n,m+1} - \varepsilon_{n,m}},$$

with $n, m = 1, 2, 3$. It can be shown that the sequence of partial sum $\varepsilon_{1,1}$, $\varepsilon_{3,1}$, $\cdots$, ε_{N+1} converges to $g(t) + E_D - C_0/2$ faster than the sequence of partial sum $(s_m, m = 1, 2, 3, \cdots)$. The actual procedure used to invert the Laplace transform consist of Eq. (5.6) together with ε-algorithm. The value of γ and l are chosen according to the criteria outlined in Honig and Hirdes [25].

6 Small-time approximations

Because of damping terms in Eqs. (3.1a)-(3.1c), the dependency of roots λ_i ($i = 1, 2, 3$) on p is complicated and hence the inversion of Laplace transform is quite difficult. These difficulties however will get reduced, if we use some approximation method. The approach used here consists of recasting the transformed functions as the Laplace transform of known functions in order to allow us to write down their inverse transforms by inspection. Mathematically this procedure is based on an elementary observation given in [27] as:

$$L^{-1}\Big(\frac{e^{-ap}}{p}\Big) = H(t-a), \qquad L^{-1}(e^{-ap}) = \delta(t-a), \tag{6.1a}$$

$$L^{-1}\Big(\frac{1}{p^2+a^2}\Big) = \frac{\sin at}{a}, \qquad L^{-1}\Big(\frac{p}{p^2+a^2}\Big) = \cos at. \tag{6.1b}$$

In order to apply this technique, Laplace transform parameter p is to be isolated as required by Eq. (6.1). Since the second sound effects are short lived (Green and Lindsay [5]), we consider the short time approximation (i.e., p large) of the solution which enables us to isolate p as required. It is found that the solution in general represents three waves propagating with finite speeds. The roots λ_i $(i=1,2,3)$ of Eq. (4.5) may be expanded binomially and after retaining positive sign only, we obtain

$$\lambda_i = \frac{1}{V_i} + \frac{\phi_i}{p} + \mathcal{O}\Big(\frac{1}{p^2}\Big), \qquad i=1,2,3, \tag{6.2}$$

with

$$\sum \frac{1}{V_1^2} = \frac{1+\bar{\omega}_b t_1 + t_0\big\{(1+\bar{a}\bar{b})(1+\varepsilon_a)+(1+\bar{\beta}\bar{b})(\varepsilon_T-\varepsilon_a)\big\}}{1-\bar{\beta}\varepsilon_C}, \tag{6.3a}$$

$$\sum \frac{1}{V_1^2V_2^2} = \frac{t_0(1+\bar{a}\bar{b})+\bar{\omega}_b t_1\{1+t_0(1+\varepsilon_T)\}}{1-\bar{\beta}\varepsilon_C}, \tag{6.3b}$$

$$\sum \frac{1}{V_1^2V_2^2V_3^2} = \frac{\bar{\omega}_b t_1 t_0}{1-\bar{\beta}\varepsilon_C}, \tag{6.3c}$$

$$\phi_i = \frac{A^*V_i^4 - B^*V_i^2 + C^*}{V_i\{(V_j^2-V_i^2)(V_k^2-V_i^2)\}}, \qquad i\neq j\neq k = 1,2,3, \tag{6.3d}$$

$$A^* = \frac{t_1+t_0}{2t_1t_0}, \tag{6.3e}$$

$$B^* = \frac{(1+\bar{a}\bar{b})+\bar{\omega}_b\{1+(t_1+t_0)(1+\varepsilon_T)\}}{2\bar{\omega}_b t_1 t_0}, \tag{6.3f}$$

$$C^* = \frac{\bar{\omega}_b t_0(1+\bar{a}\bar{b})(1+\varepsilon_a)}{2\bar{\omega}_b t_1 t_0}. \tag{6.3g}$$

The displacement, concentration change and temperature fields in the transformed domain are given by

$$\begin{aligned}&\big\{\bar{u}(x,p),\bar{T}(x,p),\bar{C}(x,p)\big\}\\ &=\sigma_0\bar{f}(p)\sum_{i=1}^{3}\frac{\Delta_{i0}}{\Delta_0}\Big[\frac{1}{p},V_i\bar{S}_{i0},\frac{\bar{W}_{i0}}{V_i}\Big]\exp\Big\{-\Big(\frac{p}{V_i}+\phi_i\Big)x\Big\},\end{aligned} \tag{6.4a}$$

$$\bar{S}_{i0} = 1-\frac{1}{V_i^2}(1+\bar{\beta}\bar{W}_{i0}), \qquad \bar{W}_{i0} = \frac{(V_i^2-1-\varepsilon_a)\bar{b}}{1+\bar{\beta}\bar{b}-\bar{\omega}_b t_1 V_i^2}, \tag{6.4b}$$

$$\Delta_{i0} = \frac{1}{V_i}\Big(\frac{\bar{S}_{k0}\bar{W}_{j0}}{V_j^2}-\frac{\bar{S}_{j0}\bar{W}_{k0}}{V_k^2}\Big), \qquad \Delta_0 = \sum_{(i,j,k)}\frac{W_{K0}}{V_k^2}(S_{j0}-S_{i0}), \tag{6.4c}$$

where $(i,j,k)=(1,2,3),(2,3,1),(3,1,2)$ and $\bar{f}(p)$ is defined in Eq. (4.14).

Inverting the Laplace transform of expression (6.4a) with the help of Eq. (6.1) the solution in the physical domain is obtained as

$$\{u,T,C\} = \sigma_0\sum_{i=1}^{3}\Big[H\Big(t-\frac{x}{V_i}\Big),\bar{S}_{i0}V_i\delta\Big(t-\frac{x}{V_i}\Big),\frac{\bar{W}_{i0}}{V_i}\delta\Big(t-\frac{x}{V_i}\Big)\Big]\frac{\Delta_{i0}}{\Delta_0}e^{-\phi_i x}, \tag{6.5a}$$

$$\{u,T,C\} = \sigma_0\sum_{i=1}^{3}\Big[\Big(t-\frac{x}{V_i}\Big),\bar{S}_{i0}V_i,\frac{\bar{W}_{i0}}{V_i}\Big]\frac{\Delta_{i0}}{\Delta_0}e^{-\phi_i x}H\Big(t-\frac{x}{V_i}\Big), \tag{6.5b}$$

$$\{u,T,C\}=\sigma_0\sum_{i=1}^{3}\Big[\frac{\sin\omega\Big(t-\frac{x}{V_i}\Big)}{\omega},\cos\omega\Big(t-\frac{x}{V_i}\Big)\bar{S}_{i0}V_i,\cos\omega\Big(t-\frac{x}{V_i}\Big)\frac{\bar{W}_{i0}}{V_i}\Big]\frac{\Delta_{i0}}{\Delta_0}e^{-\phi_i x}, \tag{6.5c}$$

in case of impulsive, continuous and periodic loads, respectively.

6.1 Approximation for thermoelastic halfspace

In the absence of mass diffusion $a=0=\beta_2$, the Eqs. (4.16) and (6.2) provide us

$$V^{*}{}_1^{-2}+V^{*}{}_2^{-2}=1+t_0(1+\varepsilon_T),\qquad V^{*}{}_1^{-2}V^{*}{}_2^{-2}=t_0,\qquad V^{*}{}_3^{-2}=\bar{\omega}_b t_1, \tag{6.6a}$$

$$\phi_i^*=\frac{V_1^*V_2^*N_1}{2(V_1^{*2}-V_2^{*2})},\qquad \phi_2^*=\frac{V_1^*V_2^*N_2}{V_1^{*2}-V_2^{*2}}, \tag{6.6b}$$

$$N_1=V_1^*(V_1^*+V_2^*)-V_2^*(1+\varepsilon_T),\quad N_2=V_2^*(V_1^*+V_2^*)-V_1^*(1+\varepsilon_T). \tag{6.6c}$$

Eq. (6.6a) implies

$$\frac{1}{V^{*2}_1},\ \frac{1}{V^{*2}_2}=\frac{1+t_0(1+\varepsilon_T)\pm\sqrt{M}}{2},$$

where

$$M=\sqrt{\{1-t_0(1+\varepsilon_T)\}^2+4\varepsilon_T t_0}=\frac{1}{V_1^{*2}}-\frac{1}{V_2^{*2}}>0. \tag{6.7}$$

Clearly $M>0$ and this implies that $V_1^*<V_2^*$. Thus V_1^* corresponds to slowest wave and V_2^* refers to fastest wave. Further in the absence of thermo-mechanical coupling ($\varepsilon_T=0$), Eq. (6.6a) leads to

$$V_2^*=\frac{1}{\sqrt{t_0}},\qquad V_1^*=1,\qquad V_3^*=\frac{1}{\sqrt{\bar{\omega}_b t_1}}, \tag{6.8}$$

where V_1^* corresponds to elastodiffusive wave and V_2^* corresponds to thermodiffusive wave because for most of the materials t_0 is quite small. Therefore $V_1^*<V_2^*$ and thus elastodiffusive wave follows thermodiffusive wave. Moreover the thermal waves have finite, though quite large, velocity of propagation. In the absence of thermal relaxation ($t_1=t_0=0$), Eq. (6.7) provides us

$$V_2^*\to\infty,\qquad V_3^*\to\infty,\qquad V_1^*=1,$$

this implies that in the absence of thermal relaxation time the longitudinal elastic wave travels with velocity equal to isothermal value

$$c_L^2=\frac{(\lambda+2\mu)}{\rho},$$

as in elastokinetics and other two waves have infinite velocity of propagation being diffusive in character. This corresponds to the case of conventional coupled thermoelasticity which predicts infinite speed of heat propagation and mass diffusion.

The displacement and temperature change in the transformed domain can be obtained from Eqs. (4.17a)-(4.17b) for small time approximation as:

$$\begin{aligned}\bar{u}(x,p) =&\frac{\sigma_0 \bar{f}(p)}{(V_1^{*2}-V_2^{*2})p}\Big[\frac{V_2^{*2}-1}{V_1^*}\exp\Big\{-x\Big(\frac{p}{V_1}+\phi_1\Big)\Big\} \\ &+\frac{V_1^{*2}-1}{V_2^*}\exp\Big\{-x\Big(\frac{p}{V_2}+\phi_2\Big)\Big\}\Big],\end{aligned} \tag{6.9a}$$

$$\begin{aligned}\bar{T}(x,p) =&\frac{\sigma_0 \bar{f}(p)}{(V_1^{*2}-V_2^{*2})p}\Big[(V_1^{*2}-1)(V_2^{*2}-1)\exp\Big\{-\Big(\frac{p}{V_1}+\phi_1\Big)x\Big\} \\ &+\exp\Big\{-\Big(\frac{p}{V_2}+\phi_2\Big)x\Big\}\Big].\end{aligned} \tag{6.9b}$$

Inverting the Laplace transform, we get

$$\begin{aligned}&T(x,t)\\ =&\frac{\sigma_0(V_1^{*2}-1)(V_2^{*2}-1)}{V_1^{*2}-V_2^{*2}}\sum_{i=1}^{2}e^{-\phi_i x}\left[\begin{array}{ll} H\big(t-\frac{x}{V_i}\big), & \text{for impulsive}\\ \big(t-\frac{x}{V_i}\big)H\big(t-\frac{x}{V_i}\big), & \text{for continuous}\\ \sin\omega\big(t-\frac{x}{V_i}\big)/\omega, & \text{for periodic}\end{array}\right],\end{aligned} \tag{6.10a}$$

$$\begin{aligned}u(x,t)=\frac{\sigma_0}{V_1^{*2}-V_2^{*2}}&\left[\begin{array}{l}\frac{V_2^{*2}-1}{V_1^*}e^{-\phi_1 x}H\big(t-\frac{x}{V_1}\big)\\ \frac{V_2^{*2}-1}{V_1^*}e^{-\phi_1 x}\big(t-\frac{x}{V_1}\big)H\big(t-\frac{x}{V_1}\big)\\ \frac{V_2^{*2}-1}{V_1^*}e^{-\phi_1 x}\sin\omega\big(t-\frac{x}{V_1}\big)\end{array}\right.\\ &\left.\begin{array}{ll}+\frac{V_1^{*2}-1}{V_2^*}e^{-\phi_2 x}H\big(t-\frac{x}{V_2}\big), & \text{for impulsive}\\ +\frac{V_1^{*2}-1}{V_2^*}e^{-\phi_2 x}\big(t-\frac{x}{V_1}\big)H\big(t-\frac{x}{V_2}\big), & \text{for continuous}\\ +\frac{V_1^{*2}-1}{V_2^*}e^{-\phi_2 x}\sin\omega\big(t-\frac{x}{V_2}\big), & \text{for periodic}\end{array}\right].\end{aligned} \tag{6.10b}$$

7 Analysis at the wave fronts

The short time solutions obtained above indicate that each of u, T and C are made up of three waves namely elastic, thermal and mass diffusion traveling with finite speeds V_1, V_2 and V_3 respectively. The terms containing $H(t-x/V_i)$ represent their contribution in vicinity of the corresponding wave fronts

$$x=V_i t, \qquad i=1,2,3.$$

Jumps at the wave fronts are denoted by $[\,]_1$, $[\,]_2$ and $[\,]_3$ respectively. Heaviside unit step function in short time solution predicts the occurrence of waves. Wave propagation velocities and position of wave fronts can be guessed from the arguments of Heaviside step function. Disturbance consists of three coupled waves one following the other. For most of the materials faster wave is elastic wave, thermal wave is slower than elastic but faster than mass diffusive wave. It is observed that displacement is continuous at each wave front in case of continuous and periodic loads. However, this function is discontinuous at the wave front in case of impulsive load. This means that a discontinuous mechanical load does generate discontinuities in the displacement which is not physically realistic. The temperature and concentration fields are found to be discontinuous at the wave fronts in case of continuous and periodic loads and exhibit Dirac delta behaviour at wave fronts in case of impulsive load. The discontinuities are given by

$$[u^+ - u^-]_{V_it} = \sigma_0 \begin{cases} \frac{\Delta_{i0}}{\Delta_0} \exp\{-\phi_i V_i t\}, \quad i=1,2,3, & \text{impulsive load}, \\ 0, & \text{continuous and periodic load}, \end{cases} \tag{7.1a}$$

$$[T^+ - T^-]_{V_it} = \sigma_0 \begin{cases} \infty, & \text{impulsive load}, \\ \frac{\Delta_{i0} S_{i0} V_i}{\Delta_0} \exp\{-\phi_i V_i t\}, \quad i=1,2,3, & \text{continuous and periodic load}, \end{cases} \tag{7.1b}$$

$$[C^+ - C^-]_{V_it} = \sigma_0 \begin{cases} \infty, & \text{impulsive load}, \\ \frac{\Delta_{i0} \bar{W}_{i10}}{V_i \Delta_0} \exp\{-\phi_i V_i t\}, \quad i=1,2,3, & \text{continuous and periodic load}. \end{cases} \tag{7.1c}$$

The jumps at the wave fronts in case of thermoelastic halfspace are given by

$$(u^+ - u^-)_{x=V_it} = \sigma_0 \begin{cases} \frac{(V_2^{*2}-1)\exp\{-\phi_1 V_1 t\}}{V_1^*(V_1^{*2}-V_2^{*2})}, \frac{(V_1^{*2}-1)\exp\{-\phi_1 V_1 t\}}{V(V_1^{*2}-V_2^{*2})}, & \text{impulsive load}, \\ 0, & \text{continuous and periodic load}, \end{cases} \tag{7.2a}$$

$$(T^+ - T^-)_{x=V_it} = \sigma_0 \begin{cases} \frac{(V_1^{*2}-1)(V_1^{*2}-1)\exp\{-\phi_1 V_1 t\}}{(V_1^{*2}-V_2^{*2})}, & \text{impulsive load}, \\ 0, & \text{continuous and periodic load}. \end{cases} \tag{7.2b}$$

Here also, the displacement and temperature change are discontinuous at the wave fronts in case of impulsive load but continuous for periodic as well as continuous loadings.

8 Numerical results and discussion

In order to illustrate and verify the analytical results obtained in the previous section we present some numerical simulation results. The material chosen for this purpose is solid solution of zinc (solute) and copper (solvent) whose physical data is given in Table 1. The dimensional value of thermal relaxation time parameter t_0 have been estimated from the relation $t_0 = 3K/\rho C_e C_L^2$ (see Chandrasekharaiah [26]) and that of t_1 is taken proportional to t_0. Consequently, the non-dimensional values of thermal relaxation times have been taken as $t_0 = 0.5$, $t_1 = 0.3$ for computation purpose. The change in non-dimensional surface displacement, temperature and concentration are computed for brass for various values of distance (x). Due to closeness of results and hence to avoid clustering of curves the variation of quantities are plotted only for two

Table 1: The physical data for Brass (70%Cu+30%Zn) material.

Quantity	Unit	Numerical Value	Reference
λ	Nm^{-2}	7.69×10^{10}	Callister, Jr [28]
μ	Nm^{-2}	3.61×10^{10}	
D	m^2s^{-1}	0.24×10^{-4}	
α_C	K^{-1}	1.8×10^{-5}	
K	W/mK	1.11×10^2	Thomas [29]
α_T	K^{-1}	2×10^{-6}	
ρ	kgm^{-3}	8.522×10^3	
T_0	K	293	
C_e	J/kg/K	385	
a	mS^{-1}	0.1521×10^2	Sherief and Saleh [17]
b	mS^{-1}	0.02×10^4	

values of time $t_0 = 0.25$ and $t_0 = 0.50$. Here solid and dotted curves are respectively for isothermal and insulated boundary of the halfspace.

Fig. 1 shows the variation of displacement (u) with distance (x) due to impulsive and continuous load acting on the isothermal boundary of the halfspace. It is observed that there is maximum displacement in the vicinity of load which decreases sharply in the range $0 \leq x \leq 2$ to become asymptotically close to each other and ultimately tends to zero at time $t = 0.25$. At time $t = 0.50$ the displacement curve shows similar behaviour as that of $t = 0.25$, but here the magnitude of displacement is relatively small. For $x \geq 2$ the profiles become quite close to be distinguished from each other which shows the existence of wave front. Variation of displacement due to continuous load acting on the isothermal boundary of the halfspace shows similar trends with small variations in magnitude. However, we have extended our analysis to the case of insulated boundary where we found that the displacement profile for both impulsive and continuous load follows similar trend of variation as that of isothermal boundary of the halfspace.

Fig. 2 presents the variation of normalized temperature change with distance due to impulsive load acting on the isothermal surface of halfspace. It is observed that the temperature is found to increase sharply in the range $0 \leq x \leq 1$ to attain its maximum value at $x = 1$ and decreases monotonically in the range $1 \leq x \leq 2$, and again increases up to $x = 2.5$, then it decreases sharply and become asymptotically close to zero with increasing distance from the vicinity of the load. At time $t = 0.50$, the temperature is found to increase in the range $0 \leq x \leq 1$, and then decreases steadily up to $x = 3.5$ and finally become close to zero for $x \geq 4$ at both considered values of time, which shows the existence of wave fronts here also.

Fig. 3 represents the variation of the concentration change with distance due to impulsive load acting at the isothermal and thermally insulated boundary of the half-space. The profiles of this quantity exhibit sinusoidal and damped behaviour at both values of the time with increasing distance from the vicinity of load for considered boundary conditions. These profiles ultimately converges to zero at $x \geq 5$ which

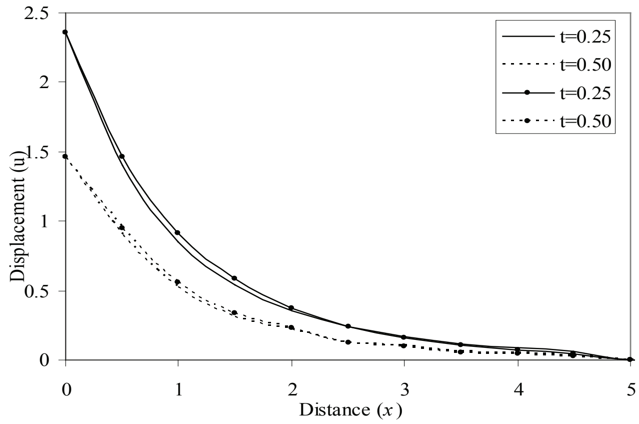

Figure 1: Displacement due to impulsive (without ball) and continuous load (with ball) in case of isothermal boundary.

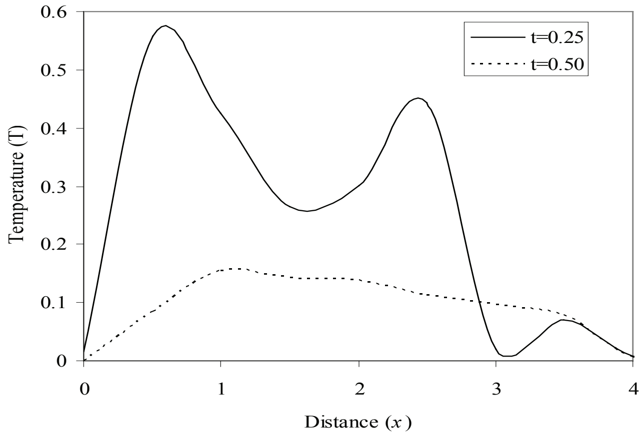

Figure 2: Temperature due to impulsive load in case of with isothermal boundary.

again shows the existence of wave front.

Fig. 4 presents the variation of temperature change due to continuous load in case of isothermal boundary. It is observed from Fig. 4 that at both considered values of time the temperature increases form its initial value in the range $0 \leq x \leq 1.5$, decreases monotonically up to $x = 4$ and become asymptotically close to zero at $x \geq 5$.

Fig. 5 represents the variation of concentration due to continuous loads with distance from the vicinity of the load at $t = 0.25$ and $t = 0.50$. At $t = 0.25$ it is revealed that the concentration follows increasing trend in the range $0 \leq x \leq 1.5$ thereafter decreases and become asymptotically close to zero at $x \geq 5$ in case of isothermal boundary. The profile of variation of this quantity at time $t = 0.05$, follows similar trend as that of $t = 0.25$, though with small magnitude. The profile in case of insulated boundary shows similar trend of variation with the exception that here the magnitude of concentration is three times less as compared with that isothermal boundary. More-

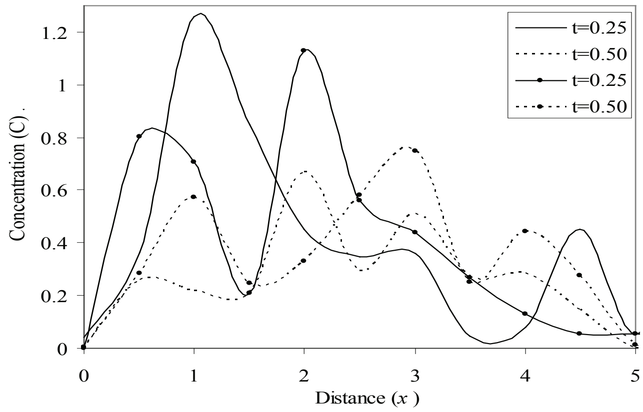

Figure 3: Concentration due to impulsive load in case of isothermal (without ball) and thermally insulated (with ball).

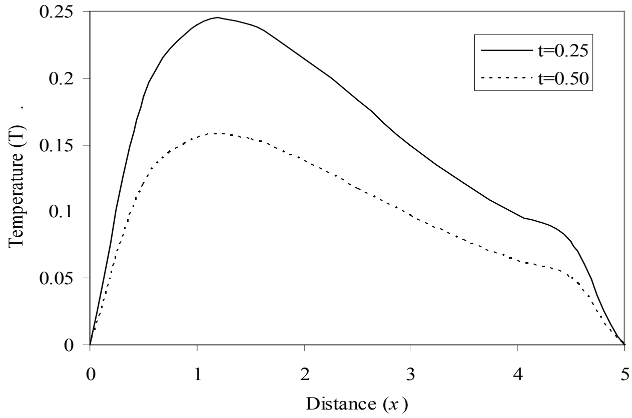

Figure 4: Temperature due to continuous load in case of isothermal boundary.

over, in this case and the variations are not steady in the range $2.5 \leq x \leq 4$ as observed in isothermal boundary of halfspace.

Fig. 6 shows variation of the temperature change for impulsive and continuous loads at the insulated and isothermal boundary of the halfspace. For insulated boundary at time $t = 0.25$ under impulsive load, the profile shows sharp increase in the range $0 \leq x \leq 0.50$ it attains maximum value at $x = 0.50$ then decreases sharply for $0.50 \leq x \leq 1$. It exhibit fluctuating behaviour in the range $1 \leq x \leq 4.5$ and then approaches to zero. At time $t = 0.50$, the temperature is found to decrease from a constant initial value and ultimately tends to zero at $x \geq 5$ after exhibiting slightly fluctuating behaviour with increasing distance from the vicinity of load. It is observed that variation of temperature has maximum value at the vicinity of continuous load acting at the boundary which decreases monotonically and becomes zero. At time $t = 0.50$ the variation profile follows similar pattern as that of $t = 0.25$ but here the temperature change is relatively small as compared to previous impulsive case.

Fig. 7 represents the variation of displacement, temperature and concentration in

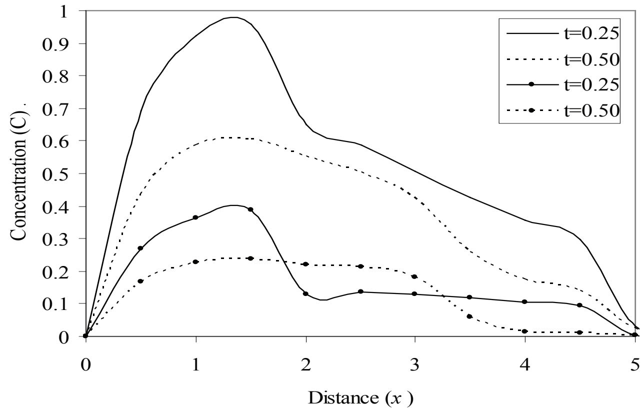

Figure 5: Concentration due to continuous load in case of isothermal (without ball) and thermally insulated (with ball).

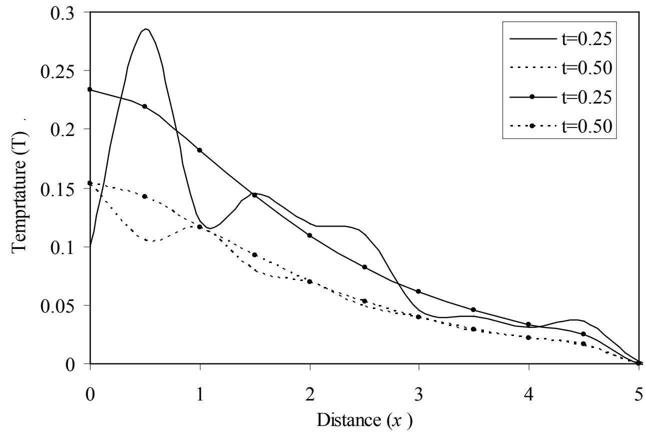

Figure 6: Temperature due to impulsive (without ball) and continuous (with ball) load in case of insulated boundary.

case of periodic load with frequency in case of isothermal and insulated boundary on logarithmic and linear scales respectively. It is observed that the displacement decreases sharply in the range $0 \leq \omega \leq 10$ both in case of isothermal and insulated boundary before the corresponding profiles become indistinguishable for $\omega \geq 10$. The temperature profile follows almost similar trend of variation as that of displacement with significant difference of magnitude in the range $1 \leq \omega \leq 10$. The concentration profiles decreases initially in the range $0 \leq \omega \leq 1$ then increases thereby follow Gaussian type behaviour in the range $1 \leq \omega \leq 10$ with maximum magnitude at $\omega = 10$ which may corresponds to the resonance condition. At higher frequency $\omega \geq 100$ of the periodic load all the quantities almost vanish in asymptotically. The comparison of various profiles shows that the physical nature and character of loads has significant effects on the considered functions which are clearly depicted by the plots. Moreover, the effect of thermal conditions, insulated or isothermal, prevailing on the boundary of the continuum, is also clearly visible from the respective profiles.

Figs. 8-10 represent small time solution in case of isothermal boundary at which the impulsive, continuous and periodic loads are acting. The slowest wave has velocity equal to $V_1 = 0.94774$ followed by $V_2 = 1.41342$ and the fastest wave has speed equal to $V_3 = 7.0114514$. Keeping in view the trend $V_1 < V_2 < V_3$, we find that the medium remains undisturbed for $x \geq V_3 t$. This means that at a given instant of time $t \geq 0$, the points of space that are beyond the faster wave front $x = V_3 t$ do not experience any disturbance. For $t = 0.25$, the corresponding wave fronts are given by $x = V_1 t = 0.23694$, $x = V_2 t = 0.3534$ and $x = V_3 t = 1.7536$. The existence of these wave fronts is clearly shown in Fig. 8. Their existence can also be observed form Figs. 9 and 10. Here all the considered quantities are found to vanish for $x \geq 1.7536$ thereby showing that the medium remains undisturbed beyond the fastest wave front. The temperature and concentration experiences infinite jumps at all the wave fronts due to impulsive load whereas these quantities remain continuous in case of continuous as well as periodic loads.

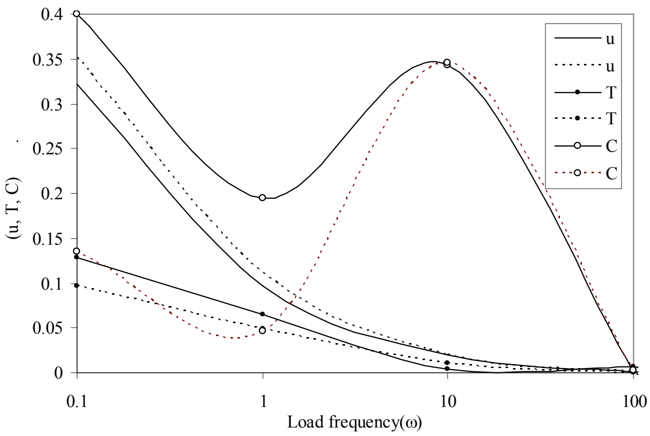

Figure 7: Displacement, Temperature and Concentration due to periodic load in case of isothermal (solid) and thermally insulated (dotted) boundary.

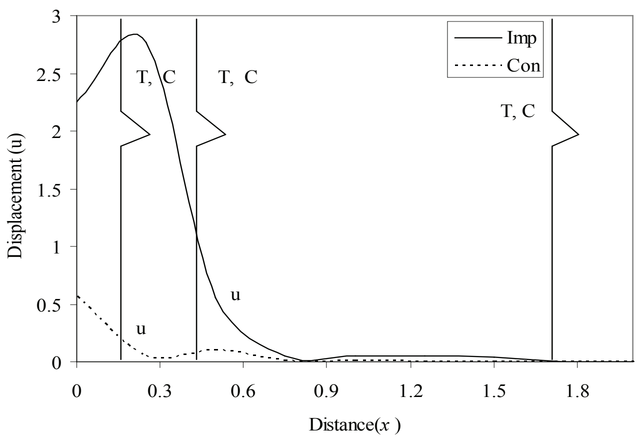

Figure 8: Profile of displacement wave fronts due to impulsive load in case of isothermal boundary.

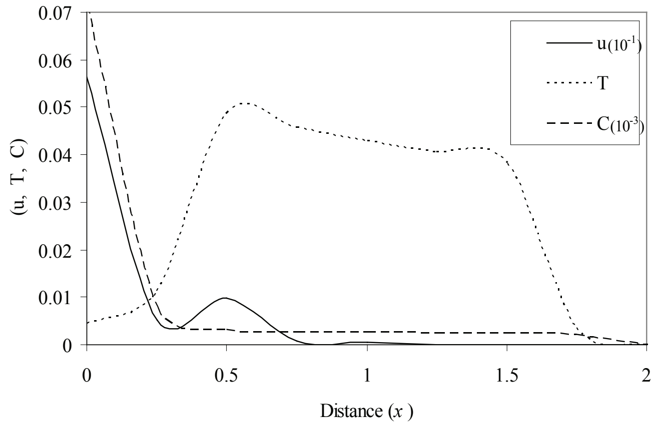

Figure 9: Profile of displacement, temperature and concentration wave fronts due to continuous load in case of isothermal boundary.

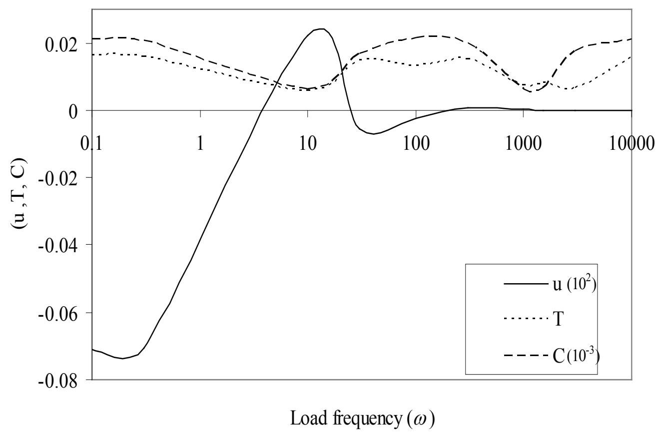

Figure 10: Profile of displacement, temperature and concentration wave fronts at $t = 0.25$ due to periodic load in case of isothermal boundary.

9 Conclusions

The present analysis reveals that maximum displacement, temperature change and concentration change occur in the vicinity of loads. It is observed that all the considered physical quantities vanish at a certain finite value of distance from the point of application of the load which indicates the existence of wave fronts. The short time solutions show that displacement is continuous at the wave fronts in case of periodic as well as continuous loads whereas it is found to be discontinuous for impulsive load. While the temperature and concentration changes experience Dirac-delta singularity at all the wave fronts in case of impulsive load in generalized elasto-thermodiffusive solid, but these are subjected to a finite jump in the absence of mass diffusion. These quantities are found to be continuous at all the wave fronts in case of periodic and continuous loads. The concentration change follows almost similar behaviour as that of temperature change in elasto-thermodiffusive halfspace. It is also observed that the displacement, temperature and concentration change are significantly affected by the load periodicity at higher frequencies. The physical nature and character of the load are clearly depicted by the plots of various considered functions. The nature of thermal boundary also exhibit significant effects on the profiles of various considered physical quantities. The study may find applications in material characterization and their fabrication in addition to electronic industry. A significant effect of mass diffusion (concentration change) has been noticed on displacement and thermal fields.

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