East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.250216.161016a

Vol. 7, No. 2, pp. 248-268
May 2017

Output Feedback Admissible Control for Singular Systems: Delta Operator (Discretised) Approach

Xin-zhuang Dong[1] and Mingqing Xiao[2,*]

[1] College of Automation and Electrical Engineering, Qingdao University, Qingdao Shandong 266071, China.
[2] Department of Mathematics, Southern Illinois University, Carbondale, IL 62901, USA.

Received 25 February 2016; Accepted (in revised version) 16 October 2016.

Abstract. Singular systems simultaneously capture the dynamics and algebraic constraints in many practical applications. Output feedback admissible control for singular systems through a delta operator method is considered in this article. Two novel admissibility conditions, derived for the singular delta operator system (SDOS) from a singular continuous system through sampling, can not only produce unified admissibility for both continuous and discrete singular systems but also practical procedures. To solve the problem of output feedback admissible control for the SDOS, an existence condition and design procedure is given for the determination of a physically realisable observer for the state estimation, and then a suitable state-feedback-like admissible controller design based on the observer is developed. All of the conditions presented are necessary and sufficient, involving strict linear matrix inequalities (LMI) with feasible solutions obtained with low computational costs. Numerical examples illustrate our approach.

AMS subject classifications: 93B07, 93B51, 93B52

Key words: Singular delta operator system (SDOS), admissibility, output feedback, state observer, linear matrix inequalities (LMI).

1. Introduction

Singular systems simultaneously capturing both dynamics and algebraic constraints arise in many practical applications such as economic systems [1], electrical networks [2], chemical processes [3], or highly interconnected large-scale systems [4]. Extensive study of singular systems during recent decades has been due to their important state variable characteristics, and many fundamental issues related to performance analysis and synthesis have been investigated — cf. [4–9] and references therein. Moreover,

*Corresponding author. *Email address:* `mingqingxiao30@gmail.com` (M. Xiao)

singular systems have also been adopted to study some state-space systems in order to incorporate state constraints [10, 11]. The fundamental requirement to implement a singular system in practice is its admissibility (its regularity, stability and impulse elimination for the continuous case or causality for the discrete case), whereas for state-space systems the main concern is usually stability.

In most digital approaches, a standard shift operator is used for discrete systems resulting from continuous dynamical systems. However, the response of the discrete system may not converge smoothly to its continuous counterpart when the sampling period tends to zero [12], mainly due to numerical ill-conditioning. A delta operator introduced to avoid this deficiency appears to be more effective than the shift operator in certain applications [13], and in particular required smaller word length when it was implemented in fixed-point digital control processors [14]. Moreover, the delta operator method has been shown to be significantly less sensitive than the shift operator setting at high sampling rates [15]. Furthermore, the delta operator model can provide a theoretically unified formulation of continuous and discrete systems [16]. Many results on state-space systems using the delta operator have been presented in the literature, including stability, sliding mode control and positive real control [16–18]. Systematic results for delta operator systems can be found in Refs. [16, 19]; and recently the delta operator method has been applied to singular systems, with progress under the framework of SDOS — e.g. see Refs. [20–26]. Specifically, the delta operator model has been established for a singular continuous system through two different approaches [22, 23], directly obtained from a suitable singular discrete model and convergent to the corresponding continuous system as the sampling period tends to zero. The delta operator model thus provides not only a practical approach both a unified description of a singular continuous system and its discrete model, including in high sample rate cases. The analysis of controllability and observability were provided in Refs. [20] and [21] respectively, and various necessary and sufficient admissibility conditions have been obtained [22]. Results on admissible control found in Refs. [23–25] are given in terms of matrix inequalities and linear matrix inequalities (LMI), respectively. Different D-admissibility conditions and design methods for a suitable D-admissible controller are derived in Ref. [26]. It is notable that the controllers obtained are all state feedback forms, with sufficient LMI-based conditions for the existence of admissible and D-admissible controllers [23–26].

State feedback control is fundamental in the controller design for dynamical systems in control system theory. If all states are available for the controller design, then the output feedback control is a full state feedback. However, in many applications not all states of the system are available for measurement due to the system limitation or inhibited high cost. Instead, the information available for controller design is usually the output measurement that has much smaller dimension than the state space, so the study of output feedback control for singular systems has both theoretical and practical significance. Some interesting results have been reported in the observer design for singular systems, in order to estimate the states [27–30]. In Ref. [24] the problem of observer-based admissible control for SDOSs was considered, where via LMI