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Unsteady Hydromagnetic Couette Flow within Porous plates in a Rotating System

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Abstract. Unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system is studied when the fluid flow within the channel is induced due to the impulsive movement of the one of the plates of the channel. The plates of the channel are considered porous and the magnetic field is fixed relative to the moving plate. Exact solution of the governing equations is obtained by Laplace transform technique. The expression for the shear stress at the moving plate is also obtained. Asymptotic behaviour of the solution is analyzed for small as well as large values of time t to highlight the transient approach to the final steady state flow and the effects of rotation, magnetic field and suction/injection. It is found that suction has retarding influence on the primary as well as secondary flow where as injection and time have accelerating influence on the primary and secondary flows.

AMS subject classifications: 76W05, 76U05

Key words: MHD Couette flow, suction/injection, magnetic field, impulsive movement of the plate, Ekman-Hartmann boundary layer, Rayleigh boundary layer, spatial oscillations, inertial oscillations.

1 Introduction

Theoretical and/or experimental investigation of the problems of the flow of an electrically conducting fluid in the presence of electromagnetic fields is carried out by many researchers under different conditions and configurations to discuss various aspects of the problems and to find its application in science and engineering. There are many natural phenomena and engineering problems susceptible to magnetohydrodynamic analysis. It is useful in Astrophysics because much of the universe is filled with

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widely spaced charged particles and permeated by magnetic fields. Geophysicists encounter MHD phenomena in the interactions of conducting fluids and magnetic fields that are present in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, pumps and flow meters, in solving space vehicle propulsion, control and reentry problems; in creating novel power generating systems and in developing confinement schemes for controlled fusion.

In general, the governing equations of MHD flow problems are inherently nonlinear. Simplified models are, therefore, studied in literature with a view to analyze different aspects of fluid flow features. Of these models, the one corresponding to MHD Couette flow is known to lead to the equations for which analytical solution can be obtained in principle [1–9]. The study of unsteady MHD Couette flow is important from practical point of view because fluid transient may be expected at the start-up time of MHD devices, namely, MHD generators, MHD pumps, MHD accelerators, flow meters and nuclear reactors. Keeping in view this fact Katagiri [2] investigated unsteady MHD Couette flow of a viscous incompressible electrically conducting fluid in the presence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to the impulsive motion of one of the plates. Katagiri [2] analyzed this problem when the magnetic field is fixed relative to fluid. Singh and Kumar [9] considered the problem studied by Katagiri [2] when the magnetic field is fixed relative to the moving plate. They also studied this problem when the fluid motion within the channel is induced due to uniformly accelerated movement of one of the plates.

The theory of rotating fluids [10] is highly important due to its occurrence in various natural phenomena and for its applications in various technological situations which are directly governed by the action of Coriolis force. The broad subjects of Oceanography, Meteorology, Atmospheric science and Limnology all contain some important and essential features of rotating fluids. The fluid flow problems in rotating medium have attracted many scholars and there appeared a number of studies in literature viz. Greenspan and Howard [11], Holton [12], Walin [13], Siegmann [14], Puri [15], Puri and Kulshrestha [16], Mazumder [17], Ganapathy [18], Das et al. [19], Hayat et al. [20] and Hayat and Hutter [21]. The study of simultaneous effects of rotation and magnetic field on MHD flow problem may find applications in the areas of Geophysics, Astrophysics and fluid engineering. Keeping in view this fact, several researchers, namely, Hide and Roberts [22], Nanda and Mohanty [23], Gupta [24], Sounalgekar and Pop [25], Gupta and Soundalgekar [26], Debnath [27, 28], Acheson [29], Seth and Jana [30], Seth and Maiti [31], Prasad Rao et al. [32], Seth et al. [33, 34], Chandran et al. [35], Singh et al. [36], Singh [37], Hayat et al. [38–44], Hayat and Abelman [45], Wang and Hayat [46] and Das et al. [47] investigated MHD flow problems in rotating medium considering different aspects of the problems.

Unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating channel is investigated by Seth et al. [33, 34], Singh [37] and Das et al. [47] when the magnetic field is fixed relative to the fluid where as Chandran et al. [35] and Singh et al. [36] considered this problem when the magnetic field is fixed relative to the moving plate. In all these investigations, the channel walls are considered non-porous. However, the study of such fluid flow problem in porous channel may find applications in petroleum, mineral and metallurgical industries, designing of cooling systems with liquid metals, MHD generators, MHD pumps, MHD accelerators and flow meters, geothermal reservoirs and underground energy transport etc. Taking into account this fact Muhuri [4], Prasad Rao et al. [32], Bhaskara Reddy and Bathaiah [48], Singh [49], Abbas et al. [50] and Hayat et al. [51, 52] considered MHD flow within a parallel plate channel with porous boundaries under different conditions, in rotating or non-rotating system.

The aim of the present paper is to study unsteady hydromagnetic Couette flow of a viscous incompressible electrically conducting fluid in a rotating system in the presence of a uniform transverse magnetic field when the magnetic field is fixed relative to the moving plate of the channel. The plates of the channel are considered porous and fluid flow within the channel is induced due to the impulsive movement of the lower plate. In fact, the problem is formulated for the general case of a moving plate with velocity proportional to t^m, t and m are, respectively, time variable and a positive integer. Exact solution of the governing equations is obtained for $m = 0$ (i.e., impulsive movement of the plate) by Laplace transform technique. The expression for the shear stress at the moving plate is also derived. The solution in dimensionless form contains three pertinent flow parameters, namely, M^2 (square of Hartmann number), K^2 (rotation parameter which is reciprocal of Ekman number) and S (suction/injection parameter). The asymptotic behaviour of the solution is analyzed for both small as well as large values of time to highlight the transient approach to the final steady state flow and the effects of rotation, magnetic field and suction/injection. For small values of time t, primary flow is independent of rotation while the secondary flow has considerable effects of rotation, magnetic field and suction/injection. However, in a slowly rotating system when the conductivity of the fluid is low and/or the applied magnetic field is weak, the secondary flow is unaffected by magnetic field for small values of time t. For large values of time t, the fluid flow is in quasi-steady state. The steady state flow is confined within an Ekman-Hartmann boundary layer of thickness

$$\mathcal{O}\big((\alpha + \frac{S}{2})^{-1}\big),$$

which becomes thinner with the increase in either M^2 or K^2 or S or all the parameters. Also steady state flow represents spatial oscillations in the flow-field. Unsteady state flow exhibits inertial oscillations in the flow-field. It is noticed that, for large values of time t, unsteady state flow is divided into two parts, namely, u_{it_1}, v_{it_1} and u_{it_2}, v_{it_2}. Inertial oscillations in u_{it_1} and v_{it_1} damp out effectively in dimensionless time of $\mathcal{O}\big((M^2 + S^2/4)^{-1}\big)$ whereas in u_{it_2} and v_{it_2} it damp out effectively in dimensionless time of $\mathcal{O}\big((M^2)^{-1}\big)$ when final steady state is developed. In the absence of rotation (i.e., $K^2 = 0$) there is no inertial oscillations in the flow-field. To study the effects of magnetic field, rotation, suction/injection and time on the flow-field the fluid velocity is depicted graphically while the numerical values of the shear stress at the moving plate are presented in tabular form for various values of M^2, K^2, S and t.

2 Formulation of the problem

Consider unsteady flow of a viscous incompressible electrically conducting fluid between two parallel porous plates $z' = 0$ and $z' = h$ of infinite length, in x' and y' directions, in the presence of a uniform transverse magnetic field H_0 applied parallel to z'– axis. The fluid and channel are in a state of rigid body rotation about z'– axis with uniform angular velocity Ω. Initially (i.e., when time $t' \leq 0$), fluid and the plates of the channel are assumed to be at rest. When time $t' > 0$ the lower plate ($z' = 0$) starts moving with time dependent velocity $U_0 t'^m$ (U_0 being a constant and m a positive integer) in x'– direction while the upper plate ($z' = h$) is kept fixed. The fluid suction/ injection takes place through the porous walls of the channel with uniform velocity W_0 which is greater than zero for suction and is less than zero for injection.

It is assumed that no applied or polarization voltages exist (i.e., $\vec{E} = 0$, $\vec{E}$ being electric field). This corresponds to the case where no energy is being added or extracted from the fluid by electrical means [53]. Since magnetic Reynolds number is very small for metallic liquids and partially ionized fluids so the induced magnetic field can be neglected in comparison to the applied one [54]. Therefore the fluid velocity $\vec{q}$ and magnetic field $\vec{H}$ are given by

$$\vec{q} \equiv (u', v', W_0), \qquad \vec{H} = (0, 0, H_0). \tag{2.1}$$

Following the studies [33–39], the governing equations for the flow of a viscous incompressible electrically conducting fluid in a rotating frame of reference are

$$\frac{\partial u'}{\partial t'} + W_0 \frac{\partial u'}{\partial z'} - 2\Omega v' = \upsilon \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} u', \tag{2.2}$$

$$\frac{\partial v'}{\partial t'} + W_0 \frac{\partial v'}{\partial z'} + 2\Omega u' = \upsilon \frac{\partial^2 v'}{\partial z'^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} v', \tag{2.3}$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial z'}. \tag{2.4}$$

Eq. (2.4) shows the constancy of pressure along axis of rotation. The absence of pressure gradient term in Eq. (2.3) implies that there is a net cross flow in y'– direction. The fluid motion is induced due to the movement of the lower plate in x'– direction, so pressure gradient term is not taken into account in Eq. (2.2).

The initial and boundary conditions are

$$u' = 0, \qquad v' = 0, \qquad 0 \leqslant z' \leqslant h, \ \ t' \leqslant 0, \tag{2.5a}$$

$$u' = U_0 t'^m, \qquad v' = 0, \qquad \text{at} \ \ z' = 0, \ \ t' > 0, \tag{2.5b}$$

$$u' = 0, \qquad v' = 0, \qquad \text{at} \ \ z' = h, \ \ t' > 0. \tag{2.5c}$$

Eq. (2.2) is valid when the magnetic field is fixed relative to the fluid. On the other hand, when the magnetic field is fixed relative to the moving plate [55], the Eq. (2.2) is replaced by

$$\frac{\partial u'}{\partial t'} + W_0 \frac{\partial u'}{\partial z'} - 2\Omega v' = v \frac{\partial^2 u'}{\partial z'^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} \left(u' - U_0 t'^m \right). \tag{2.6}$$

It is now required to find the solution of Eqs. (2.3) and (2.6) subject to the initial and boundary conditions (2.5).

Combining Eqs. (2.3) and (2.6), we obtain

$$\frac{\partial q'}{\partial t'} + W_0 \frac{\partial q'}{\partial z'} + 2i\Omega q' = v \frac{\partial^2 q'}{\partial z'^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} \left(q' - U_0 t'^m \right), \tag{2.7}$$

where $q' = u' + iv'$.

The initial and boundary conditions (2.5) become

$$q' = 0, \qquad 0 \leqslant z' \leqslant h, \quad t' \leqslant 0, \tag{2.8a}$$
$$q' = U_0 t'^m, \qquad \text{at} \quad z' = 0, \quad t' > 0, \tag{2.8b}$$
$$q' = 0, \qquad \text{at} \quad z' = h, \quad t' > 0. \tag{2.8c}$$

3 Solution of the problem

The fluid flow process described by the Eq. (2.7) subject to initial and boundary conditions (2.8) is quite general. In this case the initial velocity of the lower plate is in terms of an arbitrary function of time variable. In order to discuss the specific flow process, we consider the case when $m = 0$, which corresponds to impulsive movement of the lower plate [2,35,55]. Introducing the non-dimensional variables

$$z = \frac{z'}{h}, \quad q = \frac{q'h}{v}, \quad \text{and} \quad t = \frac{t'v}{h^2}, \tag{3.1}$$

the Eq. (2.7) after substitution of $m = 0$, in non-dimensional form, become

$$\frac{\partial q}{\partial t} + S \frac{\partial q}{\partial z} + 2iK^2 q = \frac{\partial^2 q}{\partial z^2} - M^2 \left(q - R_e \right), \tag{3.2}$$

where $S = W_0 h / v$ is suction/injection parameter ($S > 0$ for suction and $S < 0$ for injection), $M^2 = \sigma \mu_e^2 H_0^2 h^2 / \rho v$ is magnetic parameter which is the square of Hartmann number, $K^2 = \Omega h^2 / v$ is rotation parameter which is reciprocal of Ekman number and $R_e = U_0 h / v$ is the Reynolds number.

The initial and boundary conditions (2.8) after substitution of $m = 0$, in non-dimensional form, reduce to

$$q = 0, \qquad 0 \leqslant z \leqslant 1, \quad t \leqslant 0, \tag{3.3a}$$
$$q = R_e, \qquad \text{at} \quad z = 0, \quad t > 0, \tag{3.3b}$$
$$q = 0, \qquad \text{at} \quad z = 1, \quad t > 0. \tag{3.3c}$$

Using Laplace transform technique, the Eq. (3.2) with the help of (3.3a) reduces to

$$\frac{d^2\bar{q}}{dz^2} - S\frac{d\bar{q}}{dz} - \left(p + M^2 + 2iK^2\right)\bar{q} = -\frac{M^2 R_e}{p}, \tag{3.4}$$

where

$$\bar{q}(z,p) = \int_0^\infty e^{-pt} q(z,t)dt, \quad p > 0,$$

p being Laplace transform parameter.

The boundary conditions (3.3b) and (3.3c) become

$$\bar{q} = R_e/p, \qquad \text{at } z = 0, \tag{3.5a}$$

$$\bar{q} = 0, \qquad \text{at } z = 1. \tag{3.5b}$$

The solution of Eq. (3.4) subject to the boundary conditions (3.5) is given by

$$\begin{aligned} \bar{q}_i = &\sum_{n=0}^{\infty}\left[\frac{1}{p}\left(e^{-a\lambda_1} - e^{-b\lambda_1}\right) - \frac{(-1)^n M^2}{p(p+M^2+2iK^2)}\left(e^{-c\lambda_1} + e^{-d\lambda_1}\right)\right] \\ &+ \frac{M^2}{p(p+M^2+2iK^2)}, \end{aligned} \tag{3.6}$$

where

$$\bar{q}_i = \bar{q}(z,p)/R_e, \qquad a = 2n + z, \qquad b = 2 + 2n - z,$$

$$c = n + z, \qquad d = 1 + n - z, \qquad \lambda_1 = S/2 + \sqrt{p + S^2/4 + M^2 + 2iK^2}.$$

Taking inverse Laplace transform of Eq. (3.6), the solution for velocity field is expressed in the following form (McLachlan [56])

$$\begin{aligned} q_i = \frac{1}{2}\sum_{n=0}^{\infty}\Bigg\{ & e^{-a\left(\frac{S}{2}-\lambda\right)} erfc\Big(\frac{a}{2\sqrt{t}} + \lambda\sqrt{t}\Big) + e^{-a\left(\frac{S}{2}+\lambda\right)} erfc\Big(\frac{a}{2\sqrt{t}} - \lambda\sqrt{t}\Big) \\ & - e^{-b\left(\frac{S}{2}-\lambda\right)} erfc\Big(\frac{b}{2\sqrt{t}} + \lambda\sqrt{t}\Big) - e^{-b\left(\frac{S}{2}+\lambda\right)} erfc\Big(\frac{b}{2\sqrt{t}} - \lambda\sqrt{t}\Big) \\ & - \frac{(-1)^n M^2}{M^2 + 2iK^2}\Bigg[e^{-c\left(\frac{S}{2}-\lambda\right)} erfc\Big(\frac{c}{2\sqrt{t}} + \lambda\sqrt{t}\Big) + e^{-c\left(\frac{S}{2}+\lambda\right)} erfc\Big(\frac{c}{2\sqrt{t}} - \lambda\sqrt{t}\Big) \\ & + e^{-d\left(\frac{S}{2}-\lambda\right)} erfc\Big(\frac{d}{2\sqrt{t}} + \lambda\sqrt{t}\Big) + e^{-d\left(\frac{S}{2}+\lambda\right)} erfc\Big(\frac{d}{2\sqrt{t}} - \lambda\sqrt{t}\Big) \\ & - e^{-\left(M^2+2iK^2\right)t}\Bigg(erfc\Big(\frac{c}{2\sqrt{t}} + \frac{S}{2}\sqrt{t}\Big) + e^{-cS} erfc\Big(\frac{c}{2\sqrt{t}} - \frac{S}{2}\sqrt{t}\Big) \\ & + erfc\Big(\frac{d}{2\sqrt{t}} + \frac{S}{2}\sqrt{t}\Big) + e^{-dS} erfc\Big(\frac{d}{2\sqrt{t}} - \frac{S}{2}\sqrt{t}\Big)\Bigg)\Bigg]\Bigg\} \\ & + \frac{M^2}{M^2 + 2iK^2}\left(1 - e^{-\left(M^2+2iK^2\right)t}\right), \end{aligned} \tag{3.7}$$

where

$$q_i = u_i + iv_i = \frac{q(z,t)}{R_e}, \qquad \lambda = \alpha + i\beta,$$

and

$$\alpha, \beta = \frac{1}{\sqrt{2}} \left\{ \left[\left(\frac{S^2}{4} + M^2 \right)^2 + 4K^4 \right]^{\frac{1}{2}} \pm \left(\frac{S^2}{4} + M^2 \right) \right\}^{\frac{1}{2}}. \tag{3.8}$$

Solution (3.7) is the general solution for impulsively started hydromagnetic Couette flow in a porous channel in a rotating system. It demonstrates a unified representation of the initial Couette flow, the final steady Ekman-Hartmann boundary layer and the decaying oscillations excited by the interaction of the magnetic field, Coriolis force, suction/injection and initial impulsive motion when the magnetic field is fixed relative to the moving plate. In the absence of suction/injection ($S = 0$) solution (3.7) is in agreement with the solution obtained by Chandran et al. [35]. On the other hand, in the absence of rotation ($K^2 = 0$) and suction/injection ($S = 0$) it agrees with the solution obtained by Singh and Kumar [9].

4 Shear stress at the moving plate

The non-dimensional shear stress components τ_{xi} and τ_{yi} at the moving plate ($z = 0$) due to the primary and secondary flow respectively are given by

$$\begin{aligned}
&\left(\tau_{xi} + i\tau_{yi}\right)\Big|_{z=0} \\
=&\frac{1}{2}\sum_{n=0}^{\infty}\Big\{-\Big(\frac{S}{2}-\lambda\Big)e^{-a'\left(\frac{S}{2}-\lambda\right)}erfc\Big(\frac{a'}{2\sqrt{t}}+\lambda\sqrt{t}\Big) \\
&-\Big(\frac{S}{2}+\lambda\Big)e^{-a'\left(\frac{S}{2}+\lambda\right)}erfc\Big(\frac{a'}{2\sqrt{t}}-\lambda\sqrt{t}\Big) \\
&-\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{a'^2}{4t}+\frac{a'S}{2}+\lambda^2 t\right)}-\Big(\frac{S}{2}-\lambda\Big)e^{-b'\left(\frac{S}{2}-\lambda\right)}erfc\Big(\frac{b'}{2\sqrt{t}}+\lambda\sqrt{t}\Big) \\
&-\Big(\frac{S}{2}+\lambda\Big)e^{-b'\left(\frac{S}{2}+\lambda\right)}erfc\Big(\frac{b'}{2\sqrt{t}}-\lambda\sqrt{t}\Big)-\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{b'^2}{4t}+\frac{b'S}{2}+\lambda^2 t\right)} \\
&-\frac{(-1)^n M^2}{M^2+2iK^2}\Big[-\Big(\frac{S}{2}-\lambda\Big)e^{-n\left(\frac{S}{2}-\lambda\right)}erfc\Big(\frac{n}{2\sqrt{t}}+\lambda\sqrt{t}\Big) \\
&-\Big(\frac{S}{2}+\lambda\Big)e^{-n\left(\frac{S}{2}+\lambda\right)}erfc\Big(\frac{n}{2\sqrt{t}}-\lambda\sqrt{t}\Big)-\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{n^2}{4t}+\frac{nS}{2}+\lambda^2 t\right)} \\
&+\Big(\frac{S}{2}-\lambda\Big)e^{-d'\left(\frac{S}{2}-\lambda\right)}erfc\Big(\frac{d'}{2\sqrt{t}}+\lambda\sqrt{t}\Big) \\
&+\Big(\frac{S}{2}+\lambda\Big)e^{-d'\left(\frac{S}{2}+\lambda\right)}erfc\Big(\frac{d'}{2\sqrt{t}}-\lambda\sqrt{t}\Big)+\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{d'^2}{4t}+\frac{d'S}{2}+\lambda^2 t\right)}
\end{aligned}$$

$$-e^{-\left(M^2+2iK^2\right)t}\left(-\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{n^2}{4t}+\frac{nS}{2}+\frac{S^2}{4}t\right)}-Se^{-nS}erfc\Big(\frac{n}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\Big)\right.$$
$$\left.+\frac{2}{\sqrt{\pi t}}e^{-\left(\frac{d'^2}{4t}+\frac{d'S}{2}+\frac{S^2}{4}t\right)}+Se^{-d'S}\ erfc\Big(\frac{d'}{2\sqrt{t}}-\frac{S}{2}\sqrt{t}\Big)\right)\Bigg]\Bigg\},\tag{4.1}$$

where $a'=2n$, $b'=2+2n$, and $d'=1+n$.

5 Asymptotic solutions

In order to gain further insight into the flow pattern, we shall now examine the solution (3.7) for small and large values of time t.

When time t is small, we obtain the velocity components from (3.7) as

$$\begin{aligned}u_i=\sum_{n=0}^{\infty}\Bigg\{&e^{-\frac{aS}{2}}\left[\Big(1+\frac{a^2}{2}m_1\Big)erfc\Big(\frac{a}{2\sqrt{t}}\Big)-am_1\sqrt{\frac{t}{\pi}}e^{-\frac{a^2}{4t}}\right]\\&-e^{-\frac{bS}{2}}\left[\Big(1+\frac{b^2}{2}m_1\Big)erfc\Big(\frac{b}{2\sqrt{t}}\Big)-bm_1\sqrt{\frac{t}{\pi}}e^{-\frac{b^2}{4t}}\right]\\&-(-1)^nM^2\Bigg\{e^{-\frac{cS}{2}}\left[\Big(t+(1+m_1t)\frac{c^2}{2}+\frac{m_1c^4}{12}\Big)erfc\Big(\frac{c}{2\sqrt{t}}\Big)\right.\\&\left.-\Big(\frac{2c}{3\sqrt{\pi}}t^{\frac{3}{2}}m_1+c\Big(1+\frac{m_1c^2}{6}\Big)\sqrt{\frac{t}{\pi}}\Big)e^{-\frac{c^2}{4t}}\right]\\&+e^{-\frac{dS}{2}}\left[\Big(t+(1+m_1t)\frac{d^2}{2}+\frac{m_1d^4}{12}\Big)\ erfc\Big(\frac{d}{2\sqrt{t}}\Big)\right.\\&\left.-\left(\frac{2d}{3\sqrt{\pi}}t^{\frac{3}{2}}m_1+d\Big(1+\frac{m_1d^2}{6}\Big)\sqrt{\frac{t}{\pi}}\right)e^{-\frac{d^2}{4t}}\right]\Bigg\}\Bigg\}+M^2t,\end{aligned}\tag{5.1}$$

$$\begin{aligned}v_i=K^2\sum_{n=0}^{\infty}\Bigg\{&e^{-\frac{aS}{2}}\left[a^2erfc\Big(\frac{a}{2\sqrt{t}}\Big)-2a\sqrt{\frac{t}{\pi}}e^{-\frac{a^2}{4t}}\right]-e^{-\frac{bS}{2}}\left[b^2erfc\Big(\frac{b}{2\sqrt{t}}\Big)\right.\\&\left.-2b\sqrt{\frac{t}{\pi}}e^{-\frac{b^2}{4t}}\right]-(-1)^nM^2\Bigg\{e^{-\frac{cS}{2}}\left[\Big(c^2t+\frac{c^4}{6}\Big)erfc\Big(\frac{c}{2\sqrt{t}}\Big)\right.\\&\left.-\Big(\frac{4c}{3\sqrt{\pi}}t^{\frac{3}{2}}+\frac{c^3}{3}\sqrt{\frac{t}{\pi}}\Big)e^{-\frac{c^2}{4t}}\right]+e^{-\frac{dS}{2}}\left[\Big(d^2t+\frac{d^4}{6}\Big)erfc\Big(\frac{d}{2\sqrt{t}}\Big)\right.\\&\left.-\Big(\frac{4d}{3\sqrt{\pi}}t^{\frac{3}{2}}+\frac{d^3}{3}\sqrt{\frac{t}{\pi}}\Big)e^{-\frac{d^2}{4t}}\right]\Bigg\}\Bigg\},\end{aligned}\tag{5.2}$$

where $m_1=M^2+S^2/4$.

It is evident from the solutions (5.1) and (5.2) that there arises Rayleigh layer of thickness $\mathcal{O}(2\sqrt{t})$ near the moving plate ($z=0$) due to the initial impulsive movement of the plate. This layer is unaffected by rotation, magnetic field and suction/injection. It is also noticed from (5.1) and (5.2) that the primary flow $u_i(z,t)$ is independent of rotation where as secondary flow $v_i(z,t)$ is affected by rotation as well as magnetic field. When magnetic parameter M^2 and rotation parameter K^2 are very small, the solutions (5.1) and (5.2) reduce to

$$\begin{aligned}
u_i = \sum_{n=0}^{\infty} \Bigg\{ & e^{-\frac{aS}{2}} \left[\left(1+\frac{a^2}{2}m_1\right) erfc\left(\frac{a}{2\sqrt{t}}\right) - am_1\sqrt{\frac{t}{\pi}}e^{-\frac{a^2}{4t}} \right] \\
& - e^{-\frac{bS}{2}} \left[\left(1+\frac{b^2}{2}m_1\right) erfc\left(\frac{b}{2\sqrt{t}}\right) - bm_1\sqrt{\frac{t}{\pi}}e^{-\frac{b^2}{4t}} \right] \\
& - (-1)^n M^2 \left(e^{-\frac{cS}{2}} \left\{ \left(t + \left(1+\frac{S^2}{4}t\right)\frac{c^2}{2} + \frac{S^2c^4}{48} \right) erfc\left(\frac{c}{2\sqrt{t}}\right) \right.\right. \\
& \left. - \left(\frac{cS^2}{6\sqrt{\pi}}t^{\frac{3}{2}} + c\left(1+\frac{S^2c^2}{24}\right)\sqrt{\frac{t}{\pi}} \right) e^{-\frac{c^2}{4t}} \right\} \\
& + e^{-\frac{dS}{2}} \left\{ \left(t + \left(1+\frac{S^2}{4}t\right)\frac{d^2}{2} + \frac{S^2d^4}{48} \right) erfc\left(\frac{d}{2\sqrt{t}}\right) \right. \\
& \left.\left. - \left(\frac{dS^2}{6\sqrt{\pi}}t^{\frac{3}{2}} + d\left(1+\frac{S^2d^2}{24}\right)\sqrt{\frac{t}{\pi}} \right) e^{-\frac{d^2}{4t}} \right\} \right) \Bigg\},
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
v_i = \sum_{n=0}^{\infty} K^2 \Bigg\{ & e^{-\frac{aS}{2}} \left[a^2 erfc\left(\frac{a}{2\sqrt{t}}\right) - 2a\sqrt{\frac{t}{\pi}}e^{-\frac{a^2}{4t}} \right] \\
& - e^{-\frac{bS}{2}} \left[b^2 erfc\left(\frac{b}{2\sqrt{t}}\right) - 2b\sqrt{\frac{t}{\pi}}e^{-\frac{b^2}{4t}} \right] \Bigg\}.
\end{aligned} \tag{5.4}$$

Expressions (5.3) and (5.4) reveal that, in a slowly rotating system when the conductivity of the fluid is low and/or the applied magnetic field is weak, the primary flow $u_i(z,t)$ is independent of rotation while secondary flow $v_i(z,t)$ is unaffected by magnetic field. Upto this stage there is no inertial oscillations in the flow field.

When time t is large, using the asymptotic expression of $erfc(x)$, i.e.,

$$erfc\,(x) \approx \frac{\exp\,(-x^2)}{\sqrt{\pi}x}, \qquad \text{as} \quad x \to \infty,$$

together with

$$erfc(-x) = 2 - erfc(x),$$

the solution (3.7) may be represented in the following form as

$$u_i = u_{is} + u_{it}, \tag{5.5}$$

where

$$\begin{aligned}
u_{is} = \sum_{n=0}^{\infty} \Bigg\{ & e^{-a\left(\frac{S}{2}+\alpha\right)} \cos\beta a - e^{-b\left(\frac{S}{2}+\alpha\right)} \cos\beta b \\
& - \frac{(-1)^n M^2}{M^4+4K^4} \left[\left(M^2\cos\beta c + 2K^2\sin\beta c\right) e^{-c\left(\frac{S}{2}+\alpha\right)} \right. \\
& \left. + \left(M^2\cos\beta d + 2K^2\sin\beta d\right) e^{-d\left(\frac{S}{2}+\alpha\right)} \right] \Bigg\} + \frac{M^2}{M^4+4K^4},
\end{aligned} \tag{5.6}$$

$$u_{it} = u_{it_1} + u_{it_2}, \tag{5.7a}$$

$$\begin{aligned} u_{it_1} = &\frac{1}{2}\sum_{n=0}^{\infty} e^{-m_1 t}\left\{-\phi_1 + \phi_3 + \frac{(-1)^n M^2}{M^4+4K^4}\Big[M^2(\phi_5+\phi_7)\right. \\ &\left. + 2K^2(\phi_6+\phi_8) - 2\phi_9\Big]\right\}, \end{aligned} \tag{5.7b}$$

$$\begin{aligned} u_{it_2} = &\frac{M^2 e^{-M^2 t}}{M^4+4K^4}\left\{\sum_{n=0}^{\infty}\Big[(-1)^n\left(e^{-cS}+e^{-dS}\right)\Big(M^2\cos 2K^2 t\right. \\ &\left. +2K^2\sin 2K^2 t\Big)\Big] - \left(M^2\cos 2K^2 t - 2K^2 \sin 2K^2 t\right)\right\}, \end{aligned} \tag{5.7c}$$

$$v_i = v_{is} + v_{it}. \tag{5.8}$$

In (5.8), we have

$$\begin{aligned} v_{is} = &\sum_{n=0}^{\infty}\left\{-e^{-a\left(\frac{S}{2}+\alpha\right)}\sin\beta a + e^{-b\left(\frac{S}{2}+\alpha\right)}\sin\beta b\right. \\ &+ \frac{(-1)^n M^2}{M^4+4K^4}\Big[\left(2K^2\cos\beta c - M^2\sin\beta c\right)e^{-c\left(\frac{S}{2}+\alpha\right)} \\ &\left.+\left(2K^2\cos\beta d - M^2\sin\beta d\right)e^{-d\left(\frac{S}{2}+\alpha\right)}\Big]\right\} - \frac{2M^2K^2}{M^4+4K^4}, \end{aligned} \tag{5.9}$$

$$v_{it} = v_{it_1} + v_{it_2}, \tag{5.10a}$$

$$\begin{aligned} v_{it_1} = &\frac{1}{2}\sum_{n=0}^{\infty} e^{-m_1 t}\left\{\phi_2 - \phi_4 - \frac{(-1)^n M^2}{M^4+4K^4}\Big[2K^2(\phi_5+\phi_7) - M^2(\phi_6+\phi_8)\right. \\ &\left. - 2\phi_{10}\Big]\right\}, \end{aligned} \tag{5.10b}$$

$$\begin{aligned} v_{it_2} = &\frac{M^2 e^{-M^2 t}}{M^4+4K^4}\left\{\sum_{n=0}^{\infty}\Big[(-1)^{n+1}\left(e^{-cS}+e^{-dS}\right)\Big(2K^2\cos 2K^2 t\right. \\ &\left. - M^2\sin 2K^2 t\Big)\Big] + \left(2K^2\cos 2K^2 t + M^2 \sin 2K^2 t\right)\right\}, \end{aligned} \tag{5.10c}$$

$$\phi_1 = \frac{a}{\sqrt{\pi t}}\frac{1}{\xi_1}e^{-\left(\frac{a^2}{4t}+\frac{aS}{2}\right)}\Big[\left(m_1 t - a^2/4t\right)\cos 2K^2 t - 2K^2 t\sin 2K^2 t\Big],$$

$$\phi_2 = \frac{a}{\sqrt{\pi t}}\frac{1}{\xi_1}e^{-\left(\frac{a^2}{4t}+\frac{aS}{2}\right)}\Big[2K^2 t\cos 2K^2 t + \left(m_1 t - a^2/4t\right)\sin 2K^2 t\Big],$$

$$\phi_3 = \frac{b}{\sqrt{\pi t}}\frac{1}{\xi_2}e^{-\left(\frac{b^2}{4t}+\frac{bS}{2}\right)}\Big[\left(m_1 t - b^2/4t\right)\cos 2K^2 t - 2K^2 t\sin 2K^2 t\Big],$$

$$\phi_4 = \frac{b}{\sqrt{\pi t}}\frac{1}{\xi_2}e^{-\left(\frac{b^2}{4t}+\frac{bS}{2}\right)}\Big[2K^2 t\cos 2K^2 t + \left(m_1 t - b^2/4t\right)\sin 2K^2 t\Big],$$

$$\phi_5 = \frac{c}{\sqrt{\pi t}}\frac{1}{\xi_3}e^{-\left(\frac{c^2}{4t}+\frac{cS}{2}\right)}\Big[\left(m_1 t - c^2/4t\right)\cos 2K^2 t - 2K^2 t\sin 2K^2 t\Big],$$

$$
\begin{aligned}
\phi_6 &= \frac{c}{\sqrt{\pi t}}\frac{1}{\xi_3}e^{-\left(\frac{c^2}{4t}+\frac{cS}{2}\right)}\left[2K^2t\cos 2K^2t+\left(m_1t-c^2/4t\right)\sin 2K^2t\right],\\
\phi_7 &= \frac{d}{\sqrt{\pi t}}\frac{1}{\xi_4}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\left[\left(m_1t-d^2/4t\right)\cos 2K^2t-2K^2t\sin 2K^2t\right],\\
\phi_8 &= \frac{d}{\sqrt{\pi t}}\frac{1}{\xi_4}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\left[2K^2t\cos 2K^2t+\left(m_1t-d^2/4t\right)\sin 2K^2t\right],\\
\phi_9 &= 2\left(M^2\cos 2K^2t+2K^2\sin 2K^2t\right)\sqrt{\frac{t}{\pi}}\left[\frac{c}{S^2t^2-c^2}e^{-\left(\frac{c^2}{4t}+\frac{cS}{2}\right)}\right.\\
&\qquad\left.+\frac{d}{S^2t^2-d^2}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\right],\\
\phi_{10} &= 2\left(2K^2\cos 2K^2t-M^2\sin 2K^2t\right)\sqrt{\frac{t}{\pi}}\left[\frac{c}{S^2t^2-c^2}e^{-\left(\frac{c^2}{4t}+\frac{cS}{2}\right)}\right.\\
&\qquad\left.+\frac{d}{S^2t^2-d^2}e^{-\left(\frac{d^2}{4t}+\frac{dS}{2}\right)}\right],\\
\xi_1 &= \left(m_1t-a^2/4t\right)^2+4K^4t^2, \qquad \xi_2=\left(m_1t-b^2/4t\right)^2+4K^4t^2,\\
\xi_3 &= \left(m_1t-c^2/4t\right)^2+4K^4t^2, \qquad \xi_4=\left(m_1t-d^2/4t\right)^2+4K^4t^2.
\end{aligned}
$$

It is evident from the expressions (5.5) to (5.10) that the terms u_{is} and v_{is} represent final steady state flow. The steady state flow is confined within an Ekman-Hartmann boundary layer of thickness $\mathcal{O}\big((\alpha+S/2)^{-1}\big)$. From Eq. (3.8) it is observed that α increases with increasing magnetic parameter M^2, rotation parameter K^2 and suction/injection parameter S. Thus we may conclude that the thickness of Ekman-Hartmann boundary layer decreases with increasing either M^2 or K^2 or S or all the parameters. It is also noticed from (5.6) and (5.9) that steady state flow represents spatial oscillations in the flow-field excited by magnetic field, rotation and suction/injection. The unsteady part of the flow in (5.5) and (5.8), represented by u_{it} and v_{it}, exhibits inertial oscillations in the flow-field excited by rotation. The unsteady state flow represented by u_{it} and v_{it} are divided into two parts viz. u_{it_1}, v_{it_1} and u_{it_2}, v_{it_2}. The inertial oscillations in u_{it_1} and v_{it_1} damp out effectively in dimensionless time of $\mathcal{O}\big((M^2+S^2/4)^{-1}\big)$ whereas in u_{it_2} and v_{it_2}, it damp out effectively in dimensionless time of $\mathcal{O}\big((M^2)^{-1}\big)$ when final steady state is developed. This implies that the suction/ injection reduces time of damping of inertial oscillations in the major part of unsteady state flow. In the absence of rotation there are no inertial oscillations in flow-field.

6 Results and discussions

To study the effects of magnetic field, rotation, suction/injection and time on the flow-field profiles of the primary and secondary velocities are drawn in Figs. 1 to 8 for various values of magnetic parameter M^2, rotation parameter K^2, suction/injection

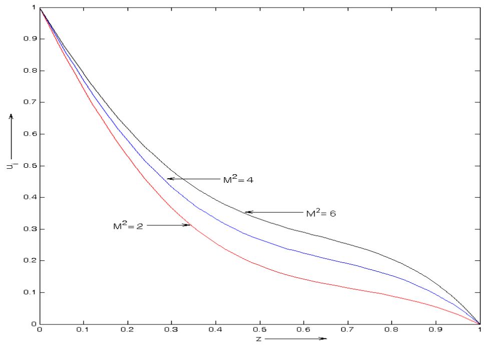

Figure 1: Primary velocity profiles when $K^2 = 3$, $S = 1$ and $t = 0.05$.

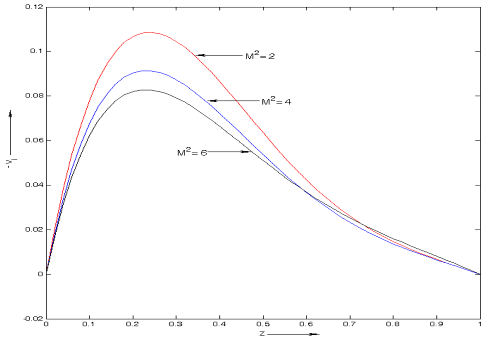

Figure 2: Secondary velocity profiles when $K^2 = 3$, $S = 1$ and $t = 0.05$.

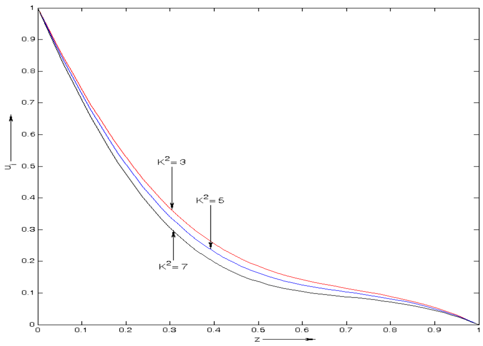

Figure 3: Primary velocity profiles when $M^2 = 2$, $S = 1$ and $t = 0.05$.

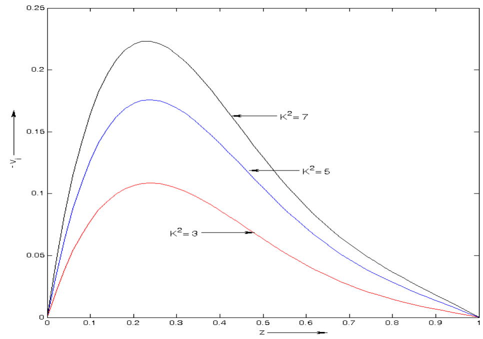

Figure 4: Secondary velocity profiles when $M^2 = 2$, $S = 1$ and $t = 0.05$.

parameter S and time t. It is evident from Figs. 1 to 4 that the primary velocity u_i increases with increasing M^2 whereas it decreases with increasing K^2. On increasing M^2, the secondary velocity v_i decreases in the lower half of the channel and is of oscillatory nature with M^2 in the upper half of the channel whereas it increases with

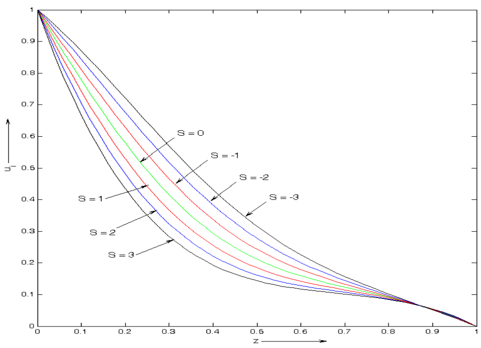

Figure 5: Primary velocity profiles when $M^2 = 2$, $K^2 = 3$ and $t = 0.05$.

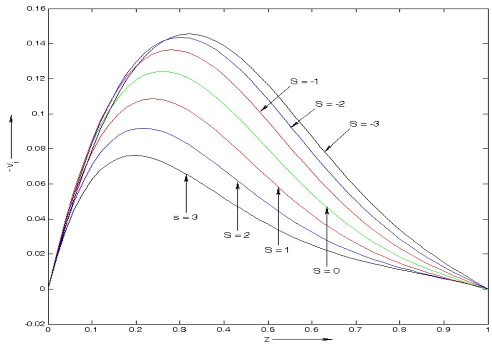

Figure 6: Secondary velocity profiles when $M^2 = 2$, $K^2 = 3$ and $t = 0.05$.

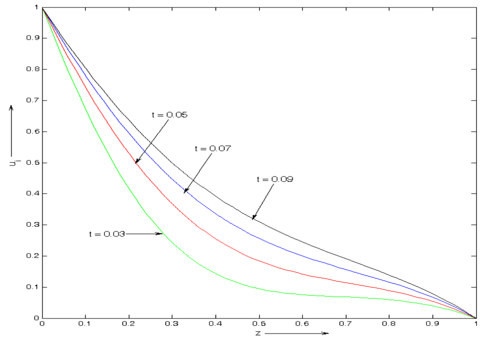

Figure 7: Primary velocity profiles when $M^2 = 2$, $K^2 = 3$ and $S = 1$.

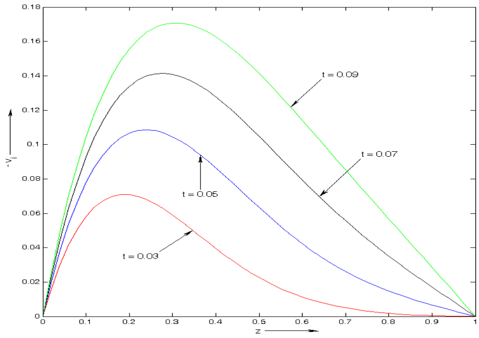

Figure 8: Secondary velocity profiles when $M^2 = 2$, $K^2 = 3$ and $S = 1$.

increasing K^2 throughout the channel. Figs. 5 and 6 reveal that the primary velocity u_i and secondary velocity v_i decrease with increasing $S(> 0)$ while it increase with increasing $S(< 0)$. Figs. 7 and 8 show that the primary and secondary velocities u_i and v_i respectively increase with increasing time t.

The numerical values of the primary and secondary shear stress components τ_{xi} and τ_{yi} at the moving plate $z = 0$ are presented in tabular form in Tables 1 to 3 for various values of M^2, K^2, S and t. It is evident from Table 1 that the primary shear stress τ_{xi} decreases where as secondary shear stress τ_{yi} increases with increasing M^2. The primary shear stress τ_{xi} and secondary shear stress τ_{yi} increase with increasing K^2. It is found from Table 2 that, on increasing S, the primary shear stress decreases when $S(< 0)$ and it increases when $S(> 0)$. Also the primary shear stress τ_{xi} decreases with increasing time t. It is observed from Table 3 that the secondary shear stress τ_{yi} increases with increasing $S(< 0)$ when $t = 0.03$ and $t = 0.05$ and it increases, attains a maximum and then decreases with increasing $S(< 0)$ when $t = 0.07$ and 0.09. Also τ_{yi} increases with increasing $S(> 0)$. The secondary shear stress τ_{yi} decreases with

Table 1: Primary and secondary shear stress at the lower plate when $t = 0.05$ and $S = 1$.

	$-\tau_{xi}$			$-\tau_{yi}$		
$M^2 \downarrow$ $K^2 \rightarrow$	3	5	7	3	5	7
2	3.6605	4.1614	4.4460	0.8820	1.0394	1.1477
4	3.1209	3.7001	4.0503	1.0912	1.3217	1.4625
6	2.6548	3.2526	3.6554	1.1633	1.5088	1.7090

Table 2: Primary shear stress $-\tau_{xi}$ at the lower plate when $M^2 = 2$ and $K^2 = 3$.

$t \downarrow$ $S \rightarrow$	-3	-2	-1	0	1	2	3
0.03	2.4416	2.8845	3.3782	3.8692	4.3202	4.7686	5.2676
0.05	1.9301	2.3251	2.7537	3.2015	3.6605	4.1379	4.6481
0.07	1.6332	2.0040	2.3955	2.8148	3.2721	3.7580	4.2645
0.09	1.3930	1.7577	2.1310	2.5317	2.9839	3.4706	3.9694

Table 3: Secondary shear stress $-\tau_{yi}$ at the lower plate when $M^2 = 2$ and $K^2 = 3$.

$t \downarrow$ $S \rightarrow$	-3	-2	-1	0	1	2	3
0.03	1.3064	1.1472	0.8186	0.6316	0.8240	1.1585	1.3240
0.05	1.1270	1.0662	0.8620	0.7405	0.8820	1.1156	1.2146
0.07	0.9353	0.9698	0.8667	0.7979	0.9174	1.1020	1.1753
0.09	0.7387	0.8609	0.8482	0.8288	0.9414	1.1016	1.1697

increasing time t when $S=-2$ and -3 while it increases, attains a maximum and then decreases with increasing time t when $S=-1$. On increasing time t, τ_{yi} increases when $S = 0$ and 1 whereas it decreases when $S = 2$ and 3.

7 Conclusions

Magnetic field has accelerating influence on the primary flow where as it has retarding influence on secondary flow in the lower half of the channel. Rotation has retarding influence on the primary flow where as it has accelerating influence on secondary flow. Suction has retarding influence on the primary as well as secondary flow where as injection and time have accelerating influence on the primary and secondary flows. Magnetic field has decreasing effect on the primary shear stress where as it has increasing effect on secondary shear stress. Rotation tends to increase the primary as well as secondary shear stress component. Suction has increasing influence on the primary shear stress where as injection and time have decreasing influence on it. Suction tends to increase secondary shear stress.

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