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Constructing Infinite Sequence Solutions for Space-Time Fractional Benjamin-Bona-Mahoney Equation

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Abstract. In this article, the space-time fractional Benjamin-Bona-Mahoney equation is investigated via applying the first kind of elliptic equation method. As a result, based on the Bäcklund transformations and some seed solutions of the first kind of elliptic equation, families of infinite sequence solutions are presented. This approach is also applicable to many other nonlinear fractional differential equations.

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Key Words: Space-time fractional Benjamin-Bona-Mahoney equation; Bäcklund transformations; infinite sequence solutions.

1 Introduction

It is well known that nonlinear fractional differential equations describe the real features in a wide variety of scientific models, including the propagation of shallow water waves, fluid dynamics, plasma physics and so on. Exact solutions to nonlinear fractional differential equations play a key role in providing more insight into the phenomena and lead to further applications. For this reason, a whole range of techniques have been developed to deal with nonlinear fractional differential equations such as fractional sub-equation method [1, 2], first integral method [3, 4], exp-function method [5-8], (G'/G)-expansion method [9, 10], Jacobi elliptic function method [11, 12] and so on. Among these methods, the fractional complex transformation [13, 14] has been suggested to convert a fractional differential equation in the sense of Jumarie's modified Riemann-Liouville derivative into its differential partner of integer order.

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It has been recognized that the first kind of elliptic equation method [15] is an effective way to acquire new explicit exact solutions of nonlinear differential equations of integer order. In this study, the first kind of elliptic equation method will be handled to construct infinite sequence exact solutions for the space-time fractional BBM equation.

The organization of this paper is as follows. In Section 2, we list the main steps of the first kind of elliptic equation method involving the fractional complex transformation. Section 3 covers some special solutions and Bäcklund transformations [16] to the first kind of elliptic equation. Section 4 is devoted to construction of the infinite sequence solutions for space-time fractional BBM equation. At last, section 5 gives the conclusion of this paper.

2 Methodology

For readability, we briefly introduce the Jumarie's modified Riemann-Liouville derivative [17] which is defined as

$$D_t^\alpha f(t)=\begin{cases} \dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_0^t (t-\xi)^{-\alpha-1}[f(\xi)-f(0)]\mathrm{d}\xi, & \alpha<0,\\ \dfrac{1}{\Gamma(1-\alpha)}\dfrac{\mathrm{d}}{\mathrm{dt}}\displaystyle\int_0^t (t-\xi)^{-\alpha}[f(\xi)-f(0)]\mathrm{d}\xi, & 0<\alpha<1,\\ [f^{(n)}(t)]^{(\alpha-n)}, \quad 1\le n\le\alpha<n+1. & \end{cases}$$

And there exist some important properties

$$\begin{aligned} &D_t^\alpha t^\delta=\frac{\Gamma(1+\delta)}{\Gamma(1+\delta-\alpha)}t^{\delta-\alpha}, \qquad \delta>0,\\ &D_t^\alpha(f(t)g(t))=g(t)D_t^\alpha f(t)+f(t)D_t^\alpha g(t),\\ &D_t^\alpha f[g(t)]=f_g'[g(t)]D_t^\alpha g(t)=D_g^\alpha f[g(t)](g'(t))^\alpha. \end{aligned}$$

Given a nonlinear fractional differential equation

$$P\big(u,D_t^\alpha u,D_x^\beta u,D_y^\gamma u,D_t^\alpha D_t^\alpha u,D_t^\alpha D_x^\beta u,\cdots\big)=0, \qquad 0<\alpha,\beta,\gamma\le 1, \tag{2.1}$$

where $u=u(t,x,y)$ is unknown function, and P is interpreted as a polynomial in u and its modified Riemann-Liouville derivatives.

The general stages are clarified as follows:

Step 1. We look for its solution in such form

$$u(t,x,y)=U(\xi)=U\left(\frac{c_0t^\alpha}{\Gamma(1+\alpha)}+\frac{k_0x^\beta}{\Gamma(1+\beta)}+\frac{l_0y^\gamma}{\Gamma(1+\gamma)}\right), \tag{2.2}$$