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Existence Theorem for a Class of Nonlinear Fourth-order Schrödinger-Kirchhoff-Type Equations

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Abstract. This paper is concerned with the existence of nontrivial solutions for the following fourth-order equations of Kirchhoff type

$$\begin{cases} \Delta^2 u - \left(a + b\int_{\mathbb{R}^N} |\nabla u|^2 \mathrm{d}x\right)\Delta u + \lambda V(x)u = f(x,u), \quad x \in \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases}$$

where a,b are positive constants, $\lambda \geq 1$ is a parameter, and the nonlinearity f is either superlinear or sublinear at infinity in u. With the help of the variational methods, we obtain the existence and multiplicity results in the working spaces.

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1 Introduction

In this paper, we consider the following nonlinear Schrödinger-Kirchhoff-type problem:

$$\begin{cases} \Delta^2 u - \left(a + b\int_{\mathbb{R}^N} |\nabla u|^2 \mathrm{d}x\right)\Delta u + \lambda V(x)u = f(x,u), \quad x \in \mathbb{R}^N, \\ u \in H^2\left(\mathbb{R}^N\right), \end{cases} \tag{P_λ}$$

where Δ^2 is the biharmonic operator and ∇u denotes the spatial gradient of u. Moreover, a, b are positive constants, $\lambda \geq 1$ is a parameter, and the potential $V(x)$ satisfies the following conditions:

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(V_1) $V(x)\in C(\mathbb{R}^N,\mathbb{R})$ and $\inf_{x\in\mathbb{R}^N} V(x)\geq A>0$, for some A;

(V_2) there exists $B>0$ such that the set $\{x\in\mathbb{R}^N : V(x)\leq B\}$ is nonempty and meas$\{x\in\mathbb{R}^N : V(x)\leq B\}<+\infty$, where "meas" means the Lebesgue measure in $\mathbb{R}^N$.

Problem (P_λ) is called nonlocal because of the presence of the term $-\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2\mathrm{d}x\right)$, which implies that equation (P_λ) is no longer a point-wise indentity. In the recent years, there are many papers about Kirchhoff-type problems without the term of biharmonic operator. On the bounded domain, positive solutions are investegated by authors such as Ma and Rivera [1] and Alves et al. [2]. Sign-changing solutions have been obtained by Zhang and Perera [3] and Mao and Zhang [4]. If the problem is set on $\mathbb{R}^N$, Some interesting results can be found in [5–7] and the references there in.

It is well-known that the following fourth-order elliptic equation of Kirchhoff type:

$$\begin{cases}\Delta^2 u-\left(a+b\int_\Omega|\nabla u|^2\mathrm{d}x\right)\Delta u=f(x,u), & x\in\Omega,\\ u=0,\nabla u=0, \quad \text{on } \partial\Omega,\end{cases}$$

is related to the stationary analogue of the equation of Kirchhoff type:

$$u_{tt}-\Delta^2 u-\left(a+b\int_\Omega|\nabla u|^2\mathrm{d}x\right)\Delta u=f(x,u), \qquad x\in\Omega,$$

which is regarded as a good approximation for describing nonlinear vibrations of beems or plates (see [8,9]). Recently, Ma [10,11] considered the fourth-order equation

$$\begin{cases}u''''-M\left(\int_0^1|u'(x)|^2\mathrm{d}x\right)u''=h(x)f(x,u), & 0\leq x\leq 1,\\ u(0)=u(1)=0, \qquad u''(0)=u''(1)=0,\end{cases}$$

and obtain the multiplicity of solutions.

Later on, Wang and An [12] get the existence of nontrivial solution of a fourth-order elliptic equation

$$\begin{cases}\Delta^2 u-M\left(\int_\Omega|\nabla u|^2\mathrm{d}x\right)\Delta u=f(x,u), & x\in\Omega,\\ u=0, \quad \Delta u=0, & \text{on } \partial\Omega,\end{cases} \tag{1.1}$$

under some conditions on the function $M(t)$ and f with Mountain Pass theorem. Moreover, Wang et al. also consider the existence of the nontrivial solutions for (1.1) with a parameter λ (see [13]), where $M=a+bt$ and $b\geq 0$.

We note that problem (P_λ) with $a=1$, $b=0$, reduces fourth-order elliptic equations

$$\begin{cases}\Delta^2 u-\Delta u+\lambda V(x)u=f(x,u), & x\in\mathbb{R}^N,\\ u\in H^2(\mathbb{R}^N).\end{cases} \tag{1.2}$$