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An Existence Result for a Class of Chemically Reacting Systems with Sign-Changing Weights

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Abstract. We prove the existence of positive solutions for the system

$$\begin{cases} -\Delta_p u = \lambda a(x) f(v) u^{-\alpha}, & x \in \Omega, \\ -\Delta_q v = \lambda b(x) g(u) v^{-\beta}, & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

where $\Delta_r z = \operatorname{div}(|\nabla z|^{r-2}\nabla z)$, for $r > 1$ denotes the r-Laplacian operator and λ is a positive parameter, Ω is a bounded domain in $\mathbb{R}^n$, $n \geq 1$ with sufficiently smooth boundary and $\alpha, \beta \in (0,1)$. Here $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe negative near the boundary and f, g are C^1 nondecreasing functions, such that $f, g\colon [0,\infty) \to [0,\infty)$; $f(s) > 0$, $g(s) > 0$ for $s > 0$, $\lim_{s\to\infty} g(s) = \infty$ and

$$\lim_{s\to\infty} \frac{f(Mg(s)^{\frac{1}{q-1}})}{s^{p-1+\alpha}} = 0, \qquad \forall M > 0.$$

We discuss the existence of positive weak solutions when f, g, $a(x)$ and $b(x)$ satisfy certain additional conditions. We employ the method of sub-supersolution to obtain our results.

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Key Words: Positive solutions; chemically reacting systems; sub-supersolutions.

1 Introduction

In this paper, we consider the existence of positive weak solutions for the nonlinear singular system

$$\begin{cases} -\Delta_p u = \lambda a(x) \frac{f(v)}{u^\alpha}, & x \in \Omega, \\ -\Delta_q v = \lambda b(x) \frac{g(u)}{v^\beta}, & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

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where $\Delta_r z=\mathrm{div}(|\nabla z|^{r-2}\nabla z)$, for $r>1$ denotes the r-Laplacian operator and Ω is a bounded domain in $\mathbb{R}^n$, $n\geq 1$ with smooth boundary, $\alpha,\beta\in(0,1)$. Here $a(x)$ and $b(x)$ are C^1 sing-changing functions that maybe negative near the boundary and f,g are C^1 nondecreasing functions such that $f,g:[0,\infty)\to[0,\infty)$; $f(s)>0$, $g(s)>0$ for $s>0$.

Systems of singular equations like (1.1) are the stationary counterpart of general evolutionary problems of the form

$$\begin{cases} u_t=\eta\Delta_p u+\lambda\frac{f(v)}{u^\alpha}, & x\in\Omega,\\ v_t=\delta\Delta_q v+\lambda\frac{g(u)}{v^\beta}, & x\in\Omega,\\ u=v=0, & x\in\partial\Omega,\end{cases}$$

where η and δ are positive parameters. This system is motivated by an interesting applications in chemically reacting systems, where u represents the density of an activator chemical substance and v is an inhibitor. The slow diffusion of u and the fast diffusion of v is translated into the fact that η is small and δ is large (see [1]).

Recently, such problems have been studied in [2–4]. Also in [2], the authors have studied the existence results for the system (1.1) in the case $a\equiv 1$, $b\equiv 1$. Here we focus on further extending the study in [2] to the system (1.1). In fact, we study the existence of positive solution to the system (1) with sign-changing weight functions $a(x),b(x)$. Due to these weight functions, the extensions are challenging and nontrivial. Our approach is based on the method of sub-super solutions (see [5–7]).

To precisely state our existence result we consider the eigenvalue problem

$$\begin{cases} -\Delta_r\phi=\lambda|\phi|^{r-2}\phi, & x\in\Omega,\\ \phi=0, & x\in\partial\Omega.\end{cases} \tag{1.2}$$

Let $\phi_{1,r}$ be the eigenfunction corresponding to the first eigenvalue $\lambda_{1,r}$ of (1.2) such that $\phi_{1,r}(x)>0$ in Ω, and $\|\phi_{1,r}\|_\infty=1$ for $r=p,q$. Let $m,\sigma,\delta>0$ be such that

$$\sigma\leq\phi_{1,r}\leq 1,\quad x\in\Omega-\overline{\Omega_\delta}, \tag{1.3}$$

$$\lambda_{1,r}\phi_{1,r}^r-\left(1-\frac{sr}{r-1+s}\right)|\nabla\phi_{1,r}|^r\leq -m,\quad x\in\overline{\Omega_\delta}, \tag{1.4}$$

for $r=p,q$, and $s=\alpha,\beta$, where $\overline{\Omega_\delta}:=\{x\in\Omega|d(x,\partial\Omega)\leq\delta\}$. (This is possible since $|\nabla\phi_{1,r}|^r\neq 0$ on $\partial\Omega$ while $\phi_{1,r}=0$ on $\partial\Omega$ for $r=p,q$. We will also consider the unique solution $\zeta_r\in W_0^{1,r}(\Omega)$ of the boundary value problem

$$\begin{cases} -\Delta_r\zeta_r=1, & x\in\Omega,\\ \zeta_r=0, & x\in\partial\Omega,\end{cases}$$

to discuss our existence result, it is known that $\zeta_r>0$ in Ω and $\partial\zeta_r/\partial n<0$ on $\partial\Omega$.

Here we assume that the weight functions $a(x)$ and $b(x)$ take negative values in $\overline{\Omega_\delta}$, but require $a(x)$ and $b(x)$ be strictly positive in $\Omega-\overline{\Omega_\delta}$. To be precise we assume that there exist positive constants a_0 , a_1 , b_0 and b_1 such that $a(x)\geq -a_0$, $b(x)\geq -b_0$ on $\overline{\Omega_\delta}$ and $a(x)\geq a_1$, $b(x)\geq b_1$ on $\Omega-\overline{\Omega_\delta}$.

2 Existence result

In this section, we shall establish our existence result via the method of sub - supersolution. A pair of nonnegative functions $(\psi_1,\psi_2)\in W^{1,p}\cap C(\overline{\Omega})\times W^{1,q}\cap C(\overline{\Omega})$ and $(z_1,z_2)\in W^{1,p}\cap C(\overline{\Omega})\times W^{1,q}\cap C(\overline{\Omega})$ are called a subsolution and supersulution of (1.1) if they satisfy $(\psi_1,\psi_2)=(0,0)=(z_1,z_2)$ on $\partial\Omega$ and

$$\begin{aligned}
&\int_\Omega |\nabla\psi_1|^{p-2}\nabla\psi_1\cdot\nabla w\,\mathrm{d}x\leq\lambda\int_\Omega a(x)\frac{f(\psi_2)}{\psi_1^\alpha}w\,\mathrm{d}x,\\
&\int_\Omega |\nabla\psi_2|^{q-2}\nabla\psi_2\cdot\nabla w\,\mathrm{d}x\leq\lambda\int_\Omega b(x)\frac{g(\psi_1)}{\psi_2^\beta}w\,\mathrm{d}x,\\
&\int_\Omega |\nabla z_1|^{p-2}\nabla z_1\cdot\nabla w\,\mathrm{d}x\geq\lambda\int_\Omega a(x)\frac{f(z_2)}{z_1^\alpha}w\,\mathrm{d}x,\\
&\int_\Omega |\nabla z_2|^{q-2}\nabla z_2\cdot\nabla w\,\mathrm{d}x\geq\lambda\int_\Omega b(x)\frac{g(z_1)}{z_2^\beta}w\,\mathrm{d}x,
\end{aligned}$$

for all $w\in W=\{w\in C_0^\infty(\Omega)|w\geq 0,x\in\Omega\}$. Then the following result holds:

Lemma 2.1. (See [5]) *Suppose there exist sub and supersolutions (ψ_1,ψ_2) and (z_1,z_2) respectively of (1.1) such that $(\psi_1,\psi_2)\leq(z_1,z_2)$. Then (1.1) has a solution (u,v) such that $(u,v)\in[(\psi_1,\psi_2),(z_1,z_2)]$.*

To state our results precisely we introduce the following hypotheses :

- **(H1)** $f,g:[0,\infty)\to[0,\infty)$ are C^1 nondecreasing functions such that $f(s)$, $g(s)>0$ for $s>0$, and $\lim_{s\to\infty}g(s)=\infty$.
- **(H2)** $\lim_{s\to\infty}\frac{f(Mg(s)^{\frac{1}{q-1}})}{s^{p-1+\alpha}}=0$, for all $M>0$.
- **(H3)** Suppose that there exists $\epsilon>0$ such that:

$$\frac{\epsilon^{\frac{p-1+\alpha}{p-1}}\lambda_{1,p}}{a_1 f(\frac{q-1+\beta}{q}\epsilon^{\frac{1}{q-1}}\sigma^{\frac{q}{q-1+\beta}})}\leq\min\left\{\frac{m\epsilon^{\frac{\alpha+p-1}{p-1}}(\frac{p-1+\alpha}{p})^\alpha}{a_0 f(\epsilon^{\frac{1}{q-1}})},\frac{m\epsilon^{\frac{\beta+q-1}{q-1}}(\frac{q-1+\beta}{q})^\beta}{b_0 g(\epsilon^{\frac{1}{p-1}})}\right\},$$

$$\frac{\epsilon^{\frac{q-1+\beta}{q-1}}\lambda_{1,q}}{b_1 g(\frac{p-1+\alpha}{p}\epsilon^{\frac{1}{p-1}}\sigma^{\frac{p}{p-1+\alpha}})}\leq\min\left\{\frac{m\epsilon^{\frac{\alpha+p-1}{p-1}}(\frac{p-1+\alpha}{p})^\alpha}{a_0 f(\epsilon^{\frac{1}{q-1}})},\frac{m\epsilon^{\frac{\beta+q-1}{q-1}}(\frac{q-1+\beta}{q})^\beta}{b_0 g(\epsilon^{\frac{1}{p-1}})}\right\}.$$

We are now ready to give our existence result .

Theorem 2.1. *Assume that*

(a) $p \geq n$ *or* (b) $p < n$ *and* $\alpha < \frac{p}{n}$,

(c) $q \geq n$ *or* (d) $q < n$ *and* $\beta < \frac{q}{n}$.

Let (H1)-(H3) *hold. Then there exists a positive solution of* (1.1) *for every* $\lambda \in [\lambda_*(\epsilon), \lambda^*(\epsilon)]$, *where*

$$\lambda^* = \min\left\{ \frac{m\epsilon^{\frac{\alpha+p-1}{p-1}}(\frac{p-1+\alpha}{p})^{\alpha}}{a_0 f(\epsilon^{\frac{1}{q-1}})}, \frac{m\epsilon^{\frac{\beta+q-1}{q-1}}(\frac{q-1+\beta}{q})^{\beta}}{b_0 g(\epsilon^{\frac{1}{p-1}})} \right\},$$

and

$$\lambda_* = \max\left\{ \frac{\epsilon^{\frac{\alpha+p-1}{p-1}}\lambda_{1,p}}{a_1 f(\frac{q-1+\beta}{q}\epsilon^{\frac{1}{q-1}}\sigma^{\frac{q}{q-1+\beta}})}, \frac{\epsilon^{\frac{\beta+q-1}{q-1}}\lambda_{1,q}}{b_1 g(\frac{p-1+\beta}{p}\epsilon^{\frac{1}{p-1}}\sigma^{\frac{p}{p-1+\alpha}})} \right\}.$$

Remark 2.1. Note that $(H3)$ implies $\lambda_* < \lambda^*$.

Example 2.1. Let $f(s) = s^4 - 1$, $g(s) = s - 1$ and ($p = q = 3$, $n < 6$, $\alpha = 1/2$, $\beta = 1/2$). Here $f(s)$, $g(s) > 0$ for $s > 0$, f, g are non-decreasing functions and

$$\lim_{s\to\infty} \frac{f(Mg(s)^{\frac{1}{2}})}{s^{\frac{5}{2}}} = 0,$$

for all $M > 0$, and $\lim_{s\to\infty} g(s) = \infty$. We can choose $\epsilon > 0$ so small that f,g satisfy (H3).

Proof of Theorem 2.1 We shall verify that

$$(\psi_1, \psi_2) = \left(\frac{p-1+\alpha}{p}\epsilon^{\frac{1}{p-1}}\phi_{1,p}^{\frac{p}{p-1+\alpha}}, \quad \frac{q-1+\beta}{q}\epsilon^{\frac{1}{q-1}}\phi_{1,q}^{\frac{q}{q-1+\beta}} \right),$$

is a sub-solution of (1.1). Let $w \in W$. Then a calculation shows that

$$\nabla\psi_1 = \epsilon^{\frac{1}{p-1}}\nabla\phi_{1,p}\phi_{1,p}^{\frac{1-\alpha}{p-1+\alpha}},$$

and we have

$$\begin{aligned}\int_\Omega |\nabla\psi_1|^{p-2}\nabla\psi_1\cdot\nabla w\,\mathrm{d}x =& \epsilon\int_\Omega \phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}}|\nabla\phi_{1,p}|^{p-2}\nabla\phi_{1,p}\nabla w\,\mathrm{d}x \\ =& \epsilon\int_\Omega |\nabla\phi_{1,p}|^{p-2}\nabla\phi_{1,p}\left\{\nabla(\phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}}w) - w\nabla(\phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}})\right\}\mathrm{d}x \\ =& \epsilon\left\{\int_\Omega \left[\lambda_{1,p}\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}} - |\nabla\phi_{1,p}|^{p-2}\nabla\phi_{1,p}\nabla(\phi_{1,p}^{1-\frac{\alpha p}{p-1+\alpha}})\right] w\,\mathrm{d}x\right\}\end{aligned}$$

$$
\begin{aligned}
&=\epsilon\left\{\int_\Omega \left[\lambda_{1,p}\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}}-|\nabla\phi_{1,p}|^p(1-\frac{\alpha p}{p-1+\alpha})\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}\right]w\mathrm{d}x\right\}\\
&=\epsilon\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}\left\{\int_\Omega \left[\lambda_{1,p}\phi_{1,p}^{p}-|\nabla\phi_{1,p}|^p(1-\frac{\alpha p}{p-1+\alpha})\right]w\mathrm{d}x\right\}.
\end{aligned}
$$

Similarly

$$
\int_\Omega |\nabla\psi_2|^{q-2}\nabla\psi_2\cdot\nabla w\mathrm{d}x=\epsilon\phi_{1,q}^{-\frac{\beta q}{q-1+\beta}}\left\{\int_\Omega [\lambda_{1,q}\phi_{1,q}^{q}-|\nabla\phi_{1,q}|^q(1-\frac{\beta q}{q-1+\beta})]w\mathrm{d}x\right\}.
$$

First we consider the case when $x\in\overline{\Omega_\delta}$. We have

$$
\lambda_{1,p}\phi_{1,p}^{p}-(1-\frac{\alpha p}{p-1+\alpha})|\nabla\phi_{1,p}|^p\le -m.
$$

Since $\lambda\le\lambda^*$ then

$$
\lambda\le\frac{m\epsilon^{\frac{\alpha+p-1}{p-1}}(\frac{p-1+\alpha}{p})^\alpha}{a_0 f(\epsilon^{\frac{1}{q-1}})}.
$$

Hence

$$
\begin{aligned}
&\epsilon\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}\left(\lambda_{1,p}\phi_{1,p}^{p}-(1-\frac{\alpha p}{p-1+\alpha})|\nabla\phi_{1,p}|^p\right)\le -m\epsilon\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}\\
&\le-\lambda a_0\frac{f(\epsilon^{\frac{1}{q-1}})(\frac{p-1+\alpha}{p})^{-\alpha}\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}}{\epsilon^{\frac{\alpha}{p-1}}}\le-\lambda a_0\frac{f(\frac{q-1+\beta}{q}\epsilon^{\frac{1}{q-1}}\phi_{1,q}^{\frac{q}{q-1+\beta}})(\frac{p-1+\alpha}{p})^{-\alpha}\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}}{\epsilon^{\frac{\alpha}{p-1}}}\\
&\le\lambda a(x)\frac{f(\psi_2)}{\psi_1^\alpha}.
\end{aligned}
$$

A similar argument shows that

$$
-m\epsilon\phi_{1,q}^{-\frac{\beta q}{q-1+\beta}}\left(\lambda_{1,q}\phi_{1,q}^{q}-(1-\frac{\beta q}{q-1+\beta})|\nabla\phi_{1,q}|^q\right)\le\lambda b(x)\frac{g(\psi_1)}{\psi_2^\beta}.
$$

On the other hand, on $\Omega-\overline{\Omega_\delta}$, we have $1\ge\phi_{1,r}\ge\sigma$ for $r=p,q$. Also $a(x)\ge a_1$, $b(x)\ge b_1$ and since $\lambda\ge\lambda_*$, we have

$$
\lambda\ge\frac{\epsilon^{\frac{\alpha+p-1}{p-1}}\lambda_{1,p}}{a_1 f(\frac{q-1+\beta}{q}\epsilon^{\frac{1}{q-1}}\sigma^{\frac{q}{q-1+\beta}})}.
$$

Hence

$$
\epsilon\phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}}\left(\lambda_{1,p}\phi_{1,p}^{p}-(1-\frac{\alpha p}{p-1+\alpha})|\nabla\phi_{1,p}|^p\right)\le\epsilon\lambda_{1,p}\phi_{1,p}^{p-\frac{\alpha p}{p-1+\alpha}}
$$

$$\leq \epsilon \lambda_{1,p} \phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}} \leq \lambda a_1 \frac{f\big(\frac{q-1+\beta}{q} \epsilon^{\frac{1}{q-1}} \sigma^{\frac{q}{q-1+\beta}}\big) \phi_{1,p}^{-\frac{\alpha p}{p-1+\alpha}} \big(\frac{p-1+\alpha}{p}\big)^{-\alpha}}{\epsilon^{\frac{\alpha}{p-1}}} \leq \lambda a(x) \frac{f(\psi_2)}{\psi_1^{\alpha}}.$$

A similar argument shows that

$$\epsilon \phi_{1,q}^{-\frac{\beta q}{q-1+\beta}} \big(\lambda_{1,q} \phi_{1,q}^{q} - (1 - \frac{\beta q}{q-1+\beta}) |\nabla \phi_{1,q}|^{q}\big) \leq \lambda b(x) \frac{g(\psi_1)}{\psi_2^{\beta}}.$$

Hence

$$\int_{\Omega} |\nabla \psi_1|^{p-2} \nabla \psi_1 \cdot \nabla w \mathrm{d}x \leq \lambda \int_{\Omega} a(x) \frac{f(\psi_2)}{\psi_1^{\alpha}} w \mathrm{d}x,$$

and

$$\int_{\Omega} |\nabla \psi_2|^{q-2} \nabla \psi_2 \cdot \nabla w \mathrm{d}x \leq \lambda \int_{\Omega} b(x) \frac{g(\psi_1)}{\psi_2^{\beta}} w \mathrm{d}x.$$

Thus, $(\psi_1,\psi_2) = \big(\frac{p-1+\alpha}{p} \epsilon^{\frac{1}{p-1}} \phi_{1,p}^{\frac{p}{p-1+\alpha}}, \frac{q-1+\beta}{q} \epsilon^{\frac{1}{q-1}} \phi_{1,q}^{\frac{q}{q-1+\beta}}\big)$ is a positive subsolution of (1.1).

Now, we construct a suupersolution $(z_1,z_2) \geq (\psi_1,\psi_2)$. When

(a) $p \geq n$ or (b) $p < n$ and $\alpha < \frac{p}{n}$,

(c) $q \geq n$ or (d) $q < n$ and $\beta < \frac{q}{n}$,

from [8], we know that are functions $w_1 \in W_0^{1,p}(\Omega) \cap C(\overline{\Omega})$ and $w_2 \in W_0^{1,q}(\Omega) \cap C(\overline{\Omega})$ such that

$$\begin{cases} -\Delta_p w_1 = \frac{1}{w_1^{\alpha}}, & x \in \Omega, \\ w_1 = 0, & x \in \partial\Omega, \end{cases}$$

and

$$\begin{cases} -\Delta_q w_2 = \frac{1}{w_2^{\beta}}, & x \in \Omega, \\ w_2 = 0, & x \in \partial\Omega, \end{cases}$$

are satisfying $w_1 \geq \theta \zeta_p$ and $w_2 \geq \theta \zeta_q$ for some $\theta > 0$. Now, we will prove there exists $c \gg 1$ such that

$$(z_1,z_2) = (c w_1, g(c\|w_1\|_{\infty})^{\frac{1}{q-1}} w_2),$$

is a supersolution of (1.1). A calculation shows that :

$$\int_{\Omega} |\nabla z_1|^{p-2} \nabla z_1 \cdot \nabla w \mathrm{d}x = c^{p-1} \int_{\Omega} |\nabla w_1|^{p-2} \nabla w_1 \cdot \nabla w \mathrm{d}x = c^{p-1} \int_{\Omega} \frac{w}{w_1^{\alpha}} \mathrm{d}x,$$

by (H2) we know that, for $c \gg 1$,

$$\frac{1}{\lambda \|a(x)\|_\infty} \geq \frac{f(\|w_2\|_\infty (g(c\|w_1\|_\infty))^{\frac{1}{q-1}})}{c^{p-1+\alpha}}.$$

Hence

$$\begin{aligned}\frac{c^{p-1}}{w_1^\alpha} &\geq \lambda \|a(x)\|_\infty \frac{f(\|w_2\|_\infty (g(c\|w_1\|_\infty))^{\frac{1}{q-1}})}{(cw_1)^\alpha} \\ &\geq \lambda a(x) \frac{f(w_2 (g(c\|w_1\|_\infty))^{\frac{1}{q-1}})}{(cw_1)^\alpha} = \lambda a(x) \frac{f(z_2)}{z_1^\alpha},\end{aligned}$$

now from (H1), we know that $g(s) \to \infty$ as $s \to \infty$. Thus, for $c \gg 1$

$$\frac{\lambda \|b(x)\|_\infty}{g(c\|w_1\|_\infty)^{\frac{\beta}{q-1}}} \leq 1,$$

and we have for $c \gg 1$,

$$\begin{aligned}&\int_\Omega |\nabla z_2|^{q-2} \nabla z_2 \cdot \nabla w \mathrm{d}x = \int_\Omega \frac{g(c\|w_1\|_\infty)}{w_2^\beta} w \mathrm{d}x \\ \geq &\lambda \|b(x)\|_\infty \int_\Omega \frac{g(cw_1)}{g(c\|w_1\|_\infty)^{\frac{\beta}{q-1}} w_2^\beta} w \mathrm{d}x \geq \lambda \int_\Omega b(x) \frac{g(z_1)}{z_2^\beta} w \mathrm{d}x,\end{aligned}$$

i.e., (z_1,z_2) is a supersolution of (1.1). Furthermore, c can be chosen large enough so that $(z_1,z_2) \geq (\psi_1,\psi_2)$, since $g(s) \to \infty$ as $s \to \infty$. Thus, there exist a positive solution (u,v) of (1) such that $(\psi_1,\psi_2) \leq (u,v) \leq (z_1,z_2)$. This completes the proof of Theorem 2.1. $\square$

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