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Dynamics Behavior of a Stochastic Predator-Prey Model with Stage Structure for Predator and Lévy Jumps*

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Abstract In order to study the effects of external environmental noise on the interaction dynamics between predator and prey populations, in this paper, we develop a predator-prey model with the stage structure for predator and Lévy noise. By constructing an appropriate Lyapunov function, we first prove that the proposed model exists the uniqueness of global positive solution. Then, we analyze the persistence and extinction of the proposed model. Finally, we perform some numerical simulations to verify the correctness of the theoretical results.

Keywords Predator-prey model, stage structure, Lévy jumps, persistence and extinction

MSC(2010) 92C10, 92C15.

1. Introduction

Predation relationships are the most common in nature, the research on predator-prey models has attracted the attention of many researchers [1–6]. These researches laid the foundation for the future work. For the typical predator-prey model, it is generally assumed that predators are equally capable of hunting prey species. But the physiology of species in nature is complex. In many species, individuals are only able to hunt when they are adults, and the immature predators have to rely on mature ones for nourishment. Thus predatory ability could be ignored, such as sparrows, penguins and so on. Recently, some scholars have paid attention to the predator-prey models with the stage structure, and they have done some work in this research direction [7–10]. In addition, scholars have developed many predator-prey models with different functional response functions, especially for the Holling type II functional response, which is the most commonly used and takes the form of $f(x) = \frac{bx}{1+mx}$, where b is the search rate and m is the search rate multiplied by the handling time [11–14].

Some research work on the predator-prey model with Holling type II functional response has been developed and investigated. For example, Wang and Chen [15] proposed and analyzed the following predator-prey model with Holling type II func-

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tional response and the stage structure for predator:

$$
\begin{cases}
\dfrac{\mathrm{d}x(t)}{\mathrm{d}t} =x(t)(r-ax(t))-\dfrac{bx(t)z(t)}{1+mx(t)}, \\
\dfrac{\mathrm{d}y(t)}{\mathrm{d}t} =\dfrac{kbx(t)z(t)}{1+mx}-(D+d_1)y(t), \\
\dfrac{\mathrm{d}z(t)}{\mathrm{d}t} =Dy(t)-d_2z(t), \\
x(0) =x_0, y(0)=y_0, z(0)=z_0,
\end{cases}
\tag{1.1}
$$

where $x=x(t)$, $y=y(t)$ and $z=z(t)$ denote the densities of prey, immature and mature predators at time t, respectively. r is the intrinsic growth rate of the prey, a represents the intraspecific competition rate of the prey, b is the consumption rate of mature predators to prey, $k(0<k<1)$ is the conversion efficiency of prey into newborn immature predators, d_1 and d_2 represent the death rates of immature and mature predators, D is the rate at which immature predators become mature predators, $x(0)=x_0$, $y(0)=y_0$ and $z(0)=z_0$ are initial values. All the parameters are positive constants. By defining the basic reproduction number of the predator $R_0=\frac{kbDr}{d_2(a+mr)(D+d_1)}$ as the average number of offsprings produced by a mature predator in its lifetime, Georgescu and Morosanu [16] showed that if $R_0\le 1$, then the prey-only equilibrium $\left(\frac{r}{a},0,0\right)$ is globally asymptotically stable on $\mathbb{R}^3_+$, while if $R_0>1$, the prey-only equilibrium $\left(\frac{r}{a},0,0\right)$ is unstable, and there exists only one positive equilibrium.

However, there exists certain limitation for the deterministic model (1.1), and it cannot reflect the effect of environmental factors on the dynamical behavior of model (1.1). Thus, by taking into account the influence of external environment noise, Liu [17] introduced the standard white noise into model (1.1) and then obtained the following stochastic model:

$$
\begin{cases}
\mathrm{d}x=\left[x(r-ax)-\dfrac{bxz}{1+mx}\right]\mathrm{d}t+\sigma_1x\mathrm{d}B_1(t), \\
\mathrm{d}y=\left[\dfrac{kbxz}{1+mx}-(D+d_1)\,y\right]\mathrm{d}t+\sigma_2y\mathrm{d}B_2(t), \\
\mathrm{d}z=[Dy-d_2z]\,\mathrm{d}t+\sigma_3z\mathrm{d}B_3(t),
\end{cases}
\tag{1.2}
$$

where σ_1^2, σ_2^2, σ_3^2 are the intensities of the environment white noise, $B_1(t)$, $B_2(t)$ and $B_3(t)$ are mutually independent standard Brownian motions with $B_1(0)=B_2(0)=B_3(0)=0$.

In addition, sudden environmental disturbance, such as hurricanes, earthquakes, floods, etc, can also have a significant impact on the predator and prey species. In order to better understand the effects of these phenomena on the dynamics of the predator-prey model, it is worth studying the predator-prey model with jumps process. Applebaum and Siakalli [18] extended Mao's techniques to the case of nonlinear stochastic differential equations driven by Lévy jumps and studied the probability stability, almost certainty stability and moment exponential stability of the stochastic differential equation. Zhao and Yuan [19] pointed out that Lévy noise can affect the optimal harvesting strategy of inshore and offshore fisheries. Liu and Bao et al. [20, 21] analyzed the Lotka-Volterra system affected by Lévy noise, and the results indicated that Lévy noise has a certain effect on the dynamics of the system. For this reason, we developed the following model incorporating the Lévy noise:

$$\begin{cases} \mathrm{d}x(t) = \left[x(t)(r - ax(t)) - \dfrac{bx(t)z(t)}{1+mx(t)}\right]\mathrm{d}t + \sigma_1 x(t)\mathrm{d}B_1(t) + \displaystyle\int_Y \gamma_1(u)x(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \mathrm{d}y(t) = \left[\dfrac{kbx(t)z(t)}{1+mx(t)} - (D+d_1)\,y(t)\right]\mathrm{d}t + \sigma_2 y(t)\mathrm{d}B_2(t) + \displaystyle\int_Y \gamma_2(u)y(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u), \\ \mathrm{d}z(t) = [Dy(t) - d_2 z(t)]\,\mathrm{d}t + \sigma_3 z(t)\mathrm{d}B_3(t) + \displaystyle\int_Y \gamma_3(u)z(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u), \end{cases} \tag{1.3}$$

where $x(t^-)$, $y(t^-)$ and $z(t^-)$ are the left limits of $x(t)$, $y(t)$ and $z(t)$, N is the Poisson counting measure with compensator $\widetilde{N}$ and characteristic measure λ on a measurable subset $\mathbb{Y}$ of $(0,\infty)$ with $\lambda(\mathbb{Y}) < \infty$, and it is assumed that λ is a Lévy measure such that $\widetilde{N}(\mathrm{d}t,\mathrm{d}u) = N(\mathrm{d}t,\mathrm{d}u) - \lambda(\mathrm{d}u)\mathrm{d}t$.

In this paper, we mainly discuss the persistence and extinction of the model (1.3). The structure of the paper is organized as follows. In section 2, we give some notations and lemmas which will be useful in the subsequent proof process. In section 3, we prove the existence and uniqueness of global positive solution of the model. In sections 4 and 5, the sufficient conditions of the persistence and extinction of model (1.3) are obtained. Finally, numerical simulations are carried out to verify the correctness and feasibility of the obtained results.

2. Preliminaries

In this section, for the sake of narration, we give some notations, lemmas and assumptions which will be used later. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ denote a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. Define

$$\mathbb{R}^3_+ = \left\{(x(t), y(t), z(t)) \in \mathbb{R}^3 : x(t) > 0, y(t) > 0, z(t) > 0\right\},$$

$$c_1 = D + d_1 - \frac{(\theta-1)\sigma_2^2}{2} - \frac{1}{\theta}\int_Y \left[(1+\gamma_2(u))^\theta - 1 - \theta\gamma_2(u)\right]\lambda(\mathrm{d}u),$$

$$c_2 = d_2 - \frac{(\theta-1)\sigma_3^2}{2} - \frac{1}{\theta}\int_Y \left[(1+\gamma_3(u))^\theta - 1 - \theta\gamma_3(u)\right]\lambda(\mathrm{d}u),$$

$$\langle f\rangle_t = \frac{1}{t}\int_0^t f(s)\mathrm{d}s, \langle f\rangle^* = \lim_{t\to\infty}\sup\frac{1}{t}\int_0^t f(s)\mathrm{d}s, \langle f\rangle_* = \lim_{t\to\infty}\inf\frac{1}{t}\int_0^t f(s)\mathrm{d}s,$$

$$M_i(t) = \int_0^t\int_Y \ln(1+\gamma_i(u))\widetilde{N}(\mathrm{d}s,\mathrm{d}u), H_i = \int_Y \left[\gamma_i(u) - \ln(1+\gamma_i(u))\right]\lambda(\mathrm{d}u), i = 1,2,3.$$

Assumption 2.1. For model (1.3), there exist two constants C_1, $C_2 > 0$ such that

$$\int_Y \left\{|\gamma_i(u)|^2 \vee [\ln(1+\gamma_i(u))]^2\right\}\lambda(\mathrm{d}u) \leq C_1 < \infty,$$

$$\int_Y \left[(1+\gamma_i(u))^\theta - 1 - \theta\gamma_i(u)\right]\lambda(\mathrm{d}u) \leq C_2 < \infty.$$

Lemma 2.1. *Assume that $\gamma_i(u)$ is a bounded function and $|\gamma_i(u)| < \frac{\delta}{\upsilon} < 1$, $u \in \mathbb{R}, i = 1, 2, 3$, where δ, υ are positive constants, $\upsilon \in (0, 1)$. By Taylor formula, we have*

$$\begin{aligned} H_i =& \int_Y \left[\gamma_i(u) - \ln(1 + \gamma_i(u))\right]\lambda(\mathrm{d}u) \\ \leq& \int_Y \left[\gamma_i(u) - \gamma_i(u) + \frac{\gamma_i^2(u)}{2!(1 - \gamma_i(u)\upsilon)^2}\right]\lambda(\mathrm{d}u) \\ \leq& \frac{\delta^2}{2(1-\delta)^2\upsilon^2}. \end{aligned}$$

Lemma 2.2 ([22]). *Suppose that $x(t) \in C(\Omega \times [0, \infty], \mathbb{R}_+)$.*

(i) If there are three positive constants T, δ and δ_0 such that

$$\ln x(t) \leq \delta t - \delta_0 \int_0^t x(t)\mathrm{d}s + \sum_{i=1}^n \alpha_i B_i(t) + \sum_{i=1}^n k_i \int_0^t \int_{\mathbb{Y}} \ln(1+\gamma_i(u))\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \ a.s.,$$

for all $t > T$, where α_i, δ and B_i are constants, then we have

$$\begin{cases} \langle x \rangle^* \leq \dfrac{\delta}{\delta_0} \ a.s., & if \delta \geq 0; \\ \lim\limits_{t\to\infty} x(t) = 0 \ a.s., & if \delta \leq 0. \end{cases}$$

(ii) If there exist three positive constants T, δ and δ_0 such that

$$\ln x(t) \geq \delta t - \delta_0 \int_0^t x(t)\mathrm{d}s + \sum_{i=1}^n \alpha_i B_i(t) + \sum_{i=1}^n k_i \int_0^t \int_{\mathbb{Y}} \ln(1+\gamma_i(u))\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \ a.s.,$$

for all $t > T$, where α_i, δ and k_i are constants, then we have $\langle x \rangle_* \geq \dfrac{\delta}{\delta_0}$ a.s.

Lemma 2.3. *For any initial value, the system* (1.3) *has the following properties.*

$$\lim_{t\to\infty} \frac{\int_0^t x(s)\mathrm{d}B_1(s)}{t} = 0, \lim_{t\to\infty} \frac{\int_0^t y(s)\mathrm{d}B_2(s)}{t} = 0,$$

$$\lim_{t\to\infty} \frac{\int_0^t z(s)\mathrm{d}B_3(s)}{t} = 0, \lim_{t\to\infty} \frac{\int_0^t \int_Y \gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u)}{t} = 0,$$

$$\lim_{t\to\infty} \frac{\int_0^t \int_Y \gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u)}{t} = 0, \lim_{t\to\infty} \frac{\int_0^t \int_Y \gamma_3(u)z(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u)}{t} = 0,$$

$$\lim_{t\to\infty} \frac{M_i(t)}{t} = 0 (i = 1, 2, 3), \lim_{t\to\infty} \frac{\int_0^t \int_Y \ln(1 + \gamma_i(u))\widetilde{N}(\mathrm{d}s, \mathrm{d}u)}{t} (i = 1, 2, 3).$$

The process of proving is similar to the references [23, 24]. Here we omit it.

3. Existence and uniqueness of the global positive solution

In this section, we will prove that system (1.3) has a unique global positive solution with any positive initial value. Then, we have the following theorem.

Theorem 3.1. *For any initial value $(x(0), y(0), z(0)) \in \mathbb{R}^3_+$, there exists a unique solution $(x(t), y(t), z(t))$ of system (1.3) on $t > 0$ and the solution will remain in $\mathbb{R}^3_+$ with probability one, namely $(x(t), y(t), z(t)) \in \mathbb{R}^3_+$ for all $t > 0$ almost surely (a.s.).*

Proof. Since the coefficients of system (1.3) satisfy the local Lipschitz condition, then for any initial value $(x(0), y(0), z(0)) \in \mathbb{R}^3_+$, there exists a unique local solution $(x(t), y(t), z(t)) \in \mathbb{R}^3_+$ on $t \in [0, \tau_e)$, where τ_e denotes the explosion time. Now we will prove the solution is global. To this end, let $n_0 > 1$ be sufficiently large such that $(x(t), y(t), z(t))$ all lie within the interval $\left[\frac{1}{n_0}, n_0\right]$. For each integer $n \geq n_0$, define the stopping time as [25]

$$\tau_n = \inf\left\{t \in [0, \tau_e) : \min\left\{x\left(t\right), y\left(t\right), z\left(t\right)\right\} \leq \frac{1}{n} \text{ or } \max\left\{x\left(t\right), y\left(t\right), z\left(t\right)\right\} \geq n\right\}.$$

Throughout this paper, we set $\inf \varnothing = \infty$ (as usual $\varnothing$ denotes the empty set). Clearly, $\lim_{n\to\infty} \tau_n = \tau_\infty$ as $n \to \infty$ and $(x(t), y(t), z(t)) \in \mathbb{R}^3_+$, $t \geq t_0$. This is to say, we need to prove $\tau_\infty = \infty$ a.s. If the assertion is not true, then here exists a pair of constants $T > 0$ and $\varepsilon \in (0, 1)$ such that $P(\tau_\infty \leq T) > \varepsilon$. Hence, there exists an integer $n_1 \geq n_0$ such that

$$P\left(\tau_n \leq T\right) \geq \varepsilon \text{ for all } n \geq n_1. \tag{3.1}$$

Define a C^2-function $V : \mathbb{R}^3_+ \to \mathbb{R}$ by

$$V(x, y, z) = \left(x - c - c\ln\frac{x}{c}\right) + \frac{1}{k}\left(y - 1 - \ln y\right) + \left(z - 1 - \ln z\right),$$

where c is a positive constant to be determined later. The nonnegativity of this function can be seen from

$$u - 1 - \ln u \geq 0 \text{ for any } u > 0.$$

Applying $It\hat{o}'s$ formula [26] to function V, we have

$$\begin{aligned}
\mathrm{d}V =& LV\mathrm{d}t + \sigma_1\left(x(t) - c\right)\mathrm{d}B_1(t) + \frac{\sigma_2}{k}\left(y(t) - 1\right)\mathrm{d}B_2(t) + \sigma_3\left(z(t) - 1\right)\mathrm{d}B_3(t) \\
&- c\int_Y\left[\ln\left(1 + \gamma_1\left(u\right)\right) - \frac{\gamma_1\left(u\right)x\left(t^-\right)}{c}\right]\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \\
&- \frac{1}{k}\int_Y\left[\ln\left(1 + \gamma_2\left(u\right)\right) - \gamma_2\left(u\right)y\left(t^-\right)\right]\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \\
&- \int_Y\left[\ln\left(1 + \gamma_3\left(u\right)\right) - \gamma_3\left(u\right)z\left(t^-\right)\right]\widetilde{N}(\mathrm{d}t, \mathrm{d}u).
\end{aligned}$$

According to the definition of the operator L, we can get

$$LV = \left(1 - \frac{c}{x}\right)\left[x\left(r - ax\right) - \frac{bxz}{1 + mx}\right] + \frac{1}{k}\left(1 - \frac{1}{y}\right)\left[\frac{kbxz}{1 + mx} - \left(D + d_1\right)y\right]$$

$$
\begin{aligned}
& + \frac{c\sigma_1^2}{2} + \frac{\sigma_2^2}{2k} + \left(1 - \frac{1}{z}\right)[Dy - d_2 z] + \frac{\sigma_3^2}{2} + cH_1 + \frac{H_2}{k} + H_3 \\
= & - ax^2 + (ac + r)\,x - \frac{bxz}{y\,(1 + mx)} - \frac{Dy}{z} + Dy - \frac{D + d_1}{k} y + \frac{cbz}{1 + mx} \\
& - d_2 z - cr + \frac{c\sigma_1^2}{2} + \frac{1}{k}\left(D + d_1 + \frac{\sigma_2^2}{2}\right) + d_2 + \frac{\sigma_3^2}{2} + cH_1 + \frac{H_2}{k} + H_3 \\
\leq & - ax^2 + (ac + r)\,x + \frac{(k - 1)\,Dy}{k} - \frac{d_1 y}{k} + cbz - d_2 z + \frac{c\sigma_1^2}{2} \\
& + \frac{1}{k}\left(D + d_1 + \frac{\sigma_2^2}{2}\right) + d_2 + \frac{\sigma_3^2}{2} + cH_1 + \frac{H_2}{k} + H_3 \\
\leq & - ax^2 + (ac + r)\,x + \frac{(k - 1)\,Dy}{k} + cbz - d_2 z + \frac{1}{k}\left(D + d_1 + \frac{\sigma_2^2}{2}\right) \\
& + \frac{c\sigma_1^2}{2} + d_2 + \frac{\sigma_3^2}{2} + cH_1 + \frac{H_2}{k} + H_3 \\
\leq & \frac{(ac + r)^2}{4a} + \frac{1}{k}\left(D + d_1 + \frac{\sigma_2^2}{2}\right) + d_2 + (cb - d_2)\,z + \frac{c\sigma_1^2}{2} + \frac{\sigma_3^2}{2} \\
& + cH_1 + \frac{H_2}{k} + H_3,
\end{aligned}
$$

where the third inequality is obtained by $0 < k \leq 1$. Letting $c = \dfrac{d_2}{b}$, we can get $cb - d_2 = 0$. Then we have

$$
LV \leq \frac{(ac + r)^2}{4a} + \frac{1}{k}\left(D + d_1 + \frac{\sigma_2^2}{2}\right) + d_2 + \frac{c\sigma_1^2}{2} + \frac{\sigma_3^2}{2} + cH_1 + \frac{H_2}{k} + H_3.
$$

From Lemma 2.1, we can get

$$
\begin{aligned}
LV \leq & \frac{(ac + r)^2}{4a} + \frac{1}{k}\left(D + d_1 + \frac{\sigma_2^2}{2}\right) + d_2 + \frac{c\sigma_1^2}{2} + \frac{\sigma_3^2}{2} + \frac{c\delta^2}{2\,(1 - \delta)^2\,\upsilon^2} \\
& + \frac{\delta^2}{2k\,(1 - \delta)^2\,\upsilon^2} + \frac{\delta^2}{2\,(1 - \delta)^2\,\upsilon^2} := K.
\end{aligned}
$$

So we obtain

$$
\begin{aligned}
\mathrm{d}V \leq & K\mathrm{d}t + \sigma_1\,(x(t) - c)\,\mathrm{d}B_1\,(t) - c\int_Y \left[\ln\left(1 + \gamma_1\,(u)\right) - \frac{\gamma_1\,(u)\,x\,(t^-)}{c}\right]\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \\
& + \frac{\sigma_2}{k}\,(y(t) - 1) - \frac{1}{k}\int_Y \left[\ln\left(1 + \gamma_2\,(u)\right) - \gamma_2\,(u)\,y\left(t^-\right)\right]\widetilde{N}(\mathrm{d}t, \mathrm{d}u) \\
& + \sigma_3\,(z(t) - 1) - \int_Y \left[\ln\left(1 + \gamma_3\,(u)\right) - \gamma_3\,(u)\,z\left(t^-\right)\right]\widetilde{N}\,(\mathrm{d}t, \mathrm{d}u)\,. \qquad (3.2)
\end{aligned}
$$

Integrating both sides of (3.2) from 0 to $\tau_n \wedge T = \min\{\tau_n, T\}$ yields

$$
\begin{aligned}
\int_0^{\tau_n \wedge T} \mathrm{d}V = & K(\tau_n \wedge T) + \sigma_1 \int_0^{\tau_n \wedge T} (x(s) - c)\,\mathrm{d}B_1\,(s) \\
& + \frac{\sigma_2}{k}\int_0^{\tau_n \wedge T} (y(s) - 1)\,\mathrm{d}B_2\,(s) + \sigma_3 \int_0^{\tau_n \wedge T} (z(s) - 1)\,\mathrm{d}B_3\,(s)
\end{aligned}
$$

$$
\begin{aligned}
&-c\int_0^{\tau_n\wedge T}\int_Y\left[\ln\left(1+\gamma_1\left(u\right)\right)-\frac{\gamma_1\left(u\right)x\left(s^-\right)}{c}\right]\widetilde{N}\left(\mathrm{d}s,\mathrm{d}u\right)\\
&-\frac{1}{k}\int_0^{\tau_n\wedge T}\int_Y\left[\ln\left(1+\gamma_2\left(u\right)\right)-\gamma_2\left(u\right)y\left(s^-\right)\right]\widetilde{N}\left(\mathrm{d}s,\mathrm{d}u\right)\\
&-\int_0^{\tau_n\wedge T}\int_Y\left[\ln\left(1+\gamma_3\left(u\right)\right)-\gamma_3\left(u\right)z\left(s^-\right)\right]\widetilde{N}\left(\mathrm{d}s,\mathrm{d}u\right).
\end{aligned}\tag{3.3}
$$

Since the solution $(x(t), y(t), z(t))$ is $\mathcal{F}_t$-adapted, taking the expectation on both sides of (3.3), we have

$$
EV\left(x,y,z\right)\leq V\left(x\left(0\right),y\left(0\right),z\left(0\right)\right)+KE\left(\tau_n\wedge T\right)\leq V\left(x\left(0\right),y\left(0\right),z\left(0\right)\right)+KT.\tag{3.4}
$$

Letting $\Omega_n=\{\omega\in\Omega:\tau_n=\tau_n\left(\omega\right)\leq T\}$ for $n\geq n_1$ and from (3.1), we have $P\left(\tau_n\leq T\right)\geq\varepsilon$. Note that for every $\omega\in\Omega_n$, there is at least one of $x\left(\tau_n,\omega\right)$ and $y\left(\tau_n,\omega\right)$ and $z\left(\tau_n,\omega\right)$ equaling either n or $\dfrac{1}{n}$. Hence, one can get that

$$
\begin{aligned}
&V\left(x\left(\tau_n,\omega\right),y\left(\tau_n,\omega\right),z\left(\tau_n,\omega\right)\right)\\
\geq&\left(n-c-c\ln\frac{n}{c}\right)\wedge\left(n-1-\ln n\right)\wedge\left(\frac{1}{n}-c+c\ln\left(nc\right)\right)\wedge\left(\frac{1}{n}-1+\ln n\right).
\end{aligned}
$$

It then follows from (3.4) that

$$
\begin{aligned}
&V(x(0),y(0),z(0))+KT\\
\geq&E\left[I_{\Omega_{n(\omega)}}V(x(\tau_n,\omega),y(\tau_n,\omega),z(\tau_n,\omega))\right]\\
\geq&\varepsilon\left\{\left(n-c-c\ln\frac{n}{c}\right)\wedge\left(n-1-\ln n\right)\wedge\left(\frac{1}{n}-c-c\ln\frac{1}{nc}\right)\wedge\left(\frac{1}{n}-1+\ln n\right)\right\},
\end{aligned}
$$

where I_{Ω_n} is the indicator function of Ω_n. Letting $n\to\infty$, then one can see that

$$
\infty>V\left(x(0),y(0),z(0)\right)+KT=\infty,
$$

which leads to the contradiction, thus we must have $\tau_\infty=\infty$ a.s. The proof is thus completed. □

4. Extinction of model (1.3)

Definition 4.1 ([27]). (1) The population x is said to be persistent in the mean if $\lim_{t\to\infty}\frac{1}{t}\int_0^t x(s)\mathrm{d}s>0$. (2) The population x is said to be extinct if $\lim_{t\to\infty}x(t)=0$, a.s.

Theorem 4.1. *Let* $(x(t), y(t), z(t))$ *be the solution of model* (1.3) *with any given positive initial value* $(x(0), y(0), z(0))$*. If* $2r<\sigma_1^2$*, then we have*

$$
\lim_{t\to\infty}x(t)=0,\lim_{t\to\infty}y(t)=0,\lim_{t\to\infty}z(t)=0,a.s.
$$

Proof. Applying *Itô's* formula to the first equation of model (1.3) yields

$$
\mathrm{d}\ln x(t)=\left[r-ax-\frac{bz}{1+mx}-\frac{\sigma_1^2}{2}+\int_Y(\ln(1+\gamma_1(u))-\gamma_1(u))\lambda(\mathrm{d}u)\right]\mathrm{d}t
$$

$$
\begin{aligned}
&+\sigma_1 \mathrm{d}B_1(t)+\int_Y \ln(1+\gamma_1(u))\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&\le\left[r-ax-\frac{bz}{1+mx}-\frac{\sigma_1^2}{2}\right]\mathrm{d}t+\sigma_1\mathrm{d}B_1(t)+\int_Y \ln(1+\gamma_1(u))\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&\le\left[r-\frac{\sigma_1^2}{2}\right]\mathrm{d}t+\sigma_1\mathrm{d}B_1(t)+\int_Y \ln(1+\gamma_1(u))\widetilde{N}(\mathrm{d}t,\mathrm{d}u),
\end{aligned}\tag{4.1}
$$

where

$$
\int_Y [\gamma_1(u)-\ln(1+\gamma_1(u))]\lambda(\mathrm{d}u)=\int_Y [1+\gamma_1(u)-1-\ln(1+\gamma_1(u))]\,\lambda(\mathrm{d}u)\ge 0.
$$

Integrating both sides of (4.1) from 0 to t, we obtain

$$
\ln x(t)-\ln x(0)\le\left[r-\frac{\sigma_1^2}{2}\right]t+\sigma_1 B_1(t)+M_1(t).\tag{4.2}
$$

Dividing both sides of (4.2) by t and taking the supremum, then by Lemma 2.2, we can get

$$
\lim_{t\to\infty}\sup\frac{\ln x(t)}{t}<r-\frac{\sigma_1^2}{2}<0.\tag{4.3}
$$

Let $W(t)=y(t)+z(t)$. Then applying $It\hat{o}^{,}s$ formula to $\ln W$ yields

$$
\begin{aligned}
\mathrm{d}\ln W=&\left[\frac{\frac{kbxz}{1+mx}-d_1y-d_2z}{y+z}-\frac{\sigma_2^2y^2+\sigma_3^2z^2}{2(y+z)^2}\right.\\
&+\int_Y\left[\ln\left(1+\frac{\gamma_2(u)y(t^-)}{y(t^-)+z(t^-)}\right)-\frac{\gamma_2(u)y(t^-)}{y(t^-)+z(t^-)}\right]\lambda(\mathrm{d}u)\\
&\left.+\int_Y\left[\ln\left(1+\frac{\gamma_3(u)z(t^-)}{y(t^-)+z(t^-)}\right)-\frac{\gamma_3(u)z(t^-)}{y(t^-)+z(t^-)}\right]\lambda(\mathrm{d}u)\right]\mathrm{d}t\\
&+\frac{\sigma_2 y}{y+z}\mathrm{d}B_2(t)+\int_Y\ln\left(1+\frac{\gamma_2(u)y(t^-)}{y(t^-)+z(t^-)}\right)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&+\frac{\sigma_3 z}{y+z}\mathrm{d}B_3(t)+\int_Y\ln\left(1+\frac{\gamma_3(u)z(t^-)}{y(t^-)+z(t^-)}\right)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
\le&[kbx-\min\{d_1,d_2\}]\mathrm{d}t+\frac{\sigma_2 y}{y+z}\mathrm{d}B_2(t)+\frac{\sigma_3 z}{y+z}\mathrm{d}B_3(t)\\
&+\int_Y\ln\left(1+\frac{\gamma_2(u)y(t^-)}{y(t^-)+z(t^-)}\right)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&+\int_Y\ln\left(1+\frac{\gamma_3(u)z(t^-)}{y(t^-)+z(t^-)}\right)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
\le&[kbx-\min\{d_1,d_2\}]\mathrm{d}t+\sigma_2\mathrm{d}B_2(t)+\sigma_3\mathrm{d}B_3(t)+\int_Y\ln(1+\gamma_2(u))\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&+\int_Y\ln(1+\gamma_3(u))\widetilde{N}(\mathrm{d}t,\mathrm{d}u).
\end{aligned}\tag{4.4}
$$

Integrating both sides of (4.4) from 0 to t yields

$$
\ln W(t)-\ln W(0)\le[kbx-\min\{d_1,d_2\}]\,t+\sigma_2\mathrm{d}B_2(t)+\sigma_3\mathrm{d}B_3(t)+M_2(t)+M_3(t).\tag{4.5}
$$

Dividing by t on both sides of (4.5) yields

$$\frac{\ln W(t)-\ln W(0)}{t} \leq kbx-\min\{d_1,d_2\}+\frac{\sigma_2 \mathrm{d}B_2(t)}{t}+\frac{\sigma_3 \mathrm{d}B_3(t)}{t}+\frac{M_2(t)}{t}+\frac{M_3(t)}{t}. \tag{4.6}$$

By inequality (4.3), we know that $x(t)\to 0$ exponentially a.s. Then there exist t_0 and a set $\Omega_\varepsilon \subset \Omega$ such that

$$P(\Omega_\varepsilon)>1-\varepsilon \text{ and} \frac{kbx}{1+mx} \leq kbx \leq kb\varepsilon \text{ for } t\geq t_0 \text{ and } \omega\in\Omega_\varepsilon.$$

Taking the supremum on both sides of (4.6), we can get

$$\limsup_{t\to\infty}\frac{W(t)}{t} \leq kb\varepsilon-\min\{d_1,d_2\}<0.$$

Letting $\varepsilon\to 0$, we have $\limsup_{t\to\infty}\frac{W(t)}{t} \leq -\min\{d_1,d_2\}<0$. Therefore we have that $\lim_{t\to\infty} y(t)=\lim_{t\to\infty} z(t)=0$, a.s. The proof is thus completed. □

5. Persistence of model (1.3)

Theorem 5.1. *For model* (1.3)*, when* $c_1, c_2>0$*, there exists a positive constant* K *such that*

$$P\{\omega:[kx(t)+y(t)+z(t)]<K\}>1-\varepsilon, \forall t\geq 0.$$

Proof. Let $M_1(t)=x^\theta(t)+y^\theta(t)+z^\theta(t)$. Applying *Itô's* formula to $M_1(t)$, we have

$$\begin{aligned}
\mathrm{d}M_1(t)=&e^t\Bigg\{x^\theta(t)+y^\theta(t)+z^\theta(t)+\theta x^{\theta-1}(t)\left[x(t)(r-ax(t))-\frac{bx(t)z(t)}{1+mx(t)}\right]\\
&+\frac{\theta(\theta-1)\sigma_1^2x^\theta(t)}{2}+x^\theta(t)\int_Y\left[(1+\gamma_1(u))^\theta-1-\theta\gamma_1(u)\right]\lambda(\mathrm{d}u)\\
&+\theta y^{\theta-1}(t)\left[\frac{kbx(t)z(t)}{1+mx(t)}-(D+d_1)y(t)\right]+\frac{\theta(\theta-1)\sigma_1^2y^\theta(t)}{2}\\
&+y^\theta(t)\int_Y\left[(1+\gamma_2(u))^\theta-1-\theta\gamma_2(u)\right]\lambda(\mathrm{d}u)+\theta z^{\theta-1}(t)[Dy(t)-d_2z(t)]\\
&+\frac{\theta(\theta-1)\sigma_3^2z^\theta(t)}{2}+z^\theta\int_Y\left[(1+\gamma_3(u))^\theta-1-\theta\gamma_3(u)\right]\lambda(\mathrm{d}u)\Bigg\}\mathrm{d}t\\
&+\theta e^t\sigma_1x^\theta(t)\mathrm{d}B_1(t)+\theta e^t\sigma_2y^\theta(t)\mathrm{d}B_2(t)+\theta e^t\sigma_3z^\theta(t)\mathrm{d}B_3(t)\\
&+e^tx^\theta(t)\int_Y\left((1+\gamma_1(u))^\theta-1\right)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&+e^ty^\theta(t)\int_Y\left((1+\gamma_2(u))^\theta-1\right)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&+e^tz^\theta(t)\int_Y\left((1+\gamma_3(u))^\theta-1\right)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
\leq& e^t\Bigg\{\theta x^\theta(t)\left(r+\frac{1}{\theta}+\frac{(\theta-1)\sigma_1^2}{2}+\frac{1}{\theta}\int_Y\left[(1+\gamma_1(u))^\theta-1-\theta\gamma_1(u)\right]\lambda(\mathrm{d}u)\right)
\end{aligned}$$

$$
\begin{aligned}
&- a\theta x^{\theta+1}(t) + \theta y^{\theta-1}(t)\frac{kbx(t)z(t)}{1+mx(t)} - c_1\theta y^{\theta}(t) + D\theta z^{\theta-1}(t)y(t) \\
&-c_2\theta z^{\theta}(t)\big\}\,\mathrm{d}t + \theta e^t\sigma_1 x^{\theta}(t)\mathrm{d}B_1(t) + \theta e^t\sigma_2 y^{\theta}(t)\mathrm{d}B_2(t) + \theta e^t\sigma_3 z^{\theta}(t)\mathrm{d}B_3(t) \\
&+ e^t x^{\theta}(t)\int_Y \big((1+\gamma_1(u))^{\theta}-1\big)\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
&+ e^t y^{\theta}(t)\int_Y \big((1+\gamma_2(u))^{\theta}-1\big)\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
&+ e^t z^{\theta}(t)\int_Y \big((1+\gamma_3(u))^{\theta}-1\big)\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
\overset{\triangle}{=}&e^t H(t)\mathrm{d}t + \theta e^t\sigma_1 x^{\theta}(t)\mathrm{d}B_1(t) + \theta e^t\sigma_2 y^{\theta}(t)\mathrm{d}B_2(t) + \theta e^t\sigma_3 z^{\theta}(t)\mathrm{d}B_3(t) \\
&+ e^t x^{\theta}(t)\int_Y \big((1+\gamma_1(u))^{\theta}-1\big)\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
&+ e^t y^{\theta}(t)\int_Y \big((1+\gamma_2(u))^{\theta}-1\big)\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
&+ e^t z^{\theta}(t)\int_Y \big((1+\gamma_3(u))^{\theta}-1\big)\,\widetilde{N}(\mathrm{d}t,\mathrm{d}u),
\end{aligned} \tag{5.1}
$$

where
$H(t) = \theta x^{\theta}(t)\left(r + \frac{1}{\theta} + \frac{(\theta-1)\sigma_1^2}{2} + \frac{1}{\theta}\int_Y \left[(1+\gamma_1(u))^{\theta} - 1 - \theta\gamma_1(u)\right]\lambda(\mathrm{d}u)\right)$
$-a\theta x^{\theta+1}(t) + \theta y^{\theta-1}(t)\frac{kbx(t)z(t)}{1+mx(t)} - c_1\theta y^{\theta}(t) + D\theta z^{\theta-1}(t)y(t) - c_2\theta z^{\theta}(t) \le g(\theta) < \infty.$
Integrating both sides of (5.1) from 0 to t and taking expectation, then we have

$$
e^t E(M_1(t)) = E(M_1(0)) + E\int_0^t e^s(H(s))\mathrm{d}s \le M_1(0) + g(\theta)e^t.
$$

According to the definition of $M_1(t)$, we have

$$
\limsup_{t\to\infty} E[x^{\theta}(t) + y^{\theta}(t) + z^{\theta}(t)] \le g(\theta).
$$

From $|x(t)+y(t)+z(t)| \le x(t)+y(t)+z(t)$, we have

$$
\limsup_{t\to\infty} E\left|x(t)+y(t)+z(t)\right| \le g(1) = g_1,
$$

$$
\limsup_{t\to\infty} E\left|x(t)+y(t)+z(t)\right|^2 \le g(2) = g_2.
$$

Let $M_2(t) = kx(t) + y(t) + z(t)$. Applying *Itô's* formula to $M_2(t)$, we have

$$
\begin{aligned}
\mathrm{d}M_2(t) =& \left[krx - krax^2 - (D+d_1)y + Dy - d_2 z\right]\mathrm{d}t + k\sigma_1 x\mathrm{d}B_1(t) + \sigma_2 y\mathrm{d}B_2(t) \\
&+ \sigma_3 z\mathrm{d}B_3(t) + k\int_Y \gamma_1(u)x(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u) + \int_Y \gamma_2(u)y(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
&+ \int_Y \gamma_3(u)z(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
\le& [krx + Dy]\mathrm{d}t + k\sigma_1 x\mathrm{d}B_1(t) + \sigma_2 y\mathrm{d}B_2(t) + \sigma_3 z\mathrm{d}B_3(t) \\
&+ k\int_Y \gamma_1(u)x(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u) + \int_Y \gamma_2(u)y(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u) \\
&+ \int_Y \gamma_3(u)z(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)
\end{aligned}
$$

$$
\begin{aligned}
\leq&[S(x+y+z)]\mathrm{d}t+k\sigma_1 x\mathrm{d}B_1(t)+\sigma_2 y\mathrm{d}B_2(t)+\sigma_3 z\mathrm{d}B_3(t)\\
&+k\int_Y \gamma_1(u)x(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)+\int_Y \gamma_2(u)y(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\
&+\int_Y \gamma_3(u)z(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u),
\end{aligned}
\tag{5.2}
$$

where $S=\max[kr,D,1]$. From (5.2), when $t\geq 0$, we have

$$
\begin{aligned}
&E\left[\sup_{t\leq s\leq t+1}[kx(s)+y(s)+z(s)]\right]\\
\leq&E[S(x(t)+y(t)+z(t))]+k\sigma_1 E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1} x(s)\mathrm{d}B_1(s)\right]\\
&+\sigma_2 E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1} y(s)\mathrm{d}B_2(s)\right]+\sigma_3 E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1} z(s)\mathrm{d}B_3(s)\right]\\
&+kE\left[\sup_{t\leq s\leq t+1}\int_t^{t+1}\int_Y \gamma_1(u)x(s)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right]\\
&+E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1}\int_Y \gamma_2(u)y(s)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right]\\
&+E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1}\int_Y \gamma_3(u)z(s)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right].
\end{aligned}
$$

By using Burkholder-Davis-Gundy inequality [28] and the Hölder inequality, we can get

$$
\begin{aligned}
&E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1} x(s)\mathrm{d}B_1(s)\right]\leq JE\left(\int_t^{t+1} x^2(s)\mathrm{d}s\right)^{\frac{1}{2}}\\
&\leq J\left(E\int_t^{t+1}|x(s)+y(s)+z(s)|^2\right)^{\frac{1}{2}},\\
&E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1} y(s)\mathrm{d}B_2(s)\right]\leq JE\left(\int_t^{t+1} y^2(s)\mathrm{d}s\right)^{\frac{1}{2}}\\
&\leq J\left(E\int_t^{t+1}|x(s)+y(s)+z(s)|^2\right)^{\frac{1}{2}},\\
&E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1} z(s)\mathrm{d}B_3(s)\right]\leq JE\left(\int_t^{t+1} z^2(s)\mathrm{d}s\right)^{\frac{1}{2}}\\
&\leq J\left(E\int_t^{t+1}|x(s)+y(s)+z(s)|^2\right)^{\frac{1}{2}}
\end{aligned}
\tag{5.3}
$$

and

$$
\begin{aligned}
&E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1}\int_Y \gamma_1(u)x(s)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right]\\
\leq& JE\left(\int_t^{t+1}\int_Y \gamma_1^2(u)x^2(s)N(\mathrm{d}s,\mathrm{d}u)\right)
\end{aligned}
$$

$$\leq J\left(\int_Y \gamma_1^2(u)\lambda(\mathrm{d}u)\right)^{\frac{1}{2}}\left(E\int_t^{t+1}|x(s)+y(s)+z(s)|^2\,\mathrm{d}s\right)^{\frac{1}{2}}. \tag{5.4}$$

Similarly, we have

$$E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1}\int_Y \gamma_2(u)y(s)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right]$$
$$\leq J\left(\int_Y \gamma_2(u)\lambda(\mathrm{d}u)\right)^{\frac{1}{2}}\left(E\int_t^{t+1}|x(s)+y(s)+z(s)|^2\,\mathrm{d}s\right)^{\frac{1}{2}},$$
$$E\left[\sup_{t\leq s\leq t+1}\int_t^{t+1}\int_Y \gamma_3(u)z(s)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\right]$$
$$\leq J\left(\int_Y \gamma_3(u)\lambda(\mathrm{d}u)\right)^{\frac{1}{2}}\left(E\int_t^{t+1}|x(s)+y(s)+z(s)|^2\,\mathrm{d}s\right)^{\frac{1}{2}}. \tag{5.5}$$

Submitting (5.3), (5.4) and (5.5) into (5.2) results in

$$E\left[\sup_{t\leq s\leq t+1}[kx(s)+y(s)+z(s)]\right]\leq Sg_1+(k\sigma_1+\sigma_2+\sigma_3)Jg_2^{\frac{1}{2}}+2C_1^{\frac{1}{2}}g_2^{\frac{1}{2}}.$$

Thus, there exists a positive constant G such that

$$E\left[\sup_{t\leq s\leq t+1}[kx(s)+y(s)+z(s)]\right]\leq G,\quad t=0,1,2,....$$

Let K be a sufficient large number such that $K=\frac{G}{\varepsilon}$. By Chebyshev inequality we obtain

$$P\left\{\omega:|kx(t)+y(t)+z(t)|>K\right\}\leq K^{-1}E\,|kx(t)+y(t)+z(t)|\leq K^{-1}G=\varepsilon.$$

Therefore, we have

$$P\left\{\omega:[kx(t)+y(t)+z(t)]\leq K\right\}>1-\varepsilon,\quad t\geq 0.$$

This completes the proof. □

Theorem 5.2. *Let$(x(t),y(t),z(t))$ be the solution of system with any initial value $(x(0),y(0),z(0))\in R_3^+$. We have*

(1) If $r-\frac{\sigma_1^2}{2}>0$, then $\limsup_{t\to\infty}\langle x\rangle_t\leq\widetilde{x^*}$; if $r-\frac{\sigma_1^2}{2}-H_1>0$, then $\liminf_{t\to\infty}\langle x\rangle_t\geq\widetilde{x_*}$. Here

$$\widetilde{x^*}=a^{-1}\left(r-\frac{\sigma_1^2}{2}\right),\ \widetilde{x_*}=\left(r-\frac{\sigma_1^2}{2}-H_1\right)(a+b(D+d_1)^{-1}d_2^{-1}Dkr)^{-1}.$$

(2) If $\kappa<\frac{r\widetilde{x_*}}{a}$, then we have

$$\limsup_{t\to\infty}\langle z\rangle_t\leq\eta_1,\quad \liminf_{t\to\infty}\langle z\rangle_t\geq\eta_2,$$

where

$$\eta_1=d_2^{-1}D(D+d_1)^{-1}kr\widetilde{x^*},\quad \eta_2=d_2^{-1}D(D+d_1)^{-1}(kr\widetilde{x_*}-ka\kappa).$$

Moreover, we have

$$\limsup_{t\to\infty}\langle y\rangle_t\leq(D+d_1)^{-1}[kr\widetilde{x_*}-ka\kappa],\quad \liminf_{t\to\infty}\langle y\rangle_t\geq(D+d_1)^{-1}ka\widetilde{x^*}.$$

Proof.

$$\begin{aligned}\mathrm{d}\ln x(t) =&\left[r - ax - \frac{bz}{1+mx} - \frac{\sigma_1^2}{2} + \int_Y (\ln(1+\gamma_1(u)) - \gamma_1(u))\lambda(\mathrm{d}u)\right]\mathrm{d}t\\ &+ \sigma_1 \mathrm{d}B_1(t) + \int_Y \ln(1+\gamma_1(u))\widetilde{N}(\mathrm{d}t,\mathrm{d}u). \end{aligned} \tag{5.6}$$

Integrating both sides of (5.6) from 0 to t, we can get

$$\begin{aligned}\ln x(t) - \ln x(0) = &\left(r - \frac{\sigma_1^2}{2}\right)t - a\int_0^t x(s)\mathrm{d}s - b\int_0^t \frac{z(s)}{1+mx(s)}\mathrm{d}s\\ &+ \sigma_1 B_1(t) - H_1 t + M_1(t).\end{aligned}$$

Then

$$\ln x(t) \le \left(r - \frac{\sigma_1^2}{2}\right)t - a\int_0^t x(s)\mathrm{d}s + F(t) := \lambda - \lambda_0\int_0^t x(s)\mathrm{d}s + F(t), \tag{5.7}$$

where $F(t) = \ln x(0) + M_1(t)$, and $\lim_{t\to\infty}\frac{F(t)}{t} = 0$. From Lemma 2.2, we know that

$$\limsup_{t\to\infty}\frac{1}{t}\int_0^t x(s)\mathrm{d}x \le \frac{\lambda}{\lambda_0},$$

which means

$$\limsup_{t\to\infty}\langle x\rangle_t \le a^{-1}\left(r - \frac{\sigma_1^2}{2}\right) = \widetilde{x^*}.$$

Let $M = Dkx + Dy + (D+d_1)z$, then we have

$$\begin{aligned}\mathrm{d}M =& Dk\mathrm{d}x + D\mathrm{d}y + (D+d_1)\mathrm{d}z\\ =&\left[Dkx(r-ax) - (D+d_1)d_2 z\right]\mathrm{d}t + Dk\sigma_1 x\mathrm{d}B_1(t) + D\sigma_2 y\mathrm{d}B_2(t)\\ &+ (D+d_1)\sigma_3 x\mathrm{d}B_3(t) + Dk\int_Y \gamma_1(u)x(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u)\\ &+ D\int_Y \gamma_2(u)y(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u) + (D+d_1)\int_Y \gamma_3(u)z(t^-)\widetilde{N}(\mathrm{d}t,\mathrm{d}u). \end{aligned}\tag{5.8}$$

Integrating (5.8) from 0 to t on both sides, we can get

$$\begin{aligned}&Dk(x(t)-x(0)) + D(y(t)-y(0)) + (D+d_1)(z(t)-z(0))\\ =&Dkr\int_0^t x(s)\mathrm{d}s - Dka\int_0^t x^2(s)\mathrm{d}s - (D+d_1)d_2\int_0^t z(s)\mathrm{d}s\\ &+ Dk\sigma_1\int_0^t x(s)\mathrm{d}B_1(s) + D\sigma_2\int_0^t y(s)\mathrm{d}B_2(s) + (D+d_1)\sigma_3\int_0^t z(s)\mathrm{d}B_3(s)\\ &+ Dk\int_0^t\int_Y \gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u) + D\int_0^t\int_Y \gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)\\ &+ (D+d_1)\int_0^t\int_Y \gamma_3(u)z(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u).\end{aligned}$$

Therefore, we can get

$$\int_0^t z(s)\mathrm{d}s = (D+d_1)^{-1}d_2^{-1}\left[Dkr\int_0^t x(s)\mathrm{d}s - Dka\int_0^t x^2(s)\mathrm{d}s\right.$$

$$
\begin{aligned}
&+ Dk\sigma_1 \int_0^t x(s)\mathrm{d}B_1(s) + Dk \int_0^t \int_Y \gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u) \\
&+ D \int_0^t \int_Y \gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u) + D\sigma_2 \int_0^t y(s)\mathrm{d}B_2(s) \\
&+ (D+d_1)\sigma_3 \int_0^t z(s)\mathrm{d}B_3(s) + (D+d_1) \int_0^t \int_Y \gamma_3(u)z(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u) \\
&- \big[Dk(x(t)-x(0)) + D(y(t)-y(0)) + (D+d_1)(z(t)-z(0))\big]\Bigg] \\
=&(D+d_1)^{-1}d_2^{-1}\left[Dkr\int_0^t x(s)\mathrm{d}x - Dka\int_0^t x^2(s)\mathrm{d}x\right] + \psi(t), \qquad (5.9)
\end{aligned}
$$

where

$$
\begin{aligned}
\psi(t) =&(D+d_1)^{-1}d_2^{-1}\Bigg[Dk\sigma_1 \int_0^t x(s)\mathrm{d}B_1(s) + D\sigma_2 \int_0^t y(s)\mathrm{d}B_2(s) \\
&+ (D+d_1)\sigma_3 \int_0^t z(s)\mathrm{d}B_3(s) + Dk \int_0^t \int_Y \gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u) \\
&+ D \int_0^t \int_Y \gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u) + (D+d_1) \int_0^t \int_Y \gamma_3(u)z(s^-)\widetilde{N}(\mathrm{d}s, \mathrm{d}u) \\
&- \big[Dk(x(t)-x(0)) + D(y(t)-y(0)) + (D+d_1)(z(t)-z(0))\big]\Bigg].
\end{aligned}
$$

Integrating both sides of (5.6) from 0 to t yields

$$
\begin{aligned}
\ln x(t) - \ln x(0) =&\left(r - \frac{\sigma_1^2}{2}\right)t - a\int_0^t x(s)\mathrm{d}s - b\int_0^t \frac{z(s)}{1+mx(s)}\mathrm{d}s + \sigma_1 B_1(t) \\
&- H_1 t + M_1(t) \\
\geq&\left(r - \frac{\sigma_1^2}{2} - H_1\right)t - a\int_0^t x(s)\mathrm{d}s - b\int_0^t z(s)\mathrm{d}s + \sigma_1 B_1(t) + M_1(t).
\end{aligned}
\tag{5.10}
$$

Substituting (5.9) into (5.10) yields

$$
\begin{aligned}
\ln x(t) - \ln x(0) \geq&\left(r - \frac{\sigma_1^2}{2} - H_1\right)t - a\int_0^t x(s)\mathrm{d}s + \sigma_1 B_1(t) + M_1(t) \\
&- b\left[(D+d_1)^{-1}d_2^{-1}\left(Dkr\int_0^t x(s)\mathrm{d}s - Dka\int_0^t x^2(s)\mathrm{d}s\right) + \psi(t)\right].
\end{aligned}
\tag{5.11}
$$

Dividing by t on both sides of (5.11), we have

$$
t^{-1}\ln\frac{x(t)}{x(0)} \geq\left(r - \frac{\sigma_1^2}{2} - H_1\right) - \left(a + b(D+d_1)^{-1}d_2^{-1}Dkr\right)\langle x\rangle_t
$$

$$
\begin{aligned}
&+ b(D+d_1)^{-1}d_2^{-1}t^{-1}Dka\int_0^t x^2(s)\mathrm{d}s + \varrho(t)\\
\geq&\left(r-\frac{\sigma_1^2}{2}-H_1\right)-\left(a+b(D+d_1)^{-1}d_2^{-1}Dkr\right)\langle x\rangle_t+\varrho(t)\\
:=&\lambda-\lambda_0\langle x\rangle_t+\varrho(t),
\end{aligned}
$$

where

$$
\varrho(t) = t^{-1}\bigg[-b\psi(t)+\sigma_1B_1(t)+M_1(t)\bigg],
$$

and $\lim_{t\to\infty}\varrho(t)=0$.
From lemma 2.2, we can get

$$
\liminf_{t\to\infty}\langle x\rangle_t \geq \left(r-\frac{\sigma_1}{2}-H_1\right)\big(a+b(D+d_1)^{-1}d_2^{-1}Dkr\big)=\widetilde{x_*}.
$$

Integrating the first two equations of the model (1.3) yields

$$
\begin{aligned}
&\frac{k(x(t)-x(0))+y(t)-y(0)}{t}\\
=&\frac{kr\int_0^t x(s)\mathrm{d}s}{t}-\frac{ka\int_0^t x^2(s)\mathrm{d}s}{t}-\frac{(D+d_1)\int_0^t y(s)\mathrm{d}s}{t}\\
&+\frac{k\sigma_1\int_0^t x(s)\mathrm{d}B_1(s)}{t}+\frac{\sigma_2\int_0^t y(s)\mathrm{d}B_2(s)}{t}+\frac{k\int_0^t\int_Y\gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}\\
&+\frac{\int_0^t\int_Y\gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}.
\end{aligned}
$$

Then we have

$$
\begin{aligned}
\frac{\int_0^t y(s)\mathrm{d}s}{t}=&(D+d_1)^{-1}\left[\frac{kr\int_0^t x(s)\mathrm{d}s}{t}-\frac{ka\int_0^t x^2(s)\mathrm{d}s}{t}+\frac{k\sigma_1\int_0^t x(s)\mathrm{d}B_1(s)}{t}\right.\\
&+\frac{\sigma_2\int_0^t y(s)\mathrm{d}B_2(s)}{t}+\frac{k\int_0^t\int_Y\gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}\\
&\left.+\frac{\int_0^t\int_Y\gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}-\frac{k(x(t)-x(0))+y(t)-y(0)}{t}\right].
\end{aligned}
\tag{5.12}
$$

Integrating the third formula of the model (1.3) yields

$$
\begin{aligned}
\frac{z(t)-z(0)}{t}=&\frac{D\int_0^t y(s)\mathrm{d}s}{t}-\frac{d_2\int_0^t z(s)\mathrm{d}s}{t}+\frac{\sigma_3\int_0^t z(s)\mathrm{d}B_3(s)}{t}\\
&+\frac{\int_0^t\int_Y\gamma_3(u)z(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}.
\end{aligned}
\tag{5.13}
$$

Substituting (5.12) into (5.13), we can get

$$
\frac{\int_0^t z(s)\mathrm{d}s}{t}=d_2^{-1}D(D+d_1)^{-1}\left[\frac{kr\int_0^t x(s)\mathrm{d}s}{t}-\frac{ka\int_0^t x^2(s)\mathrm{d}s}{t}+\frac{k\sigma_1\int_0^t x(s)\mathrm{d}B_1(s)}{t}\right.
$$

$$
\begin{aligned}
&+\frac{\sigma_2\int_0^t y(s)\mathrm{d}B_2(s)}{t}+\frac{k\int_0^t\int_Y\gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}\\
&+\left.\frac{\int_0^t\int_Y\gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}-\frac{k(x(t)-x(0))+y(t)-y(0)}{t}\right]\\
&+d_2^{-1}\left[\frac{\sigma_3\int_0^t z(s)\mathrm{d}B_3(s)}{t}+\frac{\int_0^t\int_Y\gamma_3(u)z(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}-\frac{z(t)-z(0)}{t}\right]\\
=&d_2^{-1}D(D+d_1)^{-1}\left[\frac{kr\int_0^t x(s)\mathrm{d}s}{t}-\frac{ka\int_0^t x^2(s)\mathrm{d}s}{t}\right]+\theta(t),
\end{aligned}\tag{5.14}
$$

where

$$
\begin{aligned}
\theta(t)=&d_2^{-1}D(D+d_1)^{-1}\left[\frac{k\sigma_1\int_0^t x(s)\mathrm{d}B_1(s)}{t}+\frac{\sigma_2\int_0^t y(s)\mathrm{d}B_2(s)}{t}\right.\\
&+\frac{k\int_0^t\int_Y\gamma_1(u)x(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}+\frac{\int_0^t\int_Y\gamma_2(u)y(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}\\
&-\left.\frac{k(x(t)-x(0))+y(t)-y(0)}{t}\right]\\
&+d_2^{-1}\left[\frac{\sigma_3\int_0^t z(s)\mathrm{d}B_3(s)}{t}+\frac{\int_0^t\int_Y\gamma_3(u)z(s^-)\widetilde{N}(\mathrm{d}s,\mathrm{d}u)}{t}-\frac{z(t)-z(0)}{t}\right].
\end{aligned}
$$

From Lemma 2.3, we can get $\lim_{t\to\infty}\theta(t)=0$.

Taking the limit of (5.14) as $t\to\infty$ yields

$$
\lim_{t\to\infty}t^{-1}\int_0^t z(s)\mathrm{d}s=d_2^{-1}D(D+d_1)^{-1}\left[\frac{kr\int_0^t x(s)\mathrm{d}s}{t}-\frac{ka\int_0^t x^2(s)\mathrm{d}s}{t}\right].
$$

From Theorem 5.1, there exists a positive constant κ such that

$$
0\le\left|\lim_{t\to\infty}\frac{\int_0^t x^2(s)\mathrm{d}s}{t}\right|\le\kappa.\tag{5.15}
$$

Therefore, we have

$$
\limsup_{t\to\infty}t^{-1}\int_0^t z(s)\mathrm{d}s\le d_2^{-1}D(D+d_1)^{-1}kr\widetilde{x^*}=\eta_1,
$$

$$
\liminf_{t\to\infty}t^{-1}\int_0^t z(s)\mathrm{d}s\ge d_2^{-1}D(D+d_1)^{-1}(kr\widetilde{x_*}-ka\kappa)=\eta_2.
$$

On the other hand, from equality (5.12) and Lemma 2.3, we know that

$$
\lim_{t\to\infty}\frac{\int_0^t y(s)\mathrm{d}s}{t}=(D+d_1)^{-1}\left[\frac{kr\int_0^t x(s)\mathrm{d}s}{t}-\frac{ka\int_0^t x^2(s)\mathrm{d}s}{t}\right].
$$

From (5.15), we know that

$$
\liminf_{t\to\infty}t^{-1}\int_0^t y(t)\mathrm{d}s\ge(D+d_1)^{-1}[kr\widetilde{x_*}-ka\kappa],
$$

$$\limsup_{t\to\infty} t^{-1}\int_0^t y(t)\mathrm{d}s \le (D+d_1)^{-1}ka\widetilde{x^*}.$$

This completes the proof. □

6. Numerical simulations

In this section, we will use Euler numerical approximation [29] to verify our results.

First of all, considering the absence of environmental noise, we select the parameters of the model as follows: $r = 0.6, a = 0.15, b = 0.1, m = 0.5, k = 0.3, D = 0.3, d_1 = 0.03, d_2 = 0.02, \sigma_1 = \sigma_2 = \sigma_3 = 0, \gamma_1 = \gamma_2 = \gamma_3 = 0$. At this time, model (1.3) is a deterministic system, and the dynamic behavior of system (1.3) is shown in the Fig. 1.

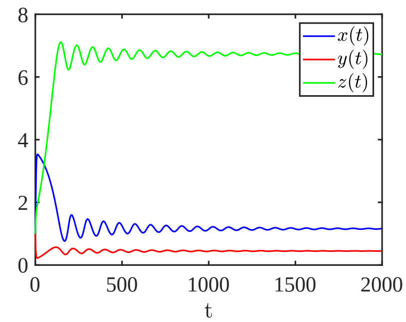


Figure 1. The time series of prey $x(t)$, immature predators $y(t)$, and mature predators $z(t)$. Here the blue line represents the prey $x(t)$, the red line represents the densities of immature predators $y(t)$ and the green line denotes mature predators $z(t)$.

Considering the influence of external environment on the system and keeping the inherent parameters unchanged, the parameters of environmental noise are selected as follows: $\sigma_1^2 = 1.44, \sigma_2^2 = 0.25, \sigma_3^2 = 0.01, \gamma_1 = 0.001, \gamma_2 = 0.002, \gamma_3 = 0.03$. According to Theorem 4.1, we can get $1.2 = 2r < \sigma_1^2 = 1.44$. From Fig. 2, we can see that system (1.3) is extinct with probability one.

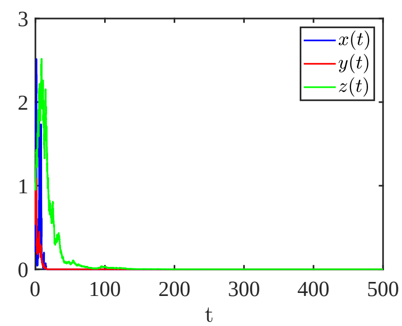


Figure 2. The time series of the extinction of immature and mature predator and prey with the $\sigma_1^2 = 1.44, \sigma_2^2 = 0.25, \sigma_3^2 = 0.01, \gamma_1 = 0.001, \gamma_2 = 0.002, \gamma_3 = 0.03$.

Now, we take the following parameter values:

$$\sigma_1^2 = 0.25, \sigma_2^2 = 0.0001, \sigma_3^2 = 0.0001, \gamma_1 = 0.001, \gamma_2 = 0.01, \gamma_3 = 0.01, \kappa = 0.4.$$

According to Theorem 5.1, we compute that

$$c_1 = D + d_1 - \frac{(\theta - 1)\sigma_2^2}{2} - \frac{1}{\theta} \int_Y \left[(1 + \gamma_2(u))^\theta - 1 - \theta\gamma_2(u)\right] \lambda(\mathrm{d}u) = 0.324658,$$

$$c_2 = d_2 - \frac{(\theta - 1)\sigma_3^2}{2} - \frac{1}{\theta} \int_Y \left[(1 + \gamma_3(u))^\theta - 1 - \theta\gamma_3(u)\right] \lambda(\mathrm{d}u) = 0.014658.$$

From Theorem 5.2, we can get

$$r - \frac{\sigma_1^2}{2} - H_1 = 0.475 > 0, \kappa < \frac{r\widetilde{x_*}}{a} = 2.2708,$$

$$\widetilde{x^*} = a^{-1} \left(r - \frac{\sigma_1^2}{2}\right) = 3.1663,$$

$$\widetilde{x_*} = \left(r - \frac{\sigma_1^2}{2} - H_1\right) (a + b(D + d_1)^{-1} d_2^{-1} Dkr)^{-1} = 0.5677,$$

$$(D + d_1)^{-1}[kr\widetilde{x_*} - ka\kappa] = 0.0909,\ (D + d_1)^{-1}ka\widetilde{x^*} = 0.4318,$$

$$\eta_1 = d_2^{-1} D(D + d_1)^{-1} kr\widetilde{x^*} = 25.9094, \eta_2 = d_2^{-1} D(D + d_1)^{-1} (kr\widetilde{x_*} - ka\kappa) = 1.3727.$$

Then, we have

$$0.5677 \le \liminf_{t\to\infty} t^{-1} \int_0^t x(s)\mathrm{d}s \le \limsup_{t\to\infty} t^{-1} \int_0^t x(s)\mathrm{d}s \le 3.1667,$$

$$0.0909 \le \liminf_{t\to\infty} t^{-1} \int_0^t y(s)\mathrm{d}s \le \limsup_{t\to\infty} t^{-1} \int_0^t y(s)\mathrm{d}s \le 0.4318,$$

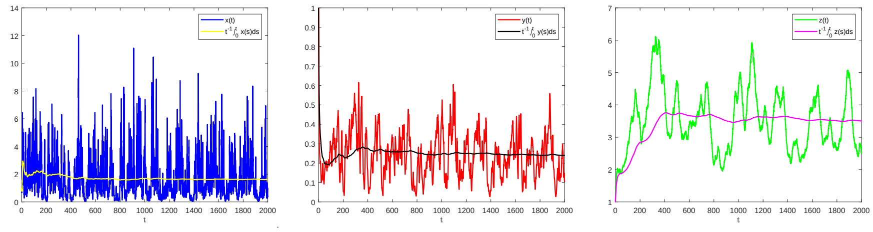
$$1.3727 \le \liminf_{t\to\infty} t^{-1} \int_0^t z(s)\mathrm{d}s \le \limsup_{t\to\infty} t^{-1} \int_0^t z(s)\mathrm{d}s \le 25.9094.$$

From Fig. 3(a), we can see that the prey x is persistent in the mean, and from Figs. 3(b) and 3(c) we can see that immature predator y and mature predator z are both persistent in the mean. This means that model (1.3) is persistent in the mean.

Finally, we keep the other parameters unchanged and change the intensity of the noise. Letting $\gamma_1 = 0.3$, $\gamma_2 = 0.11$, $\gamma_3 = 0.1$, from Fig. 4 we know that system (1.3) is extinct. Comparing Fig. 3 with Fig. 4, we can find that the Lévy jumps may suppress the survival of the species.

7. Conclusions

In this paper, we analyze a stochastic predator-prey model with the stage structure for predator and Holling type II functional reaction. By constructing appropriate Lyapunov functions, we first prove that the proposed model exists a uniqueness global positive solution. Then we obtain the sufficient conditions for the extinction and persistence in the mean of the proposed model. Finally, some numerical



(a) (b) (c)

Figure 3. The time series of the immature and mature predator and prey with $\sigma_1^2 = 0.25, \sigma_2^2 = 0.0001, \sigma_3^2 = 0.0001, \gamma_1 = 0.001, \gamma_2 = 0.01, \gamma_3 = 0.01, \kappa = 0.4$.

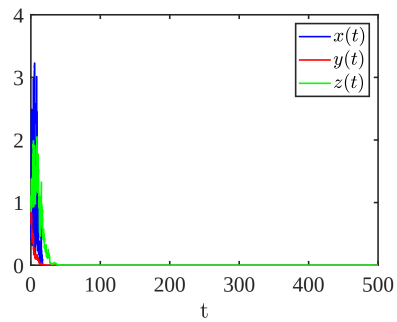


Figure 4. The time series of immature and mature predators and prey with $\gamma_1 = 0.3$, $\gamma_2 = 0.11$, $\gamma_3 = 0.1$.

simulations are carried out to verify the correctness of the theoretical results. By numerical results, we find that large environmental noise is not conducive to the survival of species, even leads to species extinction.

Some interesting questions deserve further investigation. On one hand, one can consider other functional responses of model (1.3). On the other hand, one can introduce the continuous-time Markov chain or impulsive effects into model (1.3). Of course, these investigations will be more complex, and we will devote ourselves to these investigations in the future.

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