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Nonlinear Dynamical Behaviour in a Predator-Prey Model with Harvesting

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Abstract. We investigate the stability and periodic orbits of a predator-prey model with harvesting. The model has a biologically-meaningful interior, an attractor undergoing damped oscillations, and can become destabilised to produce periodic orbits via a Hopf bifurcation. Some sufficient conditions for the existence of the Hopf bifurcation are established, and a stability analysis for the periodic solutions using a Lyapunov function is presented. Finally, some computer simulations illustrate our theoretical results.

AMS subject classifications: 92D25

Key words: Predator-prey system, Holling type IV functional response, periodic solution, bifurcation, Lyapunov function.

1. Introduction

The governing equations in the Kolmogorov predator-prey model are

$$\begin{cases} \dot{x} = xF(x,y)\,, \\ \dot{y} = yG(x,y)\,, \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ respectively denote the prey and predator populations at time t [1]. The functions $F(x,y)$ and $G(x,y)$ denote the respective per capita growth rates of the two species, and it is assumed that $\mathrm{d}F(x,y)/\mathrm{d}y < 0$ and $\mathrm{d}G(x,y)/\mathrm{d}x > 0$.

On the basis of model (1.1), Yodzis [2] proposed the corresponding generalised predator-prey model

$$\begin{cases} \dot{x} = xf(x) - yH(x,y)\,, \\ \dot{y} = yG(x,y)\,, \end{cases} \tag{1.2}$$

where $f(x)$ is the intrinsic growth rate of the prey and $H(x,y)$ is the predator response function. In this article, we assume $f(x) = r(1-x/K)$ such that the prey population grows

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logistically in the absence of predators, with the positive constants r and K denoting the maximal prey growth rate and carrying capacity, respectively. We also adopt the generalised Holling type IV response function $H(x,y) = my/(ax^2 + b_1x + 1)$ in Refs. [3,4] to describe "inhibition" in microbial dynamics and "group defence" in population dynamics, where the positive constants m and a respectively denote the capture rate and half-saturation constant, and the positive constant b_1 is such that the denominator of predator-prey model (1.2) does not vanish for nonnegative x. Finally, we choose the Lotka-Volterra type growth rate $G(x,y) = -k_1 + d_1x$, with the positive constants k_1 and d_1 respectively representing the predator mortality rate and maximal predator growth rate [4]. Consequently, the specific predator-prey model we consider is

$$\begin{cases} \dot{x} = x\left(r - \dfrac{r}{K}x - \dfrac{my}{ax^2 + b_1x + 1}\right), \\ \dot{y} = y\left(-k_1 + d_1x\right). \end{cases} \tag{1.3}$$

On scaling the variables x, y, t and the parameters such that

$$\bar{t} = rt\,, \quad \bar{x} = \frac{x}{K}\,, \quad \bar{y} = \frac{my}{rK^2}\,, \quad b = \frac{b_1}{K}\,, \quad c = \frac{1}{K^2}\,, \quad k = \frac{k_1}{r}, \quad d = \frac{d_1K}{r}\,,$$

and then omitting the bars, our model (1.3) becomes

$$\begin{cases} \dot{x} = x\left(1 - x - \dfrac{y}{ax^2 + bx + c}\right), \\ \dot{y} = y\left(-k + dx\right). \end{cases} \tag{1.4}$$

According to the economic principle of Gordon [5], we have the algebraic equation [6-9]

$$E(t)\big(px(t) - n\big) = v, \tag{1.5}$$

where $E(t)$ is the harvesting effort for the prey, v represents the economic profit, and the positive constants p and n respectively denote the harvesting reward and harvesting cost. Combining Eqs. (1.4) and (1.5), we obtain the following modified predator-prey model with the generalised Holling type IV response function:

$$\begin{cases} \dot{x} = x\left(1 - x - \dfrac{y}{ax^2 + bx + c} - E\right), \\ \dot{y} = y\left(-k + dx\right), \\ 0 = E\left(px - n\right) - v\,, \end{cases} \tag{1.6}$$

where the harvesting E is considered as a variable in this article.

We proceed to investigate the dependence of the dynamics of the model (1.6) on the economic profit, by treating v as the variable bifurcation parameter. Our work therefore complements the research undertaken in Refs. [6-9]., where related models are formulated and various issues including singularity induced bifurcation, flip bifurcation, saddle-node